Polar phase of 1D large spin bosons.

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December 2011

For large S the number of relevant interactions is reduced to 2:

(i) density-density one and
(ii) pairing.

If the paring interaction is attractive something like BCS pairing takes place,

BUT

for *bosons*.

The model: for 2S+1 >> 1 one may keep only two terms:

$$H = \frac{1}{2M}\partial_x \Psi_m^+ \partial_x \Psi_m + \frac{g_0}{2N}\left[\sum_m \Psi_m^+ \Psi_m\right]^2 - \frac{g_1}{2N}\left[\sum_m (-1)^m \Psi_m^+ \Psi_{-m}^+\right] \circ \left[\sum_n (-1)^n \Psi_n \Psi_{-n}\right]$$

where n,m = - S, …+S; N = 2S+1.

The most interesting situation occurs when $g_1 > 0$.

Hubbard-Stratonovich transformation:

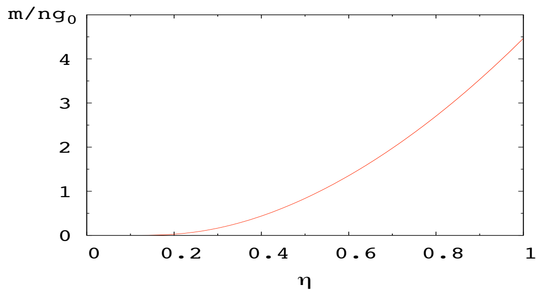
$$N\left[|\Delta|^2/2g_1 + \lambda^2/2g_0\right] + i\lambda \sum_m \Psi_m^+ \Psi_m +$$

$$\left[\Delta \sum_m (-1)^m \Psi_m^+ \Psi_{-m}^+ + h.c.\right]$$

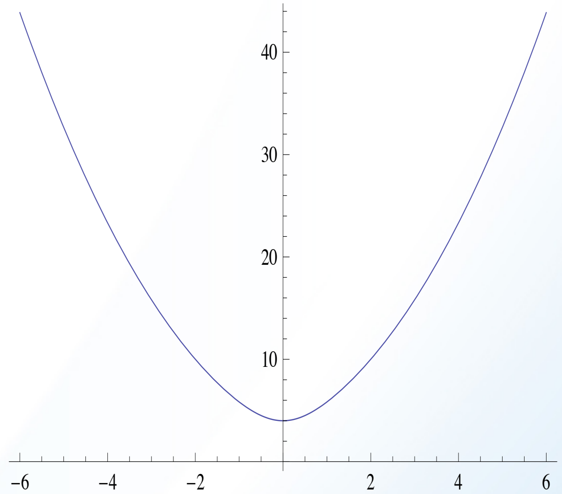
The saddle point action

$$S=\sum_{k,m,\omega}\left(\Psi^{+}_{\omega,k,m},\Psi_{-\omega,-k,-m}\right)\begin{pmatrix} i\omega-\varepsilon & (-1)^m\Delta \\ (-1)^m\Delta^{+} & -i\omega-\varepsilon\end{pmatrix}\begin{pmatrix}\Psi_{\omega,k,m}\\ \Psi^{+}_{-\omega,-k,-m}\end{pmatrix},$$

$$\varepsilon=k^2/2M-\mu,\qquad \mu=\mu_0-i\lambda_0.$$



$$\eta=\frac{1}{2\pi}\sqrt{\frac{Mg_0}{n}}$$



Mean field spectrum:

$$E(q)=\frac{1}{2M}\sqrt{(q^2+\kappa^2)(q^2+Q^2)},\qquad \kappa^2=2M(|\mu|-\Delta),Q^2=2M(|\mu|+\Delta)$$

Saddle point

$$\frac{\Delta}{g_1} = \int \frac{d\omega dk}{(2\pi)^2} \frac{\Delta}{\omega^2 + E^2(k)},$$

$$n = \int \frac{d\omega dk}{(2\pi)^2} \frac{i\omega + \varepsilon(k)}{\omega^2 + E^2(k)},$$

