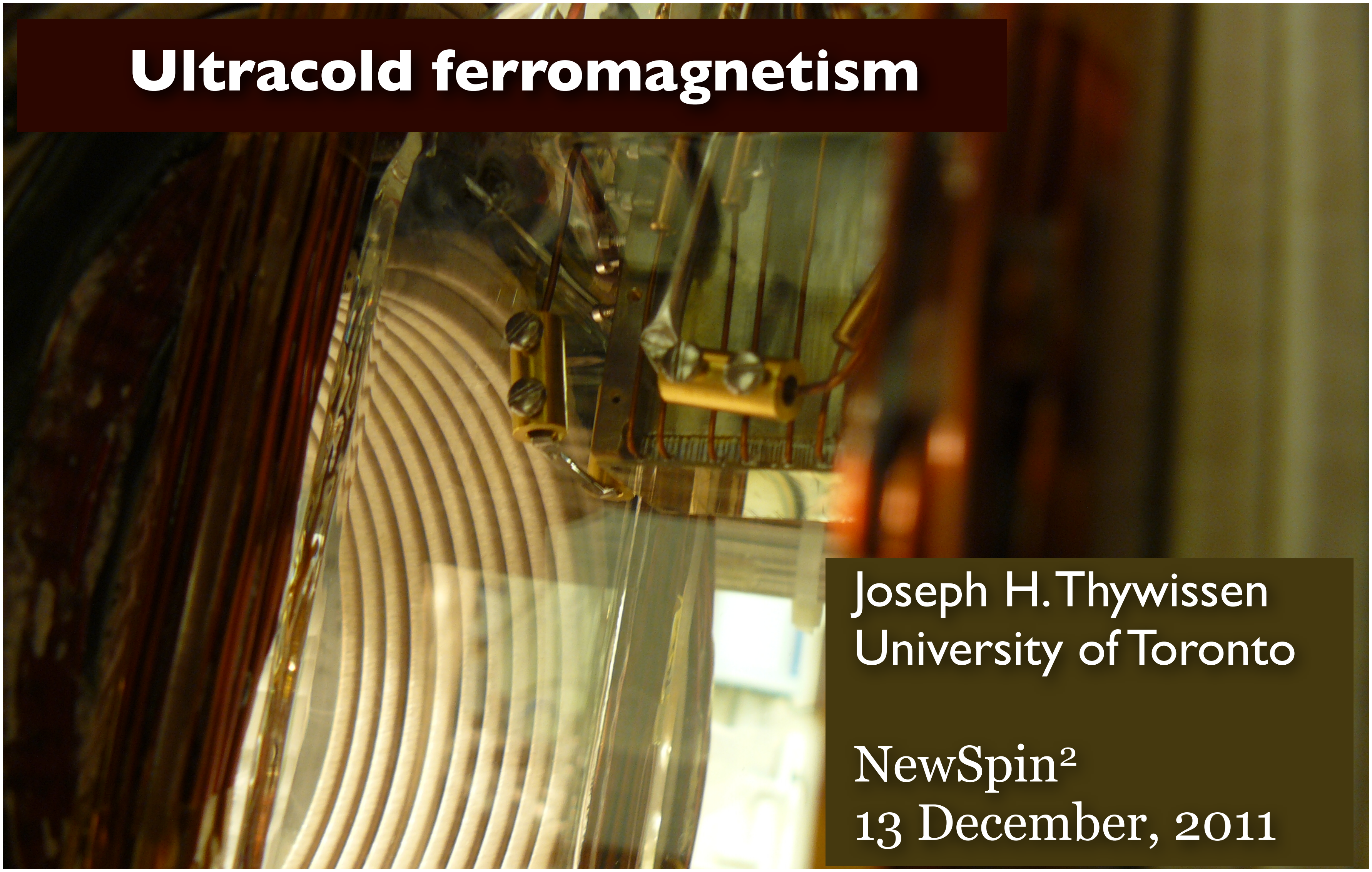
Ultracold ferromagnetism



Joseph H. Thywissen
University of Toronto

NewSpin^2
13 December, 2011

in the beginning...

PRL **95,** 230403 (2005) PHYSICAL REVIEW LETTERS week ending 2 DECEMBER 2005

Itinerant Ferromagnetism in an Ultracold Atom Fermi Gas

R. A. Duine* and A. H. MacDonald†

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(Received 18 April 2005; published 29 November 2005)

We address the possible occurrence of ultracold atom ferromagnetism by evaluating the free energy of a spin polarized Fermi gas to second order in its interaction parameter. We find that Hartree-Fock theory

- *attractive* interactions ➔ **superfluidity**
- *repulsive* interactions ➔ **magnetism**

Repulsive gases (and magnetism) largely ignored in studies of Feshbach gases, which have focused on attractive gases.

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PHYSICAL REVIEW A

Repulsive Fermi gas in a harmonic trap

L. J. LeBlanc,[1] J. H. Thywissen,[1] A. A
[1]*Department of Physics, University of Toront*
[2]*Department of Physics, University of Waterlo*
(Received

We study ferromagnetism in a repuls
a local density approximation, the two

What is the minimal set of ingredients for ferromagnetism?

Itinera

Gas of Ultracold Atoms

Gyu-Boong Jo,[1]* Ye-Ryoung Lee,[1] Jae-Hoon Choi,[1] Caleb A. Christensen,[1] Tony H. Kim,[1] Joseph H. Thywissen,[2] David E. Pritchard,[1] Wolfgang Ketterle[1]

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Correlations and Pair Formation in a Repulsively Interacting Fermi Gas

Christian Sanner, Edward J. Su, Wujie Huang, Aviv Keshet, Jonathon Gillen, and Wolfgang Ketterle
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, Massachusetts Institute of Technology, Cambridge Massachusetts 02139, USA

A degenerate Fermi gas is rapidly quenched into the regime of strong effective repulsion near a

Ultracold Ferromagnetism:

The good, the bad, & the ugly.

NewSpin2
12-17 December 2011

J. H. Thywissen
University of Toronto


?
The shepherd Magnes
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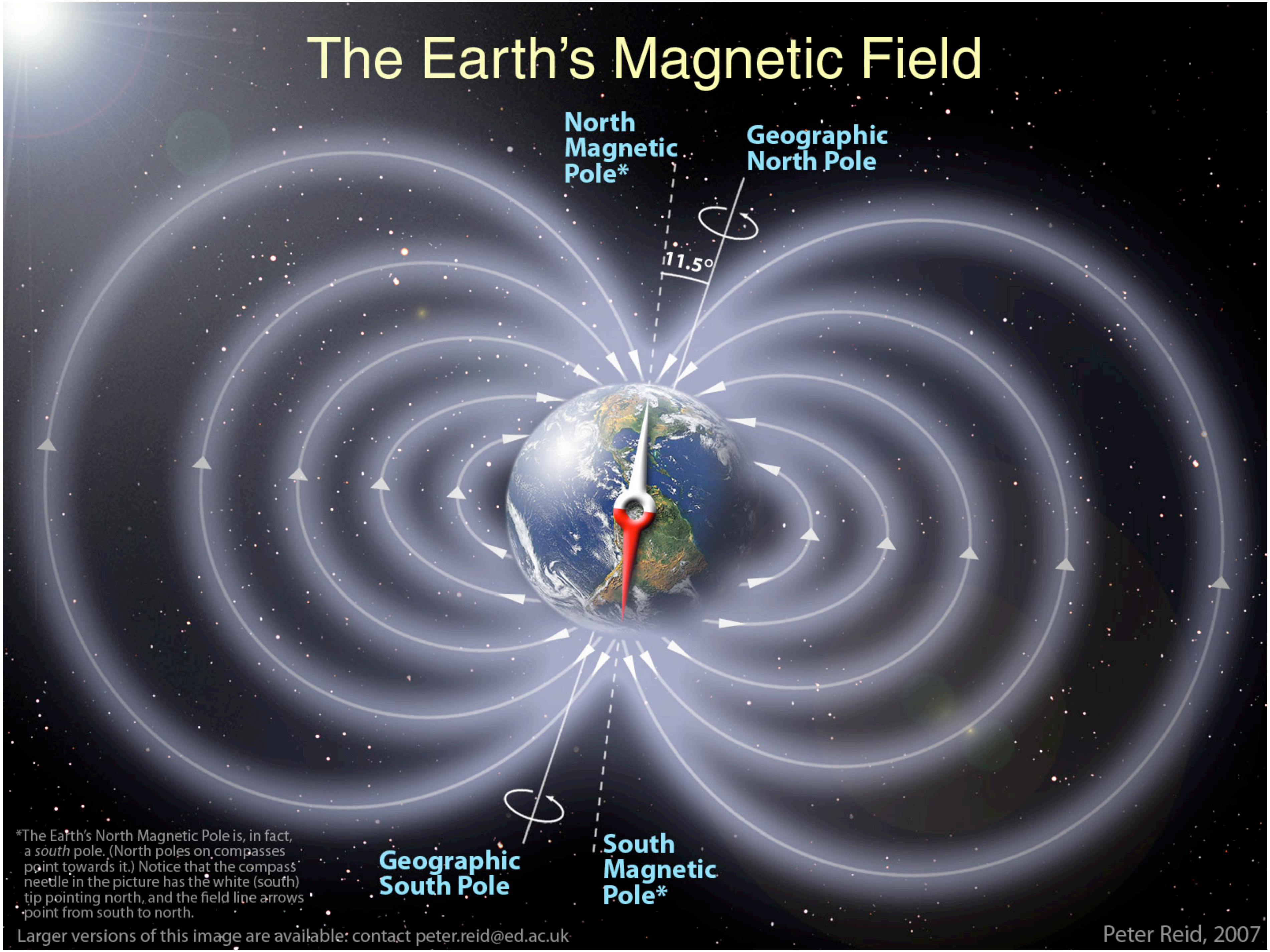

The Earth's Magnetic Field
North Magnetic Pole*
Geographic North Pole
11.5°
Geographic South Pole
South Magnetic Pole*
*The Earth's North Magnetic Pole is, in fact, a *south* pole. (North poles on compasses point towards it.) Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.
Larger versions of this image are available: contact peter.reid@ed.ac.uk
Peter Reid, 2007

Long history of magnetism

- Greeks (600 BC): lodestone attracts iron
- Gilbert (England, 16 c.): Earth is a weak magnet
- Gauss (Germany, 18 c.): theory…
- Coulomb (France, 18 c.): inverse square law
- Oersted (Denmark, 19c.): connection to electricity
- Ampere, Faraday (19 c.): how E-fields relate to B-fields
- Maxwell (Scotland, 19 c.): E&M unification
- Curie, Weiss (19 c.): effect of T on magnet
- Ising, Heisenberg, Bloch, Stoner (20 c.): quantum theory
- Weinberg, Salam (20 c.): electroweak unification

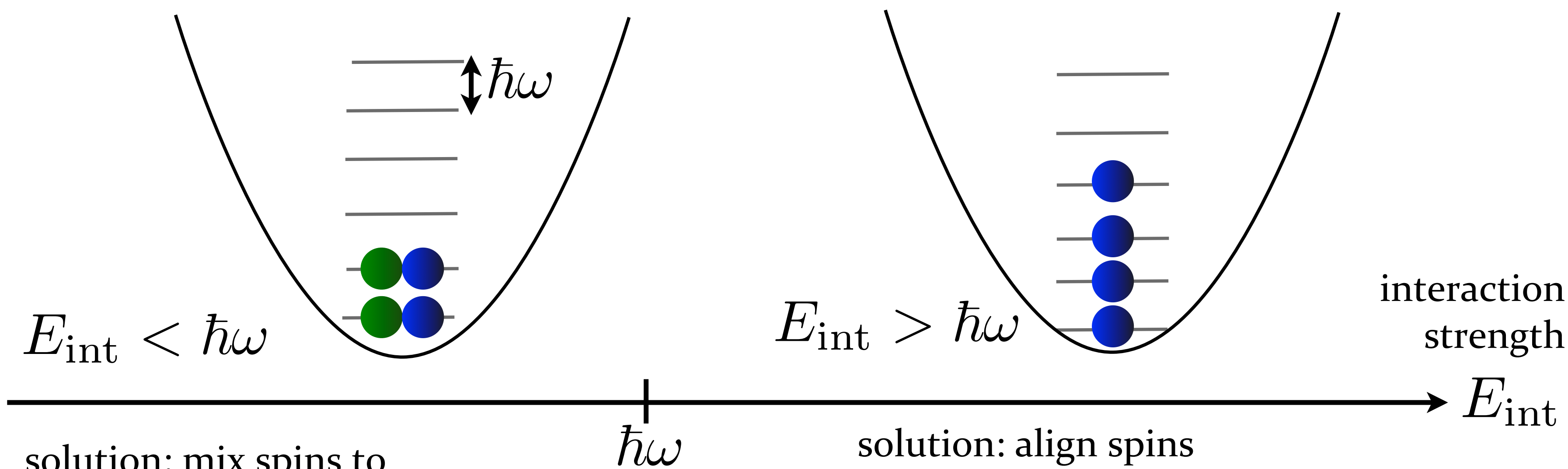
Applications

Basic energetics of ferromagnetism

Total energy = single-particle energy + interaction energy

$$E_{\text{tot}} = \hbar\omega \sum n_i + E_{\text{int}} N_{\uparrow} N_{\downarrow}$$

For example, what configuration minimizes energy for 4 particles ?



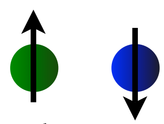
solution: mix spins to minimize maximum occupied level.

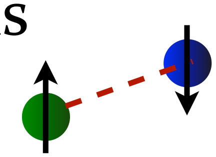
solution: align spins (polarize) to minimize interaction energy.

Ferromagnetic configuration is *strongly interacting*:
Interaction energy must be higher than single-particle energy.

Ingredients

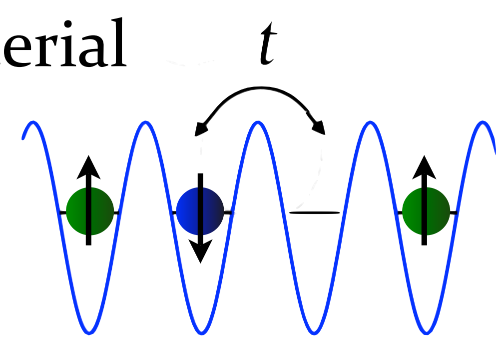
that we find in ferromagnetic materials

1. Fermions 
   -unpaired electrons

2. Repulsive interactions
   -Coulomb repulsion 

Necessary to energetics.

3. Lattice
   -structure of material



Necessary?
What about a gas?

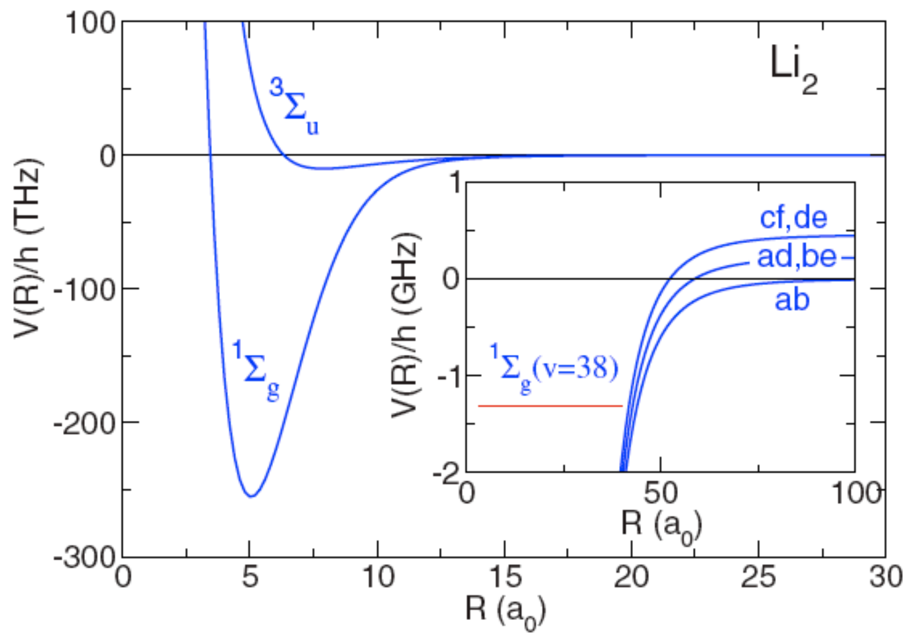
What is the simplest condition in which permanent magnetism can occur?

Condensed matter physics with trapped ultracold atoms

Neutral atom Hamiltonian

$$\hat{H} = \int dr\, \hat{\Psi}^\dagger(r) \left[-\frac{\hbar^2}{2m}\nabla^2 + U(r) \right] \hat{\Psi}(r) + \frac{1}{2}\int dr dr'\, \hat{\Psi}^\dagger(r)\hat{\Psi}^\dagger(r')V(r-r')\hat{\Psi}(r')\hat{\Psi}(r)$$

- V: *Inter-atomic potential* is deep, complex, and unique to each atom pair)
- U: *Trapping potential* not reminiscent of textbooks, where we typically worked "in a box" (U=0)

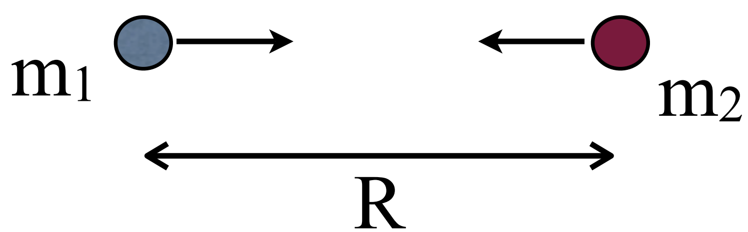


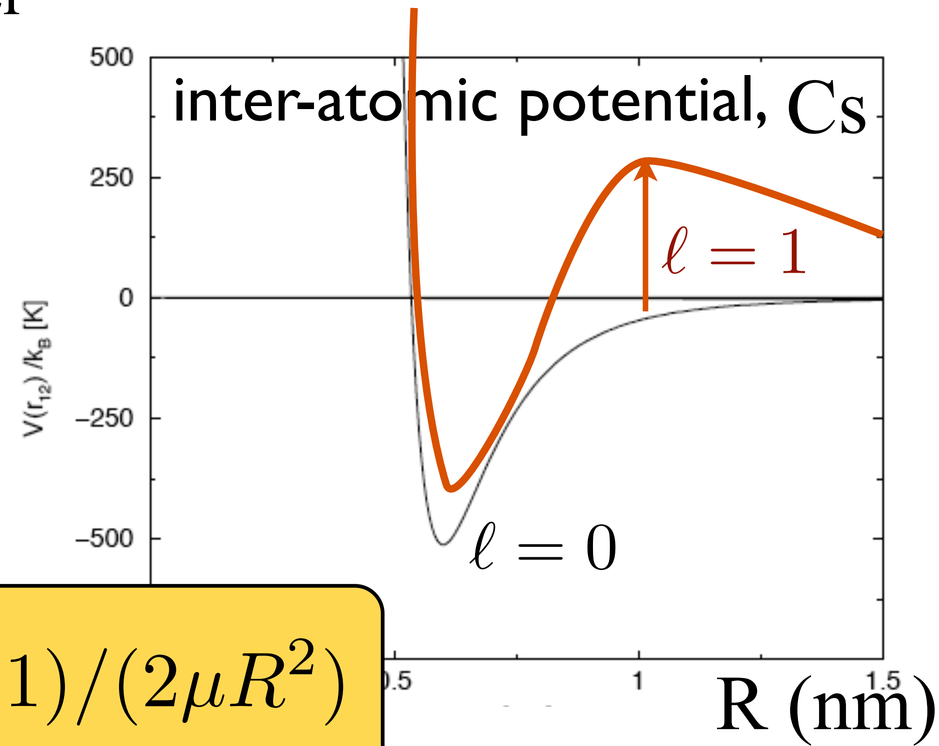
How could this Hamiltonian be useful to understand other systems?

1st simplification: low-energy limit

- Dilute atoms scatter pair-wise, because their typical spacing $\overline{R} = n^{-1/3}$ is much smaller than the potential range r_0
- Below 0.1mK, atom pairs do not have enough E to overcome the p-wave centrifugal barrier

Two-body collision





$$V_\ell(R) = V(R) + \hbar^2 \ell(\ell+1)/(2\mu R^2)$$

Measurement of p-Wave Threshold Law Using Evaporatively Cooled Fermionic Atoms

B. DeMarco, J. L. Bohn, J. P. Burke, Jr., M. Holland, and D. S. Jin*
JILA, National Institute of Standards and Technology and University of Colorado, Boulder, Colorado 80309
and Physics Department, University of Colorado, Boulder, Colorado 80309
(Received 21 December 1998)

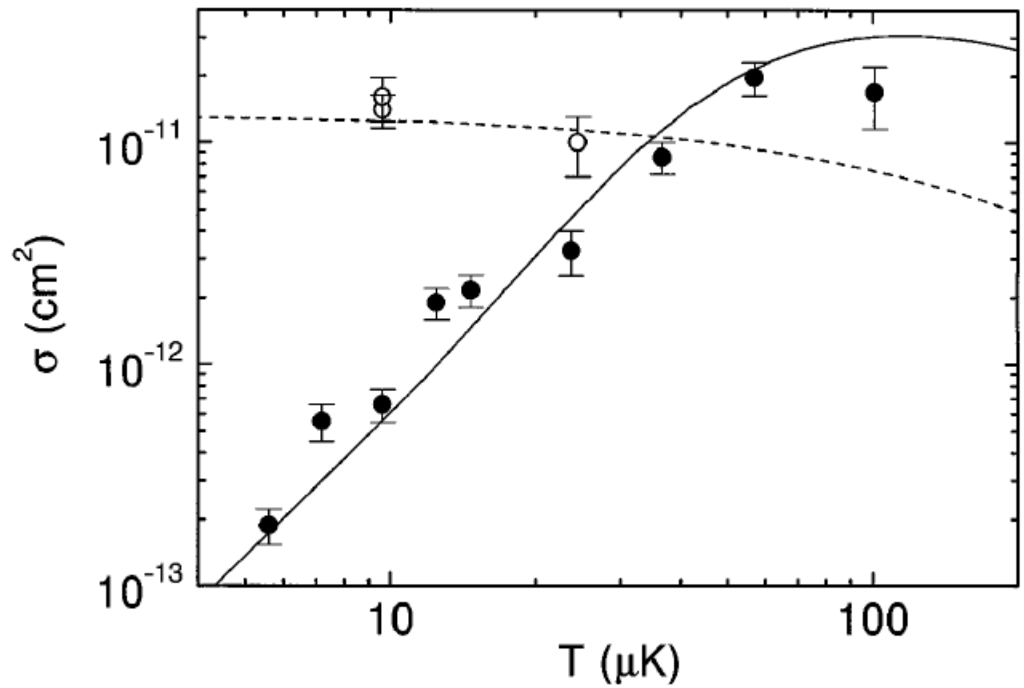
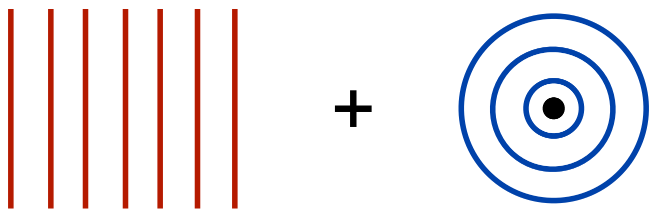


FIG. 2. Elastic cross sections vs temperature. The s-wave cross section (○), measured using a mixture of spin states, shows little temperature dependence. However, the p-wave cross section (●), measured using spin-polarized atoms, exhibits the expected threshold behavior and is seen to vary by over 2 orders of magnitude. The lines are a fit to the data, as described in the text, yielding $a_t = (157 \pm 20)a_0$.

S-wave ($\ell = 0$) scattered wave function

For elastic scattering, must be

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{a}{1+ika}\frac{e^{ikr}}{r}$$



plane wave

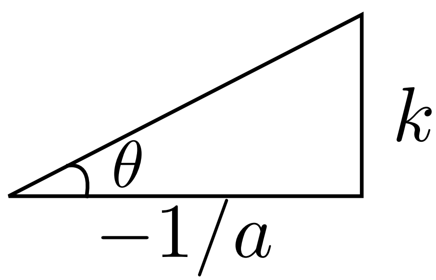
spherical wave

Only one free parameter!
“scattering length” **a**

The scattering term has an amplitude

$$f_k = -[1/a + ik]^{-1}$$ “scattering amplitude”

from which you find the phase



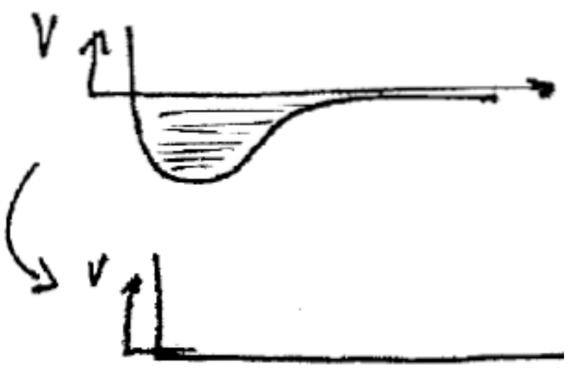
& cross-section

$$\sigma = 4\pi |f_{\vec{k}}(\vec{n})|^2$$

Pseudo-potential

- Two interaction potentials V and V' are equivalent if they have the same scattering length

- So: after measuring **a** for the real system, we can model with a very simple potential.



$V(\vec{R})$

Replace interaction potential with delta function!

$V(\vec{R}) = g\delta(\vec{R})$

where $\boxed{g = \frac{4\pi\hbar^2}{m}a}$

- Actually, to avoid divergences you need

$$\boxed{V(\vec{R}) = g\delta(\vec{R})\partial_R(R\,\cdot)}$$ "regularized"

Neutral atom Hamiltonian (revisited)

$$\hat{H} = \int dr\, \hat{\Psi}^\dagger(r) \left[-\frac{\hbar^2}{2m}\nabla^2 + U(r) \right] \hat{\Psi}(r) + \frac{1}{2}\int dr dr'\, \hat{\Psi}^\dagger(r)\hat{\Psi}^\dagger(r')V(r-r')\hat{\Psi}(r')\hat{\Psi}(r)$$

Can write V(..) as pseudo potential:

$$V(\vec{R}) = g\delta(\vec{R})\partial_R(R\,\cdot)$$

in limit of dilute ($\overline{R} \gg r_0$) and ultracold ($T \ll 100\mu\mathrm{K}$).

✓

Neutral atom Hamiltonian (revisited)

$$\hat{H} = \int dr\, \hat{\Psi}^\dagger(r) \left[-\frac{\hbar^2}{2m}\nabla^2 \boxed{+ U(r)} \right] \hat{\Psi}(r) + \underline{\frac{1}{2} \int dr dr'\, \hat{\Psi}^\dagger(r) \hat{\Psi}^\dagger(r') V(r-r') \hat{\Psi}(r') \hat{\Psi}(r)}$$

Can write V(..) as pseudo potential:

$$V(\vec{R}) = g\delta(\vec{R})\partial_R(R\,\cdot)$$

in limit of dilute ($\overline{R} \gg r_0$) and ultracold ($T \ll 100\mu\mathrm{K}$).

✓

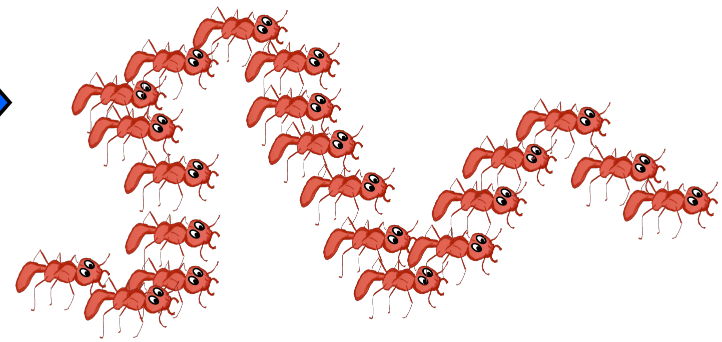
What about the trap?

2nd simplification: Local chemical potential

- What if a cold gas were a distribution of *local* creatures?

snake

line of ants



{scare the ant at the front of the line, and the last ant won't rattle its tail...}

- Recipe:

$$\boxed{\mu \longrightarrow \mu_{\text{local}} = \mu - U(\vec{r})}$$

Local chemical potential: $\mu_{\mathrm{local}} = \mu - U(\vec{r})$
"how to use your Stat. Mech. textbook"

- Thomas Fermi density profiles:

textbook

$$n = \frac{1}{6\pi^2}\left[\frac{2mE_F}{\hbar^2}\right]^{3/2}$$

local μ

$$n_{\mathrm{TF}} = \frac{(2m)^{3/2}}{6\pi^2\hbar^3}\left[E_F - U(\vec{r})\right]^{3/2}$$

for zero-temperature fermions in semiclassical limit.

- ideal quantum gas functions: $n = \lambda_T^{-3} f_{3/2}(z)$

textbook **local μ**

$$z = e^{\beta\mu} \longrightarrow z = e^{\beta(\mu - U(\vec{r}))}$$

at finite temperature ($\beta = 1/k_{\mathrm{B}}T$), where z=fugacity.