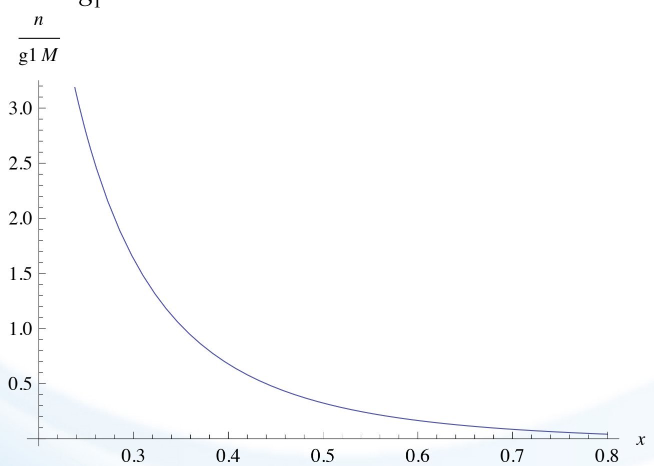
$$\mu = \mu_0 - g_0 n.$$

After taking the integrals we obtain:

$$2\pi n = \frac{\kappa^2 + Q^2}{2\kappa} K\left(1 - \frac{\kappa^2}{Q^2}\right) - QE\left(1 - \frac{\kappa^2}{Q^2}\right),$$

$$\frac{\pi}{Mg_1} = \frac{1}{\kappa} K\left(1 - \frac{\kappa^2}{Q^2}\right).$$

$\frac{n}{Mg_1}$



K(x),E(x) –elliptic functions.

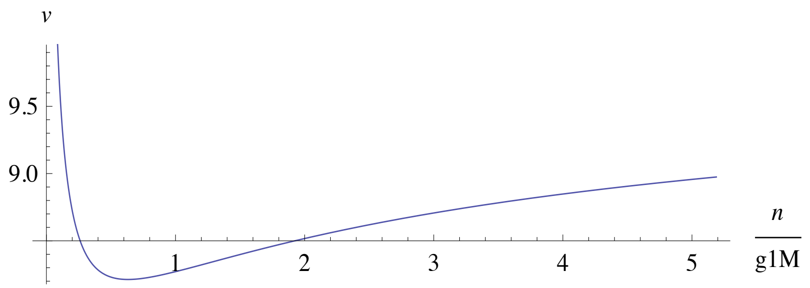
$$Q \approx \sqrt{4nMg_1}$$

$$x = \frac{\kappa}{Q}$$

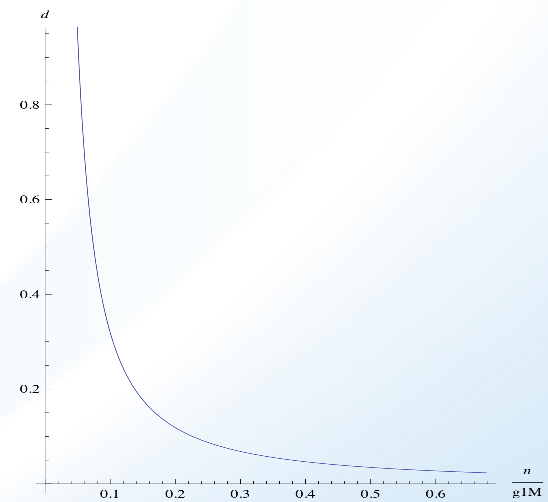
Low energy effective theory: Luttinger liquid.

$$H = \frac{v}{2}\int dx\left[K_s\Pi^2 + K_s^{-1}(\partial_x\Phi)^2\right], \qquad \left[\Pi(x),\Phi(y)\right] = i\delta(x-y).$$

$$\Delta = |\Delta| e^{i\Phi(t,x)} \qquad \varepsilon(k) = v|k|$$



Velocity 10V in units of $\sqrt{12Q/M}$



$d = K_s/4\pi$

Scaling dimension of order parameter

Conclusions of part 1:

- At $2S+1 >> 1$ and $g_1 > 0$ the predominant interaction is pairing. The phase is a condensate of singlet pairs with quasi-long-range order.
- Since the scaling dimension of the *order parameter* quickly becomes <<1 with increase of density, the saddle point approximation is robust.