- Similar Thomas Fermi expression for bosons:

textbook

$$\mu = gn$$

local μ

$$n_{\mathrm{TF}} = \frac{1}{g}\left[\mu - U(\vec{r})\right]$$

Validity of local chemical potential

- A “local density approximation” (LDA).
- Not a good approximation when:
  -tunneling can occur through barriers
  -long-range order affected (eg, phase coherence)
  -gradients perturb states (eg, localized states)
  -long-range interactions (Coulomb etc)
- In those cases, *model must include trapping potential.*
- However in some important cases works well:
  -important length scales (eg, Fermi length or lattice constant) much smaller than trap size
  -Far from edges, compared to healing length ξ:

$$\xi = 1/\sqrt{8\pi n a} \quad \text{such that} \quad \frac{\hbar^2}{2m\xi^2} = gn$$

Cold neutral gases: length scales

- inter-atomic potential range, r_0: 2 nm
- scattering length, a
  -low-field (background) 5 nm
  -near a Feshbach resonance 100 nm to 1000 nm
- thermal de Broglie wavelength: 100 nm
- average inter-particle spacing: 100 nm
  -same length scale as $1/k_F$

Quantum degeneracy

- lattice constant: 400 nm

Effective Hamiltonian

- ground state width:
  1μm @ 100Hz (typ. magnetic trap)
  100nm @ 10kHz (single site of optical lattice)
- cloud size: 1-100 μm

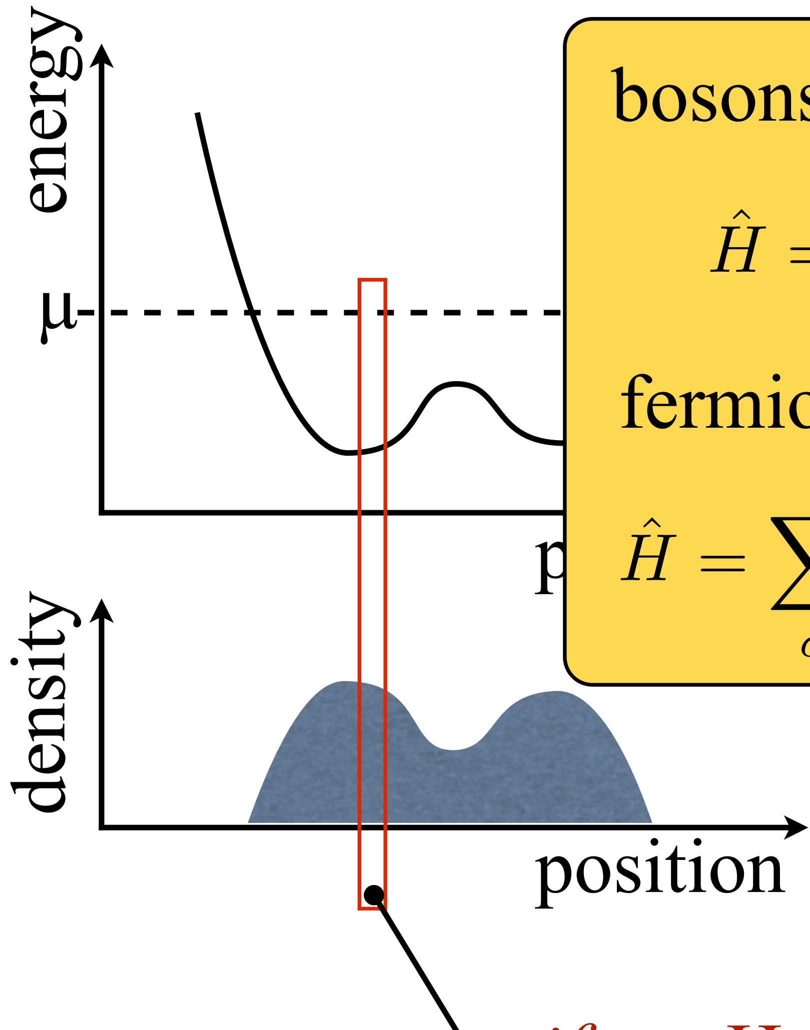
Cold neutral gases: length scales (in traps)

- inter-atomic potential range, r_0: 2 nm
- scattering length, a
  -low-field (background) 5 nm
  -near a Feshbach resonance 100 nm to 1000 nm
- thermal de Broglie wavelength: 100 nm
- average inter-particle spacing: 100 nm
  -same length scale as $1/k_F$

Effective Hamiltonian

- ground state width:
  1μm @ 100Hz (typ. magnetic trap)
- cloud size: 1-100 μm

Physics in local μ picture:



bosons (for single component):

$$\hat{H} = \hat{\Psi}^\dagger \left[-\frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi} + \frac{g}{2} \hat{n}^2$$

fermions (for 2-component gas):

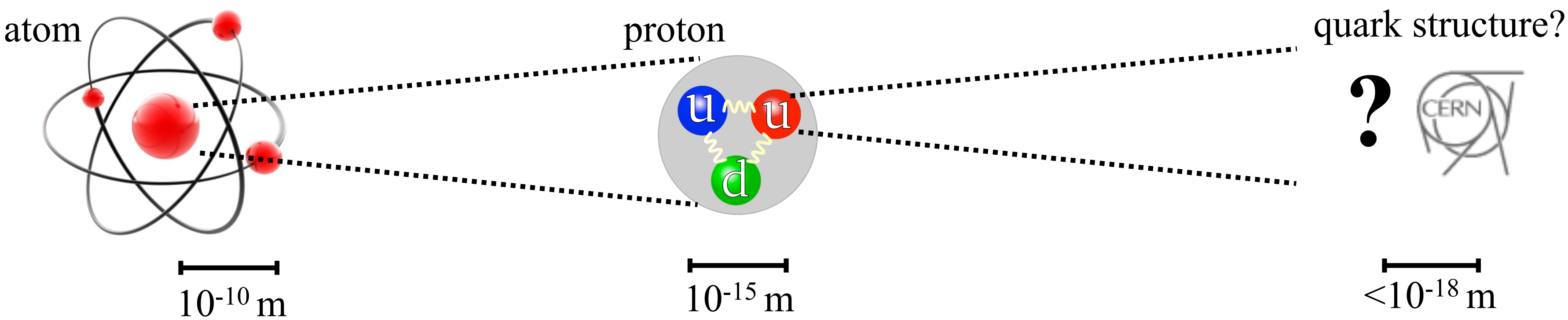
$$\hat{H} = \sum_\sigma \hat{\Psi}_\sigma^\dagger \left[-\frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}_\sigma + g\, \hat{n}_\uparrow \hat{n}_\downarrow$$

uniform H, simulated with local μ & T.

Why use gases to study CM physics?

A: Separation of length scales

Example: Standard model physics



Atomic Physics

Quantum Chromodynamics

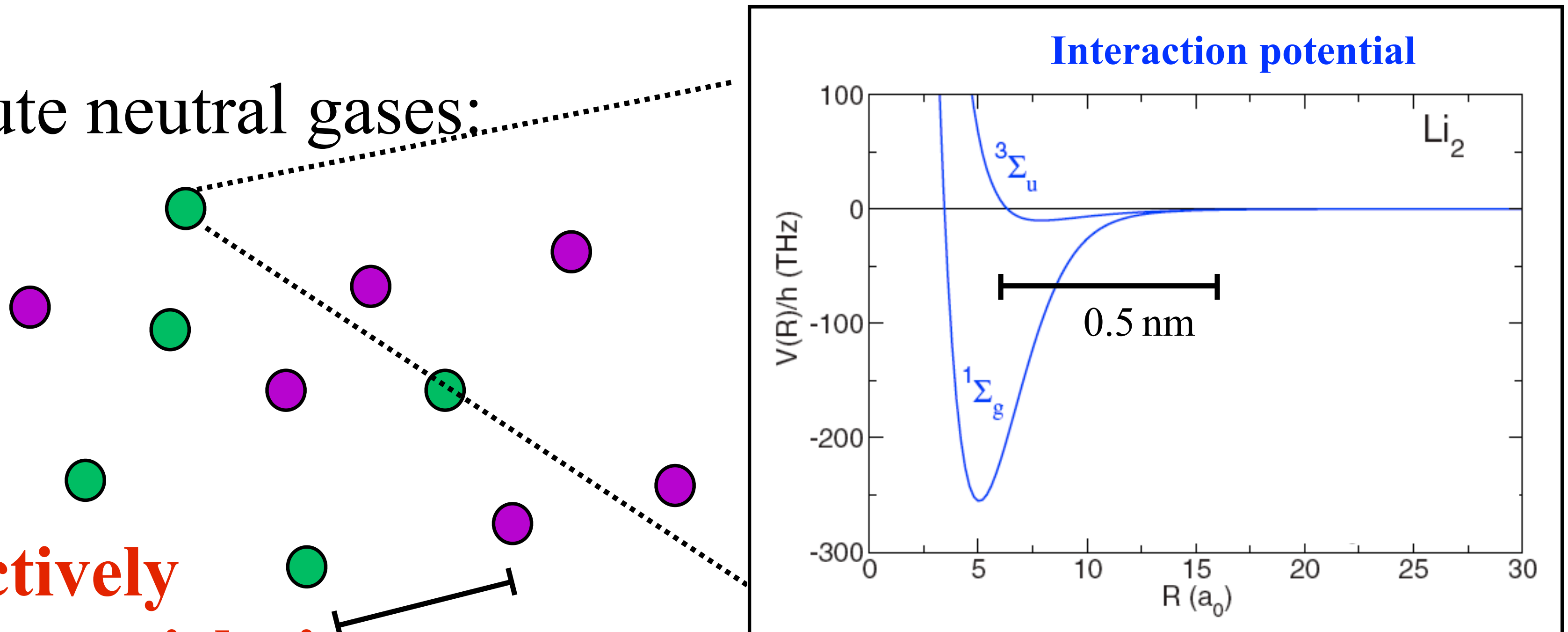
(string theory?)

Theories effective at each length scale.

Why use gases to study CM physics?

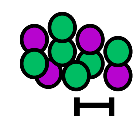
A: Separation of length scales

Dilute neutral gases:



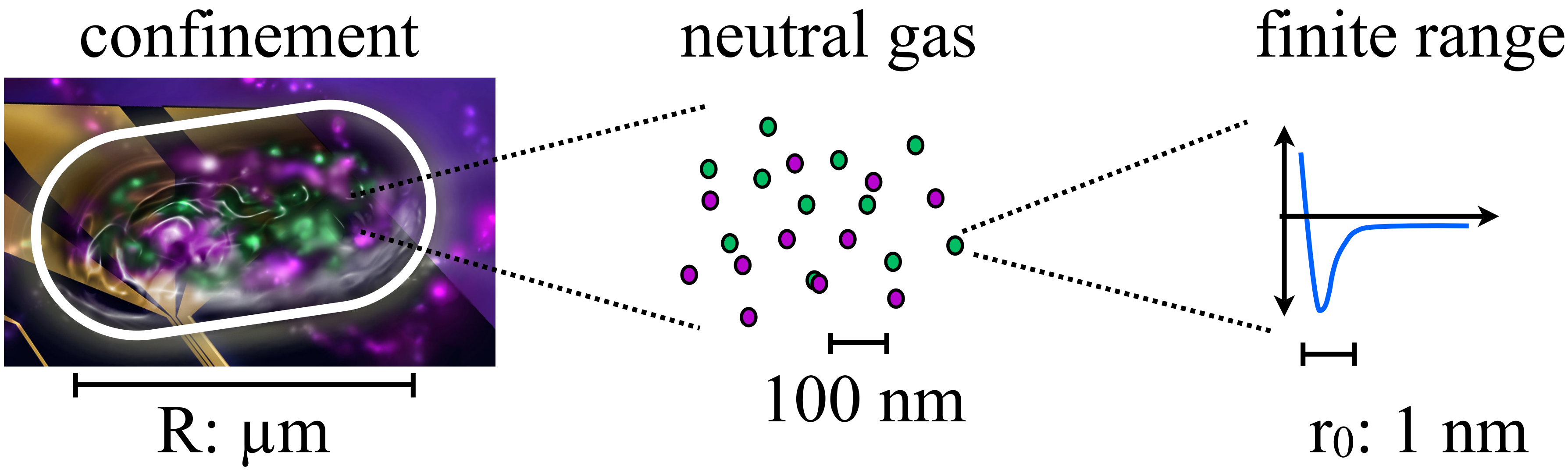
Effectively point particles!

In contrast to solids & liquids, where interactions depend on details: *chemistry*.



$n^{-1/3} \lesssim 1$ nm

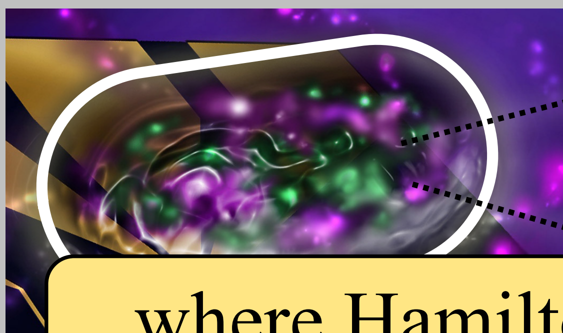
Why use gases to study CM physics?



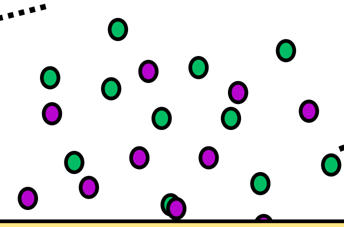
Why use gases to study CM physics?

SIMULATION SPACE

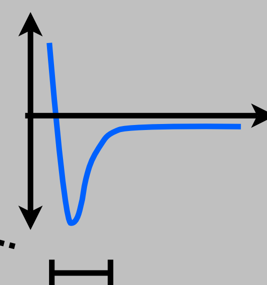
confinement



ultracold atoms



finite range



r_0: 1 nm

...where Hamiltonians are generic!

bosons (particles with integer spin):

$$\hat{H} = \hat{\Psi}^{\dagger}\left[-\frac{\hbar^2}{2m}\nabla^2\right]\hat{\Psi} + \frac{g}{2}\hat{n}^2$$

fermions (particles with half-integer spin):

$$\hat{H} = \sum_{\sigma}\hat{\Psi}^{\dagger}_{\sigma}\left[-\frac{\hbar^2}{2m}\nabla^2\right]\hat{\Psi}_{\sigma} + g\,\hat{n}_{\uparrow}\hat{n}_{\downarrow}$$

"Let's simulate!"

ce

gth:

erate

s

$$g = \frac{4\pi\hbar}{m}a_S$$

Spinful Fermi gases

Perturbative treatment

Energy of repulsive gas:
Universal to second order (ind. of details of short-range int.)

$$f(x) = \frac{1}{2}(\eta_\uparrow^5 + \eta_\downarrow^5) + \frac{10 k_F a}{9\pi}\eta_\uparrow^3\eta_\downarrow^3 + \frac{(k_F a)^2}{21\pi^2}\xi(\eta_\uparrow, \eta_\downarrow)$$

ideal gas mean field beyond mean field

where $\mathcal{E} = \frac{3}{5}E_F f(x)$ and [Kanno, 1970]:

$$\xi = 22\eta_\uparrow^3\eta_\downarrow^3(\eta_\uparrow + \eta_\downarrow) - 4\eta_\uparrow^7 \ln\frac{\eta_\uparrow + \eta_\downarrow}{\eta_\uparrow} - 4\eta_\downarrow^7 \ln\frac{\eta_\uparrow + \eta_\downarrow}{\eta_\downarrow}$$
$$+ \frac{1}{2}(\eta_\uparrow - \eta_\downarrow)^2\eta_\uparrow\eta_\downarrow(\eta_\uparrow + \eta_\downarrow)[15(\eta_\uparrow^2 + \eta_\downarrow^2) + 11\eta_\uparrow\eta_\downarrow]$$
$$+ \frac{7}{4}(\eta_\uparrow - \eta_\downarrow)^4(\eta_\uparrow + \eta_\downarrow)(\eta_\uparrow^2 + \eta_\downarrow^2 + 3\eta_\uparrow\eta_\downarrow)\ln\left|\frac{\eta_\uparrow - \eta_\downarrow}{\eta_\uparrow + \eta_\downarrow}\right|.$$

Perturbative treatment

Energy of repulsive gas:
Universal to second order (ind. of details of short-range int.)