Part II: Magnetic field. Exact solution in Low Energy sector.

Assumption: thermodynamic equilibrium.

According to Jiang et.al. (2010) the model has U(1)xSO(2S+1) symmetry.

Hence at low energies it is described by *Luttiger liquid + O(2S+1) nonlinear sigma model:*

$$L = \frac{1}{2g}\int dx\left[c^{-1}\left(\partial_t\vec{n}\right)^2 - c\left(\partial_x\vec{n}\right)^2\right], \qquad \vec{n} = (n_1, \ldots n_{2S+1}), \qquad \vec{n}^2 = 1$$

The sigma model is exactly solvable even in magnetic field (Zamolodchikov, Zamolodchikov 1978).

$$c \approx \sqrt{ng_0/M}$$

Bethe Ansatz equations

$$E = \sum_{i=1,\dots n} \Delta \cosh\theta_i - h(nS - \sum m_i)$$

$$\exp\left[iL(\Delta/v)\sinh\theta_i\right] = \prod_{j=1}^{n} S_0^{-1}(\theta_i - \theta_j) \prod_{a_1=1}^{m_1} \frac{\theta_i - \lambda_{a_1}^{(1)} - i\pi}{\theta_i - \lambda_{a_1}^{(1)} + i\pi},$$

$$\prod_{i=1}^{n} \frac{\lambda_{a_1}^{(1)} - \theta_i - i\pi}{\lambda_{a_1}^{(1)} - \theta_i + i\pi} \prod_{a_2=1}^{m_2} \frac{\lambda_{a_1}^{(1)} - \lambda_{a_2}^{(2)} - i\pi}{\lambda_{a_1}^{(1)} - \lambda_{a_2}^{(2)} + i\pi} = \prod_{b=1}^{m_1} \frac{\lambda_{a_1}^{(1)} - \lambda_{b}^{(1)} - 2i\pi}{\lambda_{a_1}^{(1)} - \lambda_{b}^{(1)} + 2i\pi},$$

$$\dots,$$

$$\prod_{a_{S-1}=1}^{m_{S-1}} \frac{\lambda_{a_S}^{(S)} - \lambda_{a_{S-1}}^{(S-1)} - i\pi}{\lambda_{a_S}^{(S)} - \lambda_{a_{S-1}}^{(S-1)} + i\pi} = \prod_{b=1}^{m_S} \frac{\lambda_{a_S}^{(S)} - \lambda_{b}^{(S)} - i\pi}{\lambda_{a_S}^{(S)} - \lambda_{b}^{(S)} + i\pi}.$$

S+1 coupled algebraic equations

$$\frac{1}{2i\pi} \frac{d\ln S_0(\theta)}{d\theta} = \delta(\theta) - K(\theta),$$

$$K(\theta) = \int e^{i\theta\omega/\pi} \frac{1 - \exp\left[-2|\omega|/(2S-1)\right]}{1 + \exp\left[-|\omega|\right]} d\omega/2\pi$$

Magnetization

$$S^z / L = \frac{S}{\pi v}\left[2\Delta(hS - \Delta)\right]^{1/2}\left[1 + const\left(hS/\Delta - 1\right)^{1/2} + \ldots\right]$$

When the magnetic field exceeds the critical value, a second condensate (of uncoupled pairs) appears.

As a result, we have *two gapless modes*.

Conclusions II.

- In a finite magnetic field there is an interval of fields where ferromagnetic liquid of unpaired bosons coexists with a polar condensate.