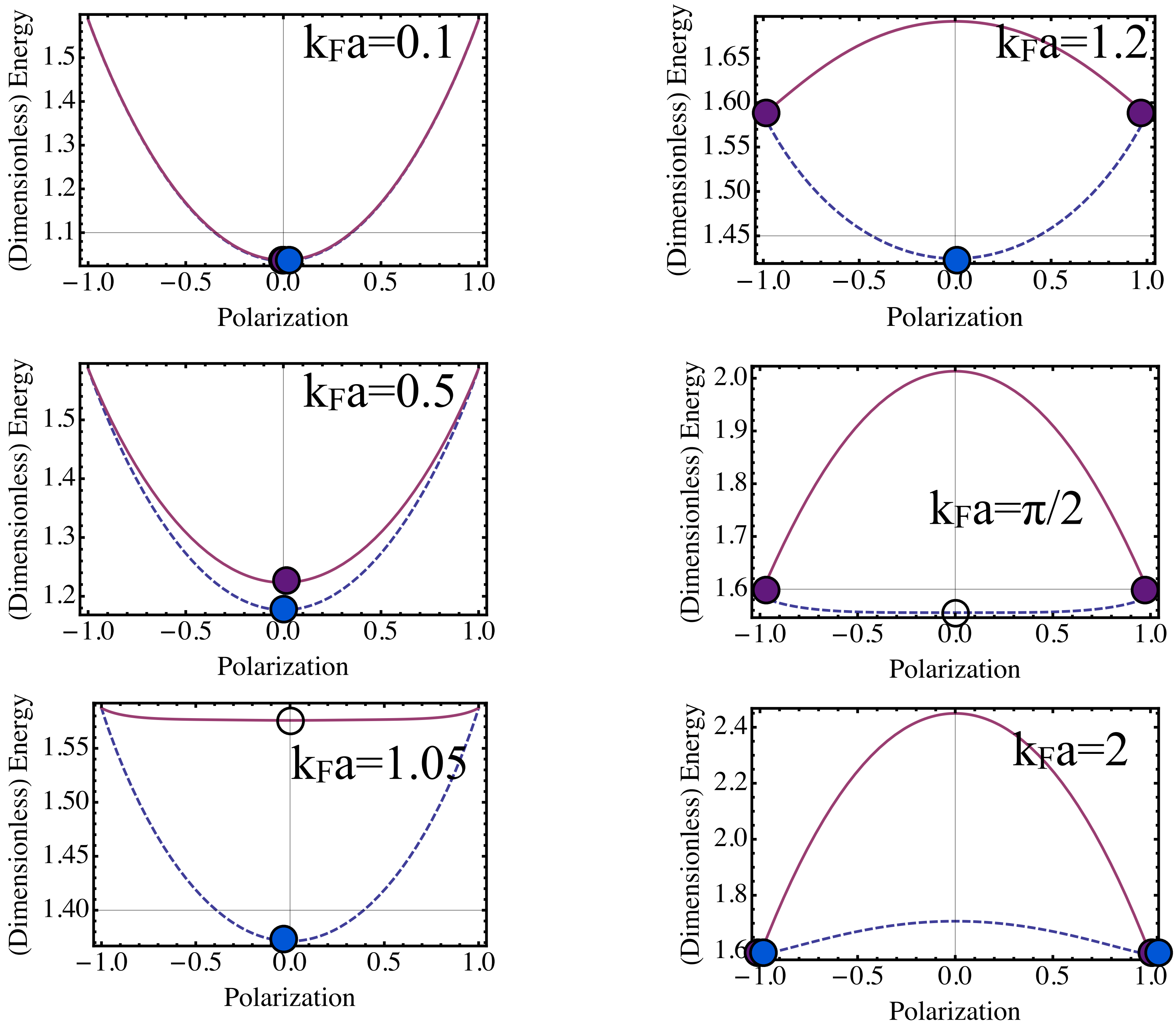
$$f(x) = \frac{1}{2}(\eta_\uparrow^5 + \eta_\downarrow^5) + \frac{10 k_F a}{9\pi}\eta_\uparrow^3 \eta_\downarrow^3 + \frac{(k_F a)^2}{21\pi^2}\xi(\eta_\uparrow, \eta_\downarrow)$$

ideal gas mean field beyond mean field

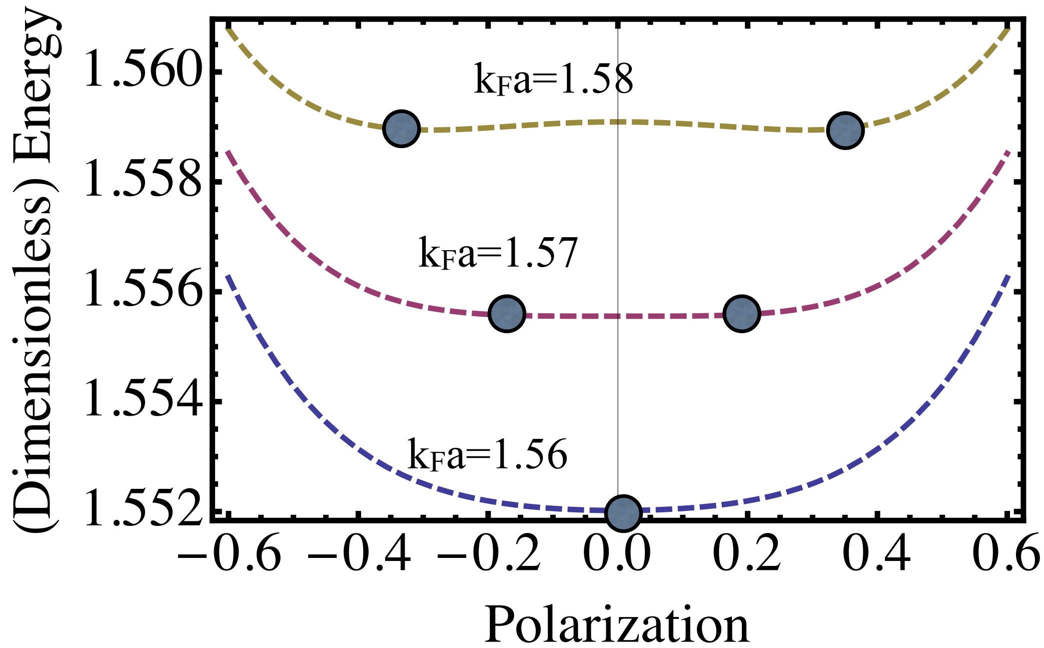
Check balanced (x=0) limit:

$$f(x) = 1 + \frac{10}{9\pi}k_F a + \frac{4(11 - 2\ln 2)}{21\pi^2}(k_F a)^2$$

✓ K. Huang and C.N. Yang, Phys. Rev. 105, 767 (1957);
T. D. Lee and C. N. Yang, Phys. Rev. 105, 1119 (1957).



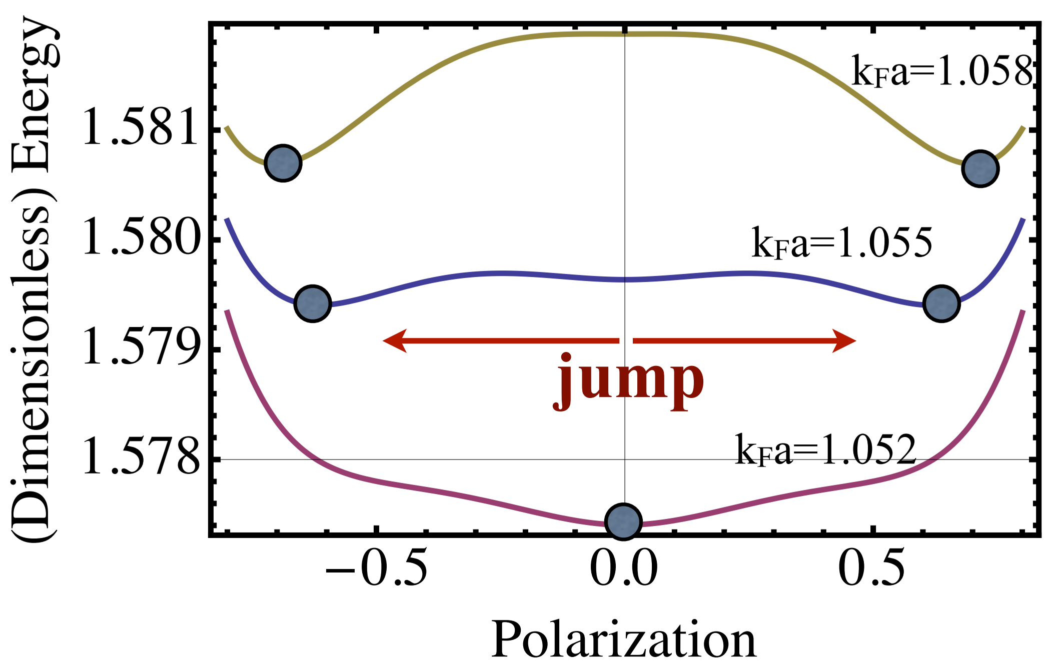
kFa=0.1
kFa=0.5
kFa=1.05
kFa=1.2
kFa=π/2
kFa=2
(Dimensionless) Energy
Polarization



Order of phase transition?

mean field:

2nd order

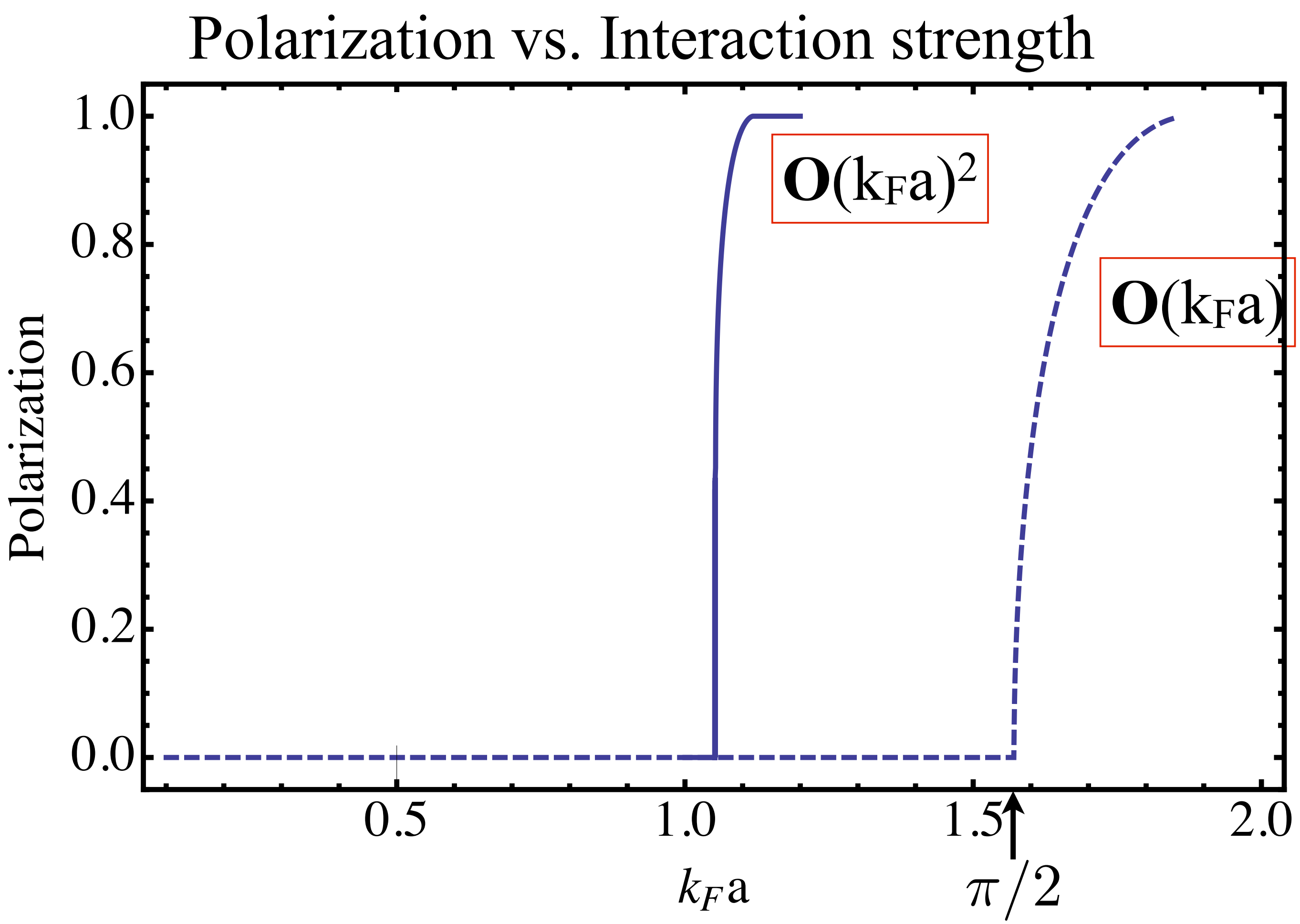


including $(k_F a)^2$:

1st order

Spontaneous polarization: Ferromagnetic phase transition

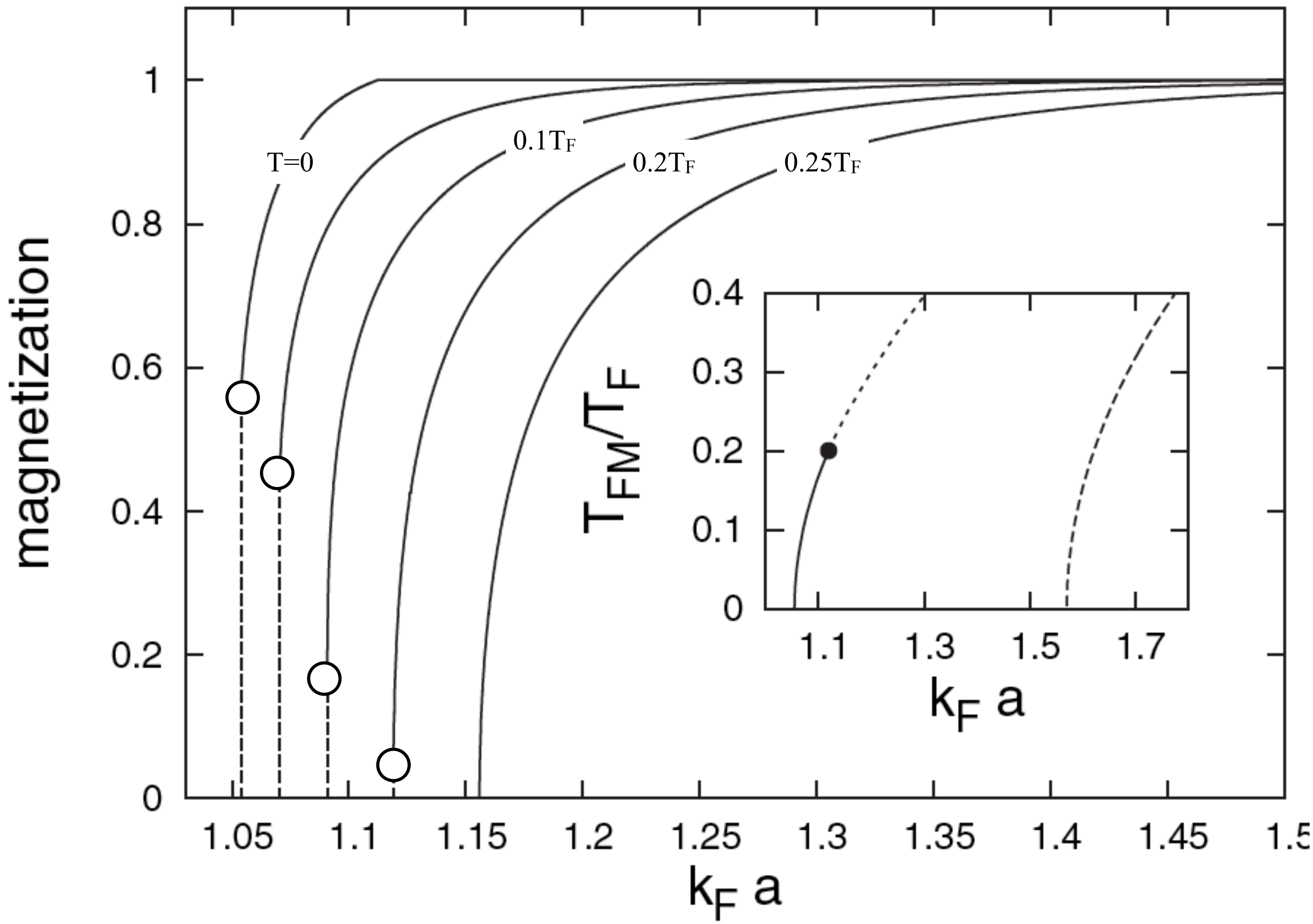
(uniform gas, T=0, perturbation theory)



Duine & MacDonald, PRL **95**, 230403 (2005).

Ferromagnetism

(uniform gas, finite temperature, 2nd-order treatment)



What about conservation laws?

- Trapped clouds are isolated. Should they be considered micro-canonical?
- Quantum statistical distributions valid for grand canonical ensemble (thermal & diffusive eq):

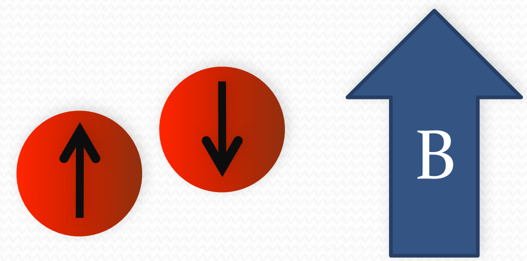
$$\overline{n}_{\mathrm{FD}} = \frac{1}{e^{(\epsilon-\mu)/k_{\mathrm{B}}T} + 1} \qquad \overline{n}_{\mathrm{BE}} = \frac{1}{e^{(\epsilon-\mu)/k_{\mathrm{B}}T} - 1}$$

- Is the system free to choose lowest energy spin configuration?

spin: metals vs. atoms

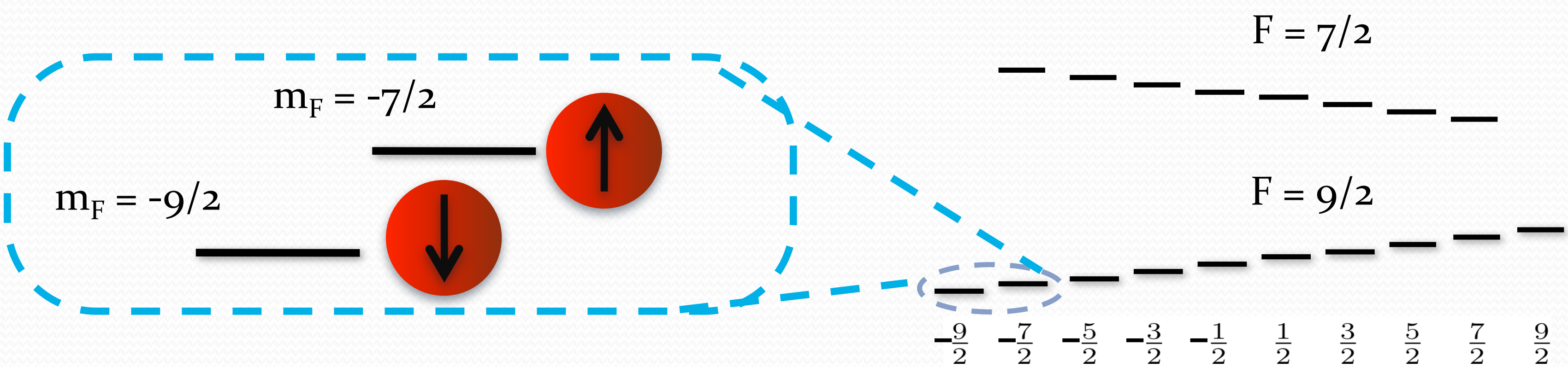
metals → electron spin:

- parallel or antiparallel to magnetic field
- spin flips allowed through interactions with the lattice



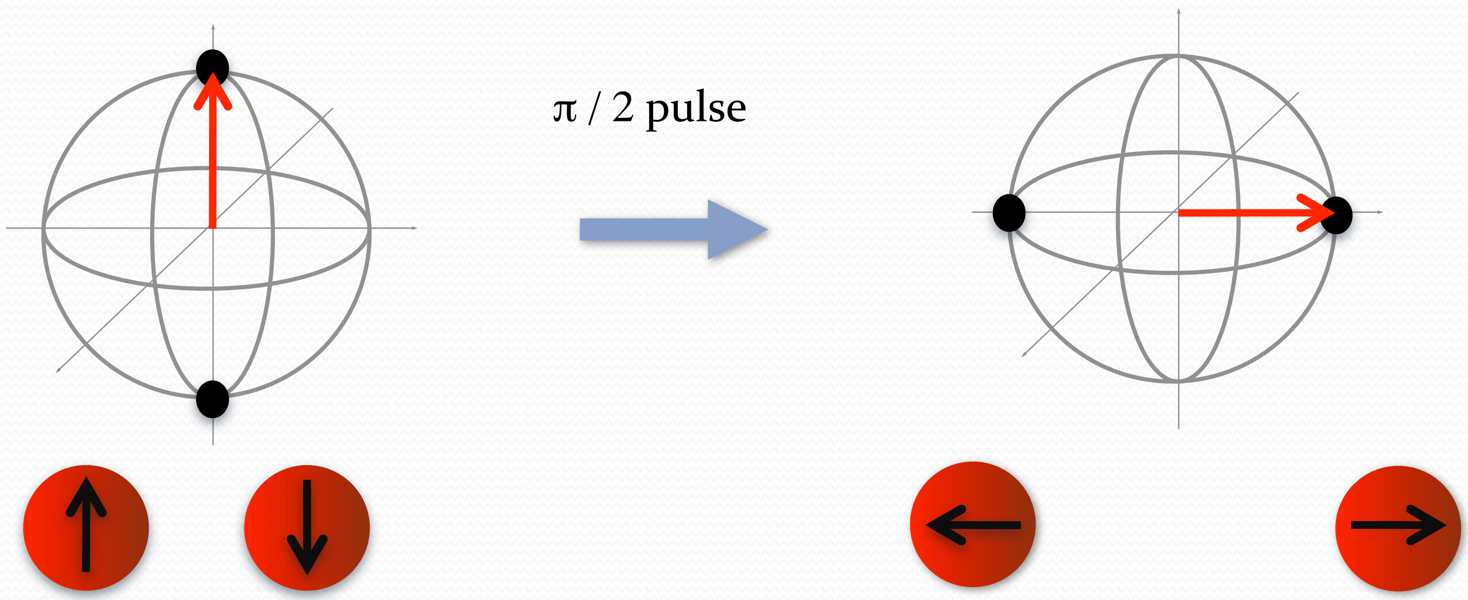
atoms → atomic substructure

- any two Zeeman sublevels are valid "spins"
- spin must be conserved = no spin flips



spin: atoms

- any orthogonal superpositions of Zeeman sublevels are also valid as pseudospin states:



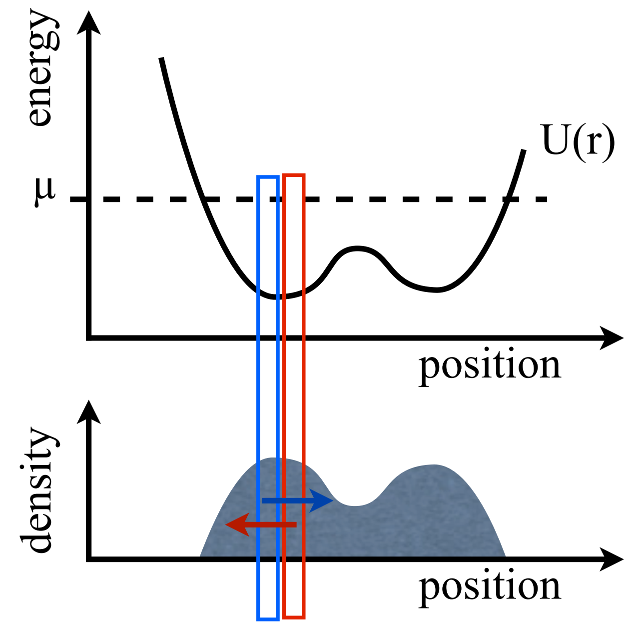
superposition states

- maintain (controllable) interactions between the two states
- spin flips are now **allowed**

What about conservation laws?

Local density picture:

Each volume element can be thought of a "system."

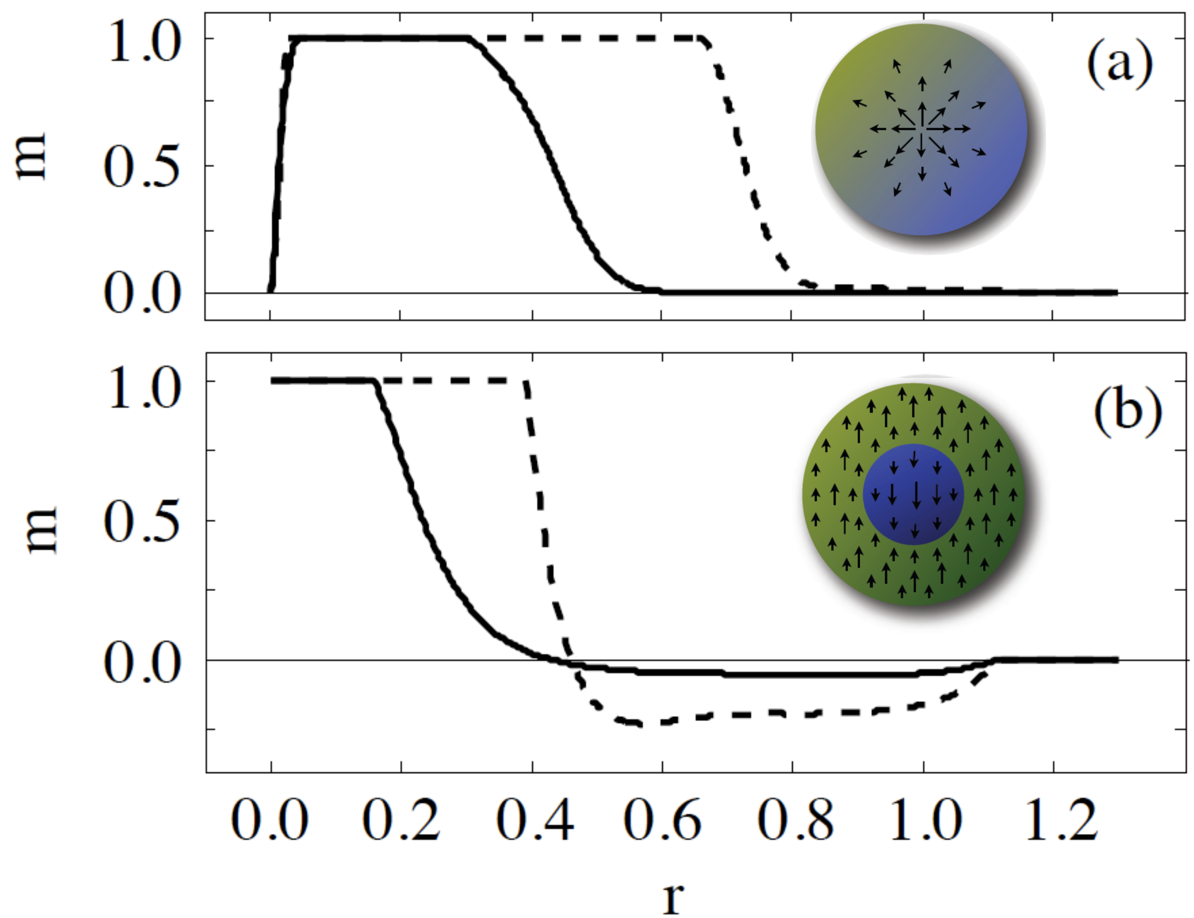


- T and μ-V(r) are both in equilibrium with neighboring elements.
- Energy, number, and total angular momentum exchanged.
- Thermodynamic identities:

Spin textures

LeBlanc, Thywissen, Burkov, & Paramekanti PRA **80**, 013607 (2009).

- Spin conserved only globally
- Approach here: minimization of mean field energy functional
- Magnetization is treated (properly) as a *vector* quantity.
- Include gradient terms: beyond LDA



“Hedgehog”

“Domain wall”

Energy functional (in LDA)

A “first pass” at weakly interacting Fermions

Fermions:

$$E = \int d^3\mathbf{R} \left[\frac{3}{5}\alpha \sum_\sigma \rho_\sigma^{5/3} + g\rho_\uparrow\rho_\downarrow + V\sum_\sigma \rho_\sigma - \sum_\sigma \mu_\sigma\rho_\sigma \right]$$

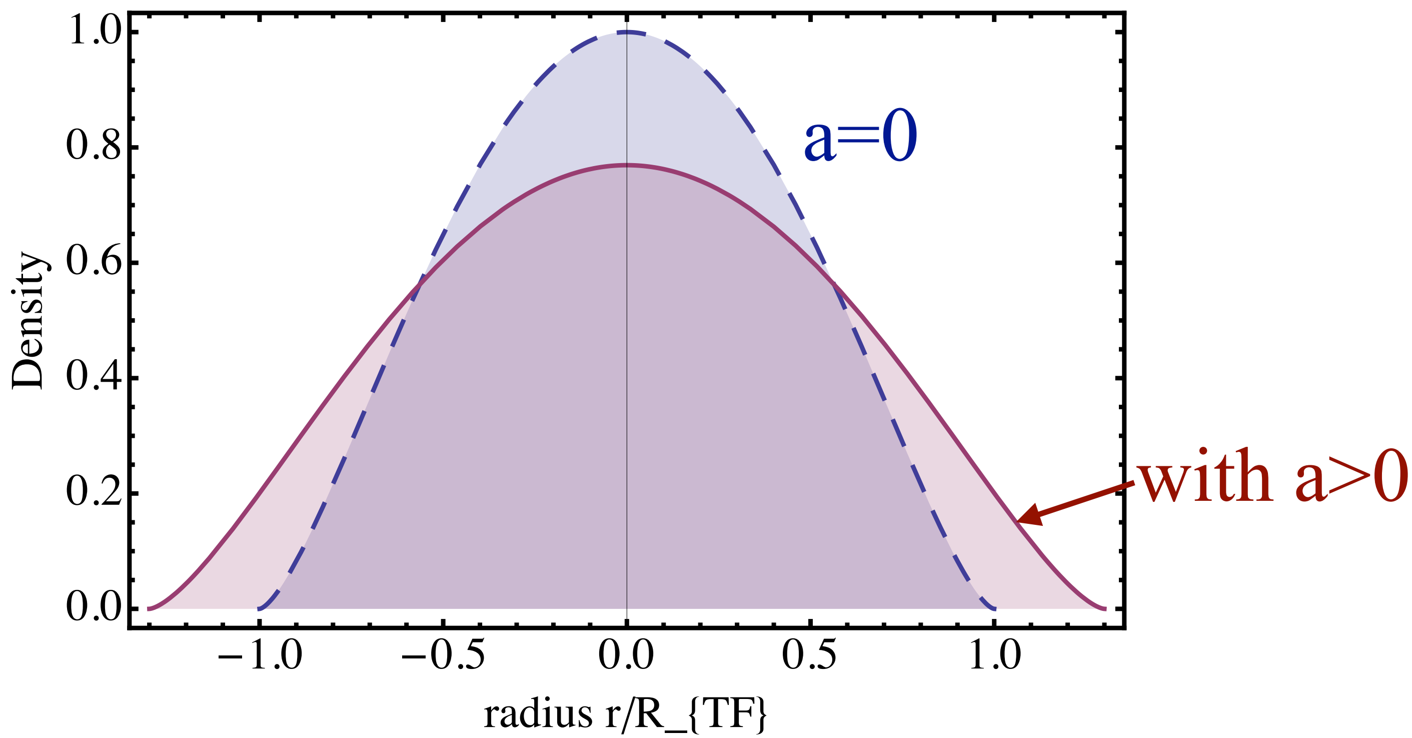
...or for equal populations everywhere:

$$E = \int d^3\mathbf{R} \left[\frac{6}{5}\alpha\rho^{5/3} + g\rho^2 + 2V\rho - 2\mu\rho \right]$$

(fixes number)

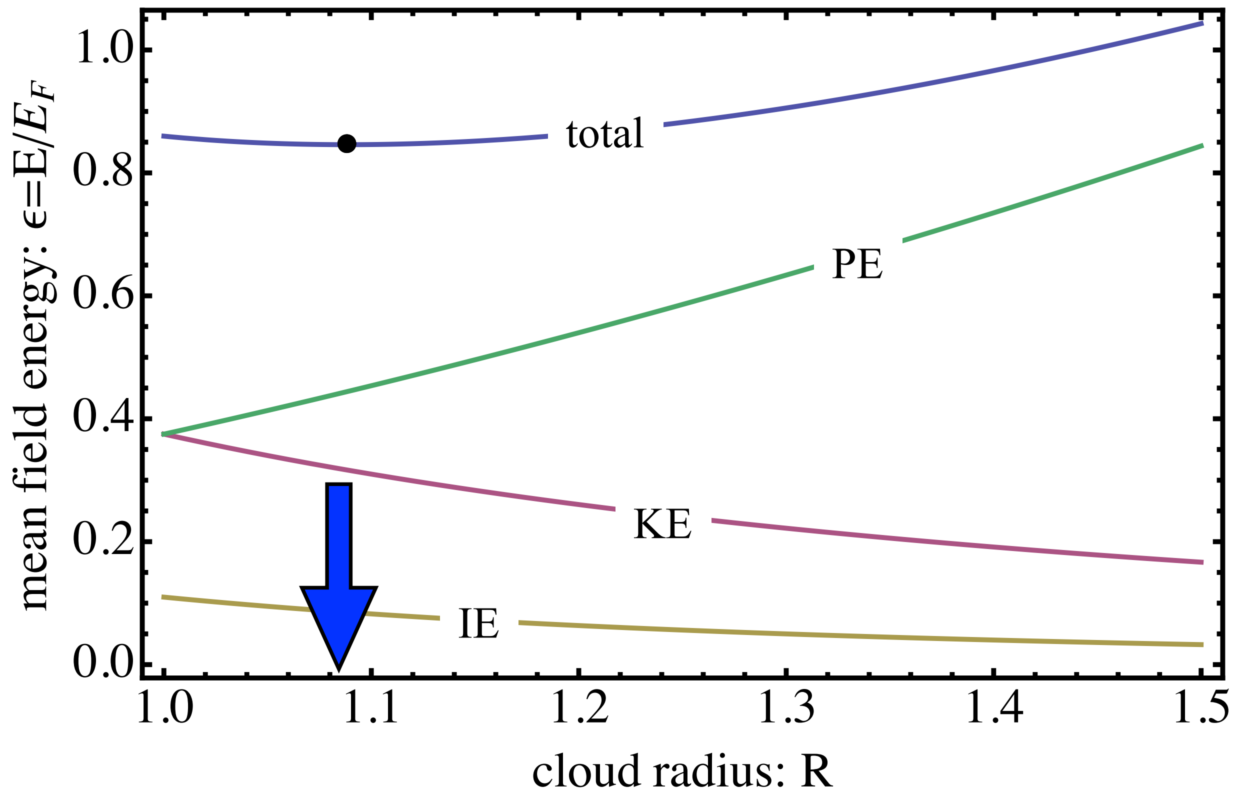
Mean field variational solution

Thomas-Fermi ansatz: $n(r) = n_0(1 - r^2/R^2)^{3/2}$



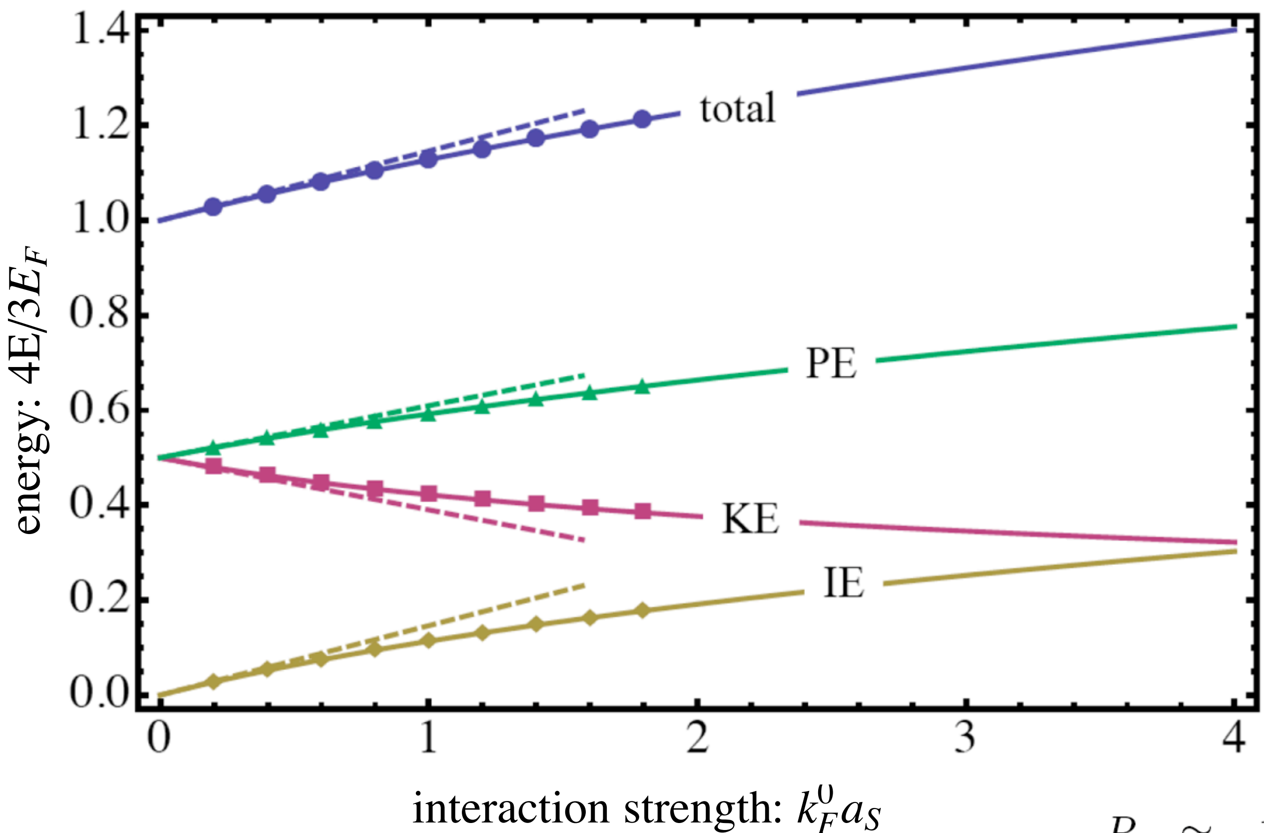
Variational soln: example of minimization

$$\varepsilon \quad = \quad A/R^2 + B(k_F^0 a_S)/R^3 + AR^2 - 1$$



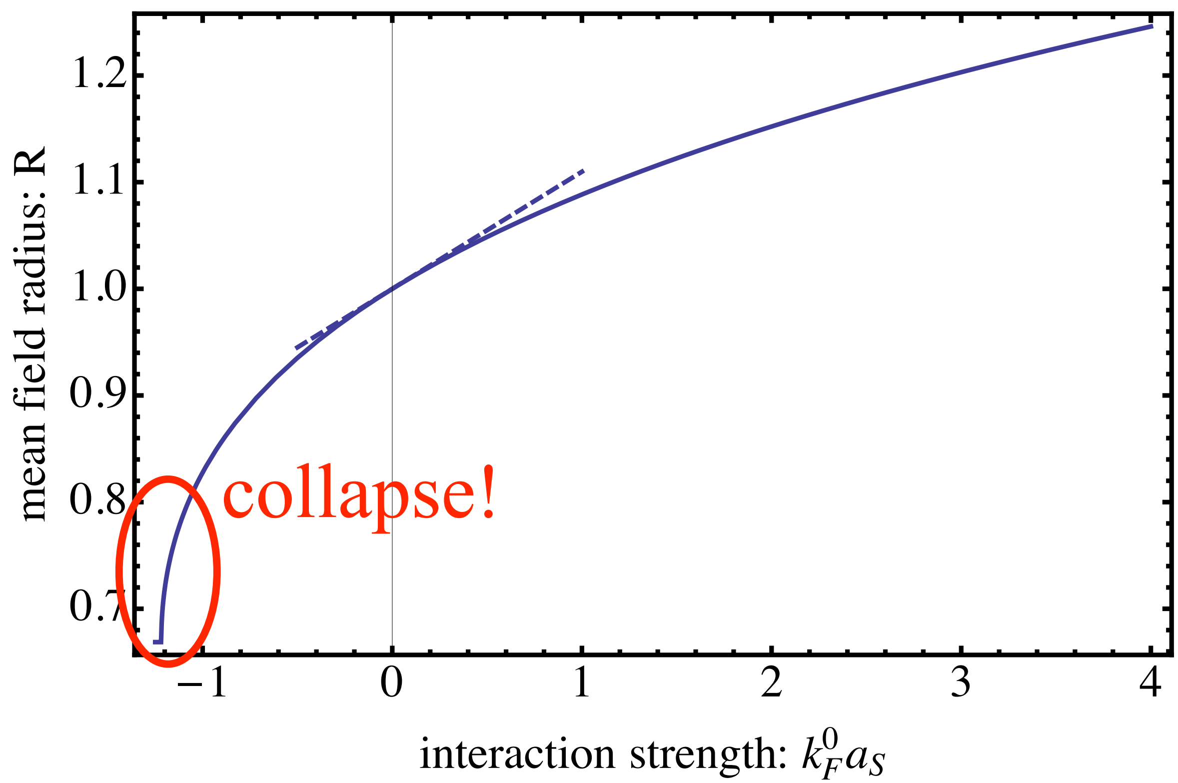
@k_Fa=1

TF ansatz vs. numerical minimization



$$
\begin{aligned}
R &\approx 1 + B k_F^0 a_S, \\
\mathrm{KE} &\approx \mathrm{KE}_0 - \frac{3}{4} B k_F^0 a_S \\
\mathrm{IE} &\approx B k_F^0 a_S \\
\mathrm{PE} &\approx \mathrm{PE}_0 + \frac{3}{4} B k_F^0 a_S
\end{aligned}
$$

Radius of TF ansatz soln

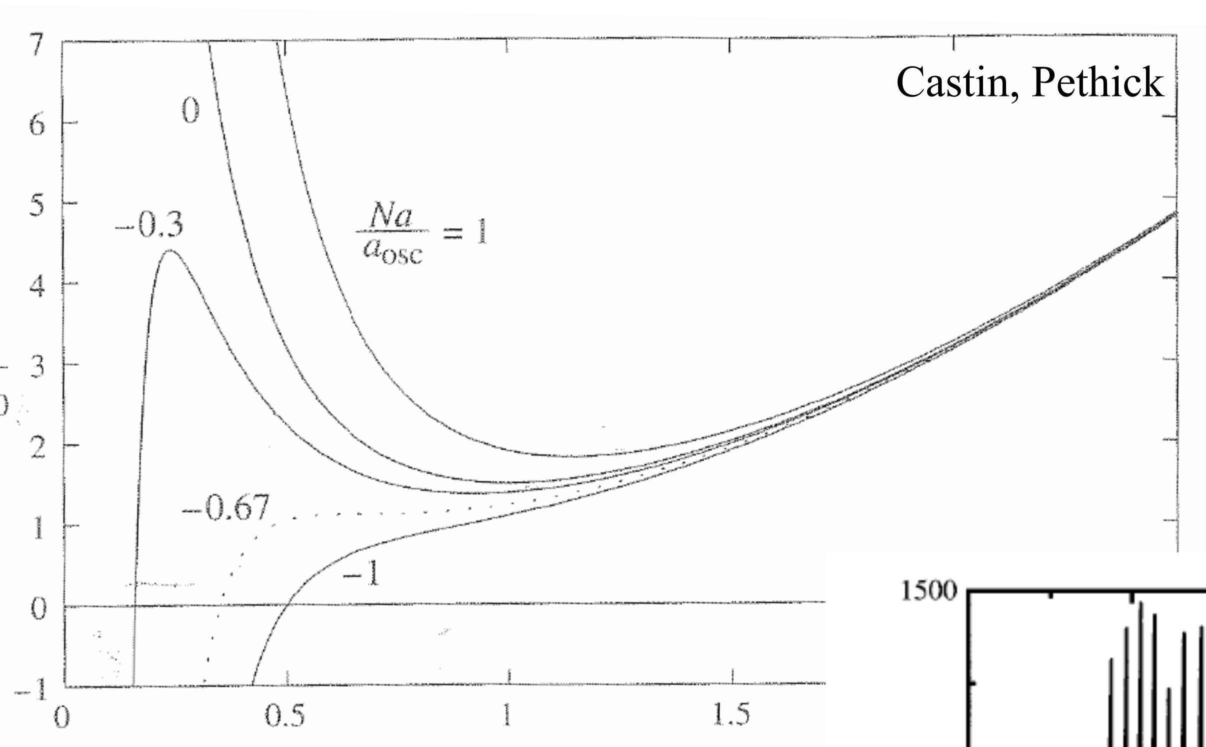


Realized in 1996. Can a cold atom superfluid be stable?

H. Stoof et al, *PRL* **76**, 10 (1996); *PRA* **56**, 4864 (1997)

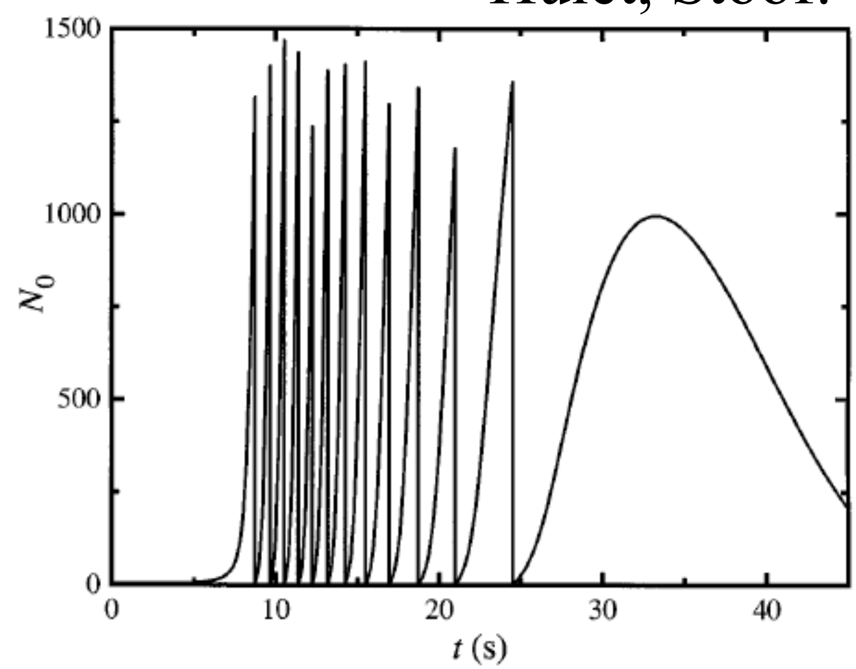
(recall collapse in attractive BECs:)

energy



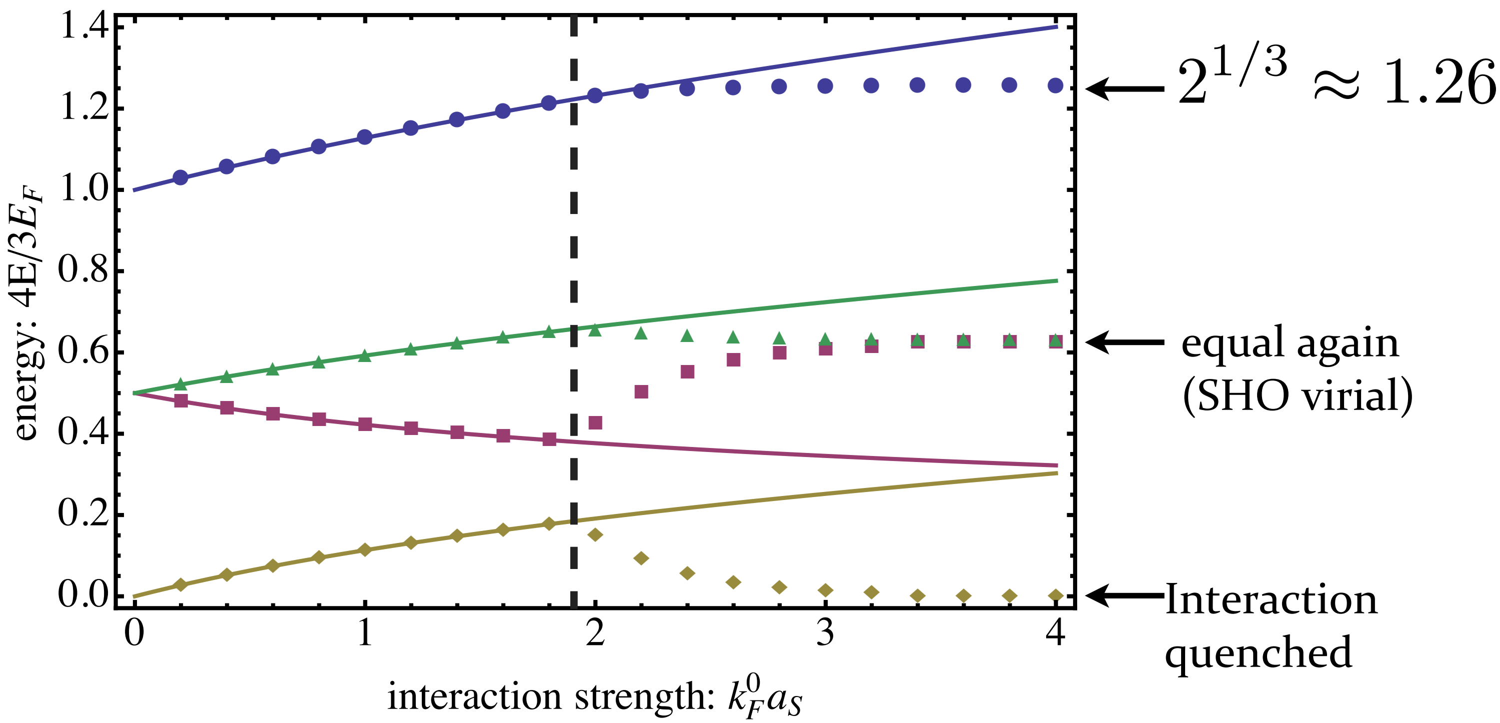
size

Hulet, Stoof:



Now allow spin degree of freedom

$$E = \int d^3\mathbf{R}\left[\frac{3}{5}\alpha\sum_\sigma \rho_\sigma^{5/3} + g\rho_\uparrow\rho_\downarrow + V\sum_\sigma \rho_\sigma - \sum_\sigma \mu_\sigma\rho_\sigma\right]$$



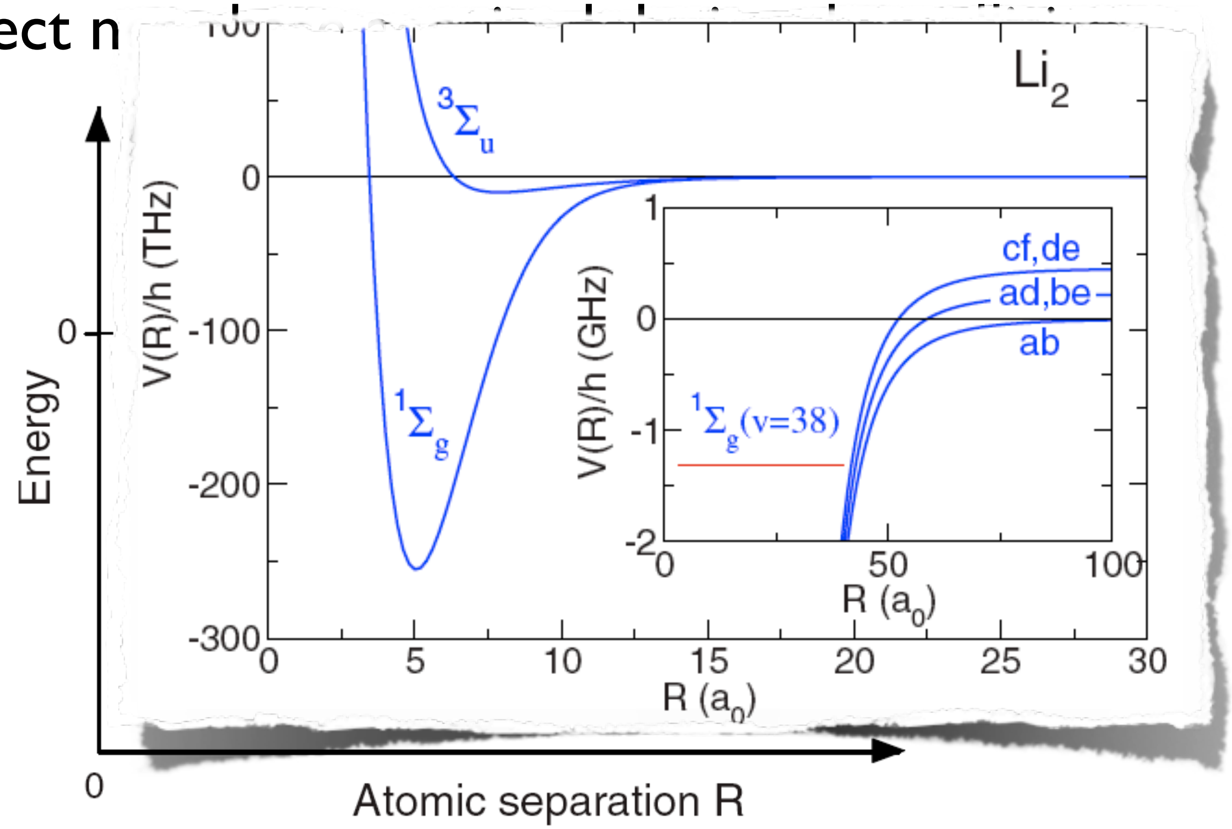
Ferromagnetic instability for repulsive gases.

Strong interactions

Feshbach resonances

*How can we tune the scattering length **a**?*

We can tune a molecular bound state into resonance with the free atoms, and affect n

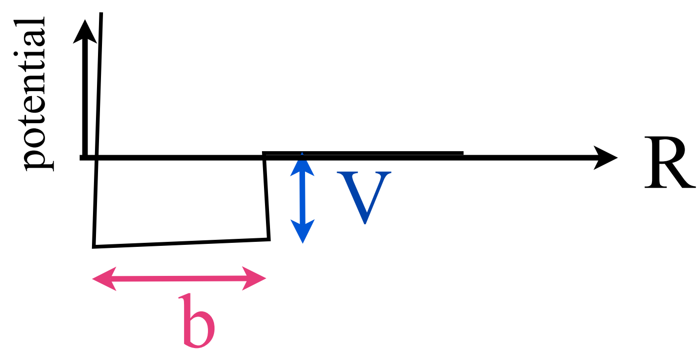


Result is indistinguishable from tuning the single-channel square well: it's only the phase that matters.

Feshbach resonances

single-channel model

Tune the square well potential & calculate a:



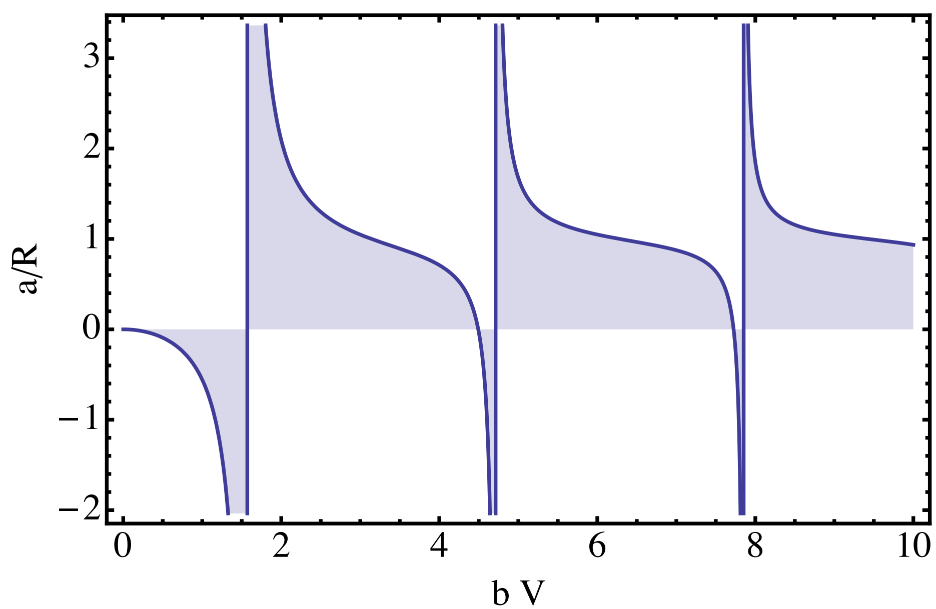
We find:

1. Resonances at

$$bV = (n + 1/2)\pi$$

when each new bound state appears.

2. Mostly a>0. Near a resonance when a<0 (eg, Li.)



Feshbach resonances

Near resonance the scattering length can be described as

$$a(B) = a_{\mathrm{bg}} \left(1 - \frac{\Delta}{B - B_0}\right)$$

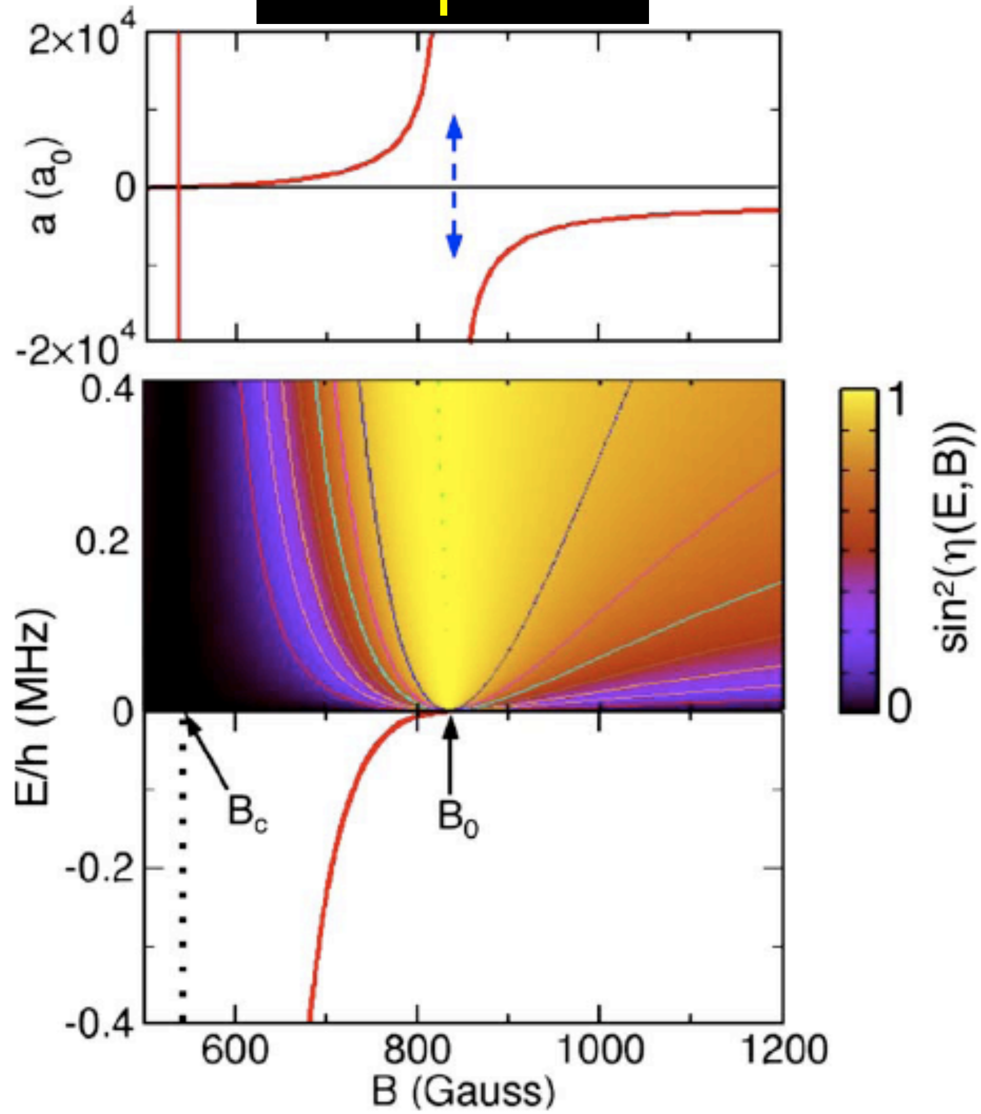
s-wave cross section is

$$\sigma_0 = \frac{4\pi}{k^2} \sin^2 \eta_0$$

For a>0, a bound state exists with binding energy

$$E_b = \frac{\hbar^2}{2\mu a^2}$$

Example: ^{6}Li



Tuning the simulation

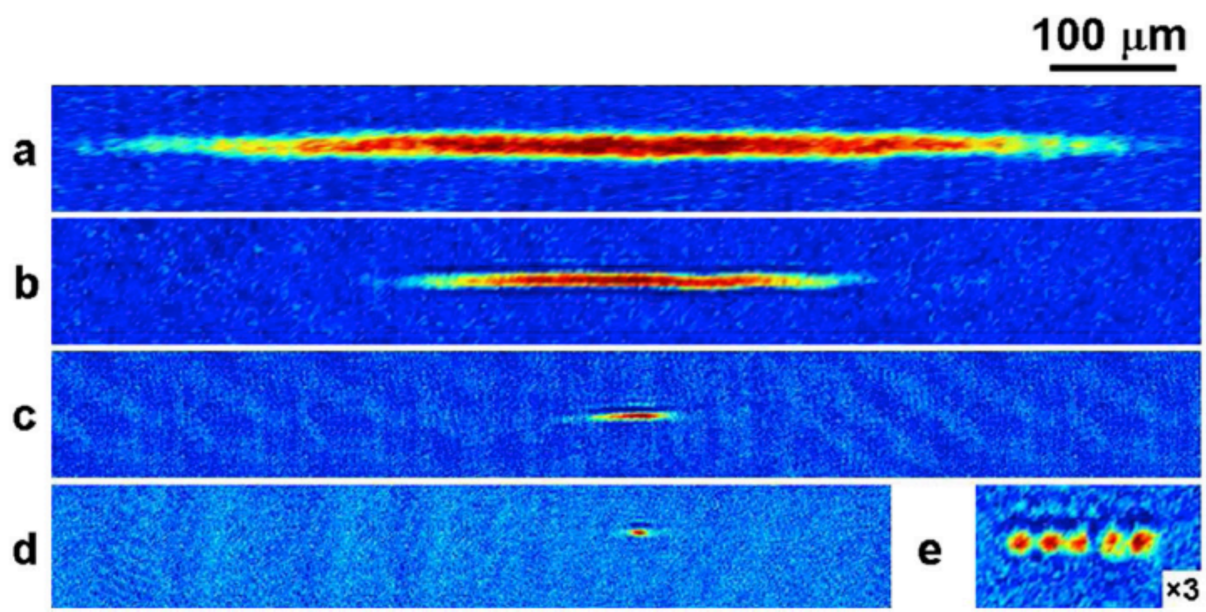
PRL 102, 090402 (2009) PHYSICAL REVIEW LETTERS week ending 6 MARCH 2009

Extreme Tunability of Interactions in a ^{7}Li Bose-Einstein Condensate

S. E. Pollack, D. Dries, M. Junker,* Y. P. Chen,† T. A. Corcovilos, and R. G. Hulet
Department of Physics and Astronomy and Rice Quantum Institute, Rice University, Houston, Texas 77005, USA
(Received 26 November 2008; published 6 March 2009)

Trapped atom clouds:

stronger interaction "g"



Tuning the simulation

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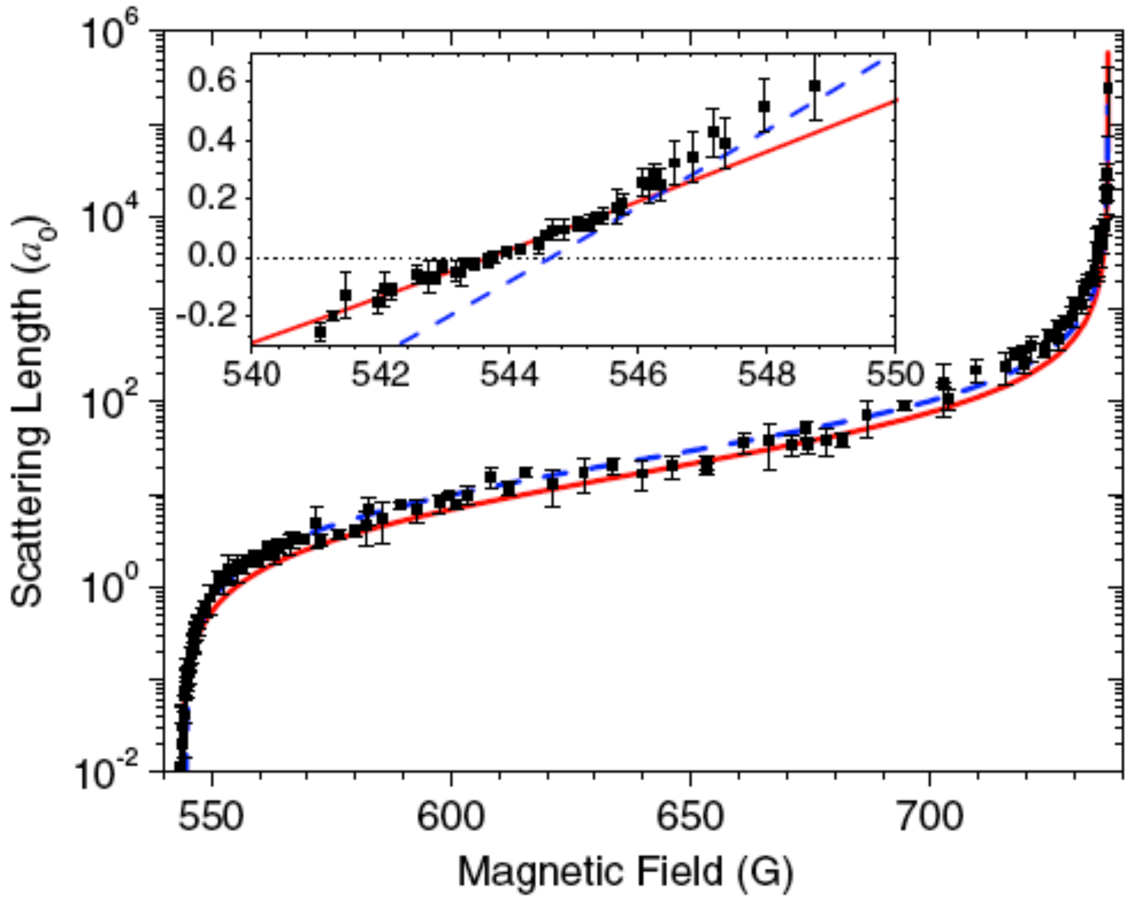
Department of Physics and Astronomy and Rice Quantum Institute, Rice University, Houston, Texas 77005, USA
(Received 26 November 2008; published 6 March 2009)

interaction strength "g"



Unitarity

Near a Feshbach resonance, |a| diverges. The scattering cross section departs from its low-ka form:

$$\sigma = \frac{4\pi a^2}{1 + k^2 a^2} \rightarrow \frac{4\pi}{k^2}$$

This is just a manifestation of the optical theorem, which says that complete reflection corresponds to a finite scattering length. In terms of the de Broglie wavelength,

$$\sigma_{\rm res} = \lambda_{dB}^2/\pi$$

You may be more familiar with the resonant atom-photon cross section (which has different constants because it is a vector instead of scalar field):

$$\sigma_{\rm res} = \frac{3}{2\pi}\lambda_L^2$$

Unitarity

For a many-body system, resonant interactions also saturate but are less easy to quantify. Certainly it is the case that a divergent **a** can no longer be a relevant physical quantity to the problem.

For fermions, the only remaining length scale is k_F^{-1}.

This means that interaction energies must scale with the Fermi E. In particular, for resonant attractive interactions,

$$\mu_{\text{Local}} = (1+\beta)\epsilon_F$$

where $\beta \approx -0.58$ has been measured in various experiments. Using the LDA to integrate over the profile, we find

$$\begin{aligned} \mu_U &= \sqrt{1+\beta} E_{\text{F}} \\ &\approx 0.65 E_{\text{F}} \end{aligned}$$

for $a \longrightarrow -\infty$

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can n ... n.

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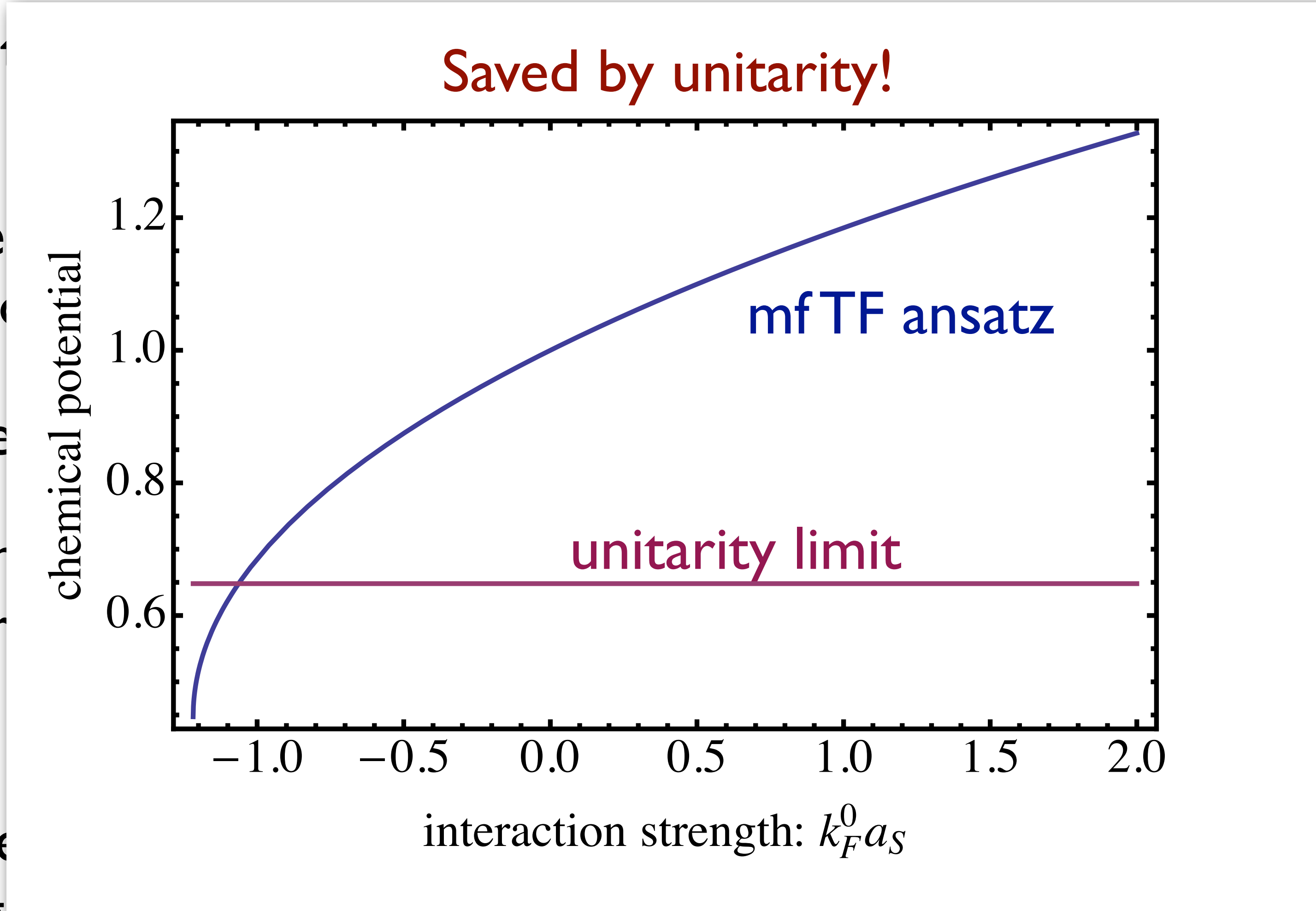


where ... ts.
Using the LDA to integrate over the profile, we find

$$\begin{aligned}\mu_U &= \sqrt{1+\beta}E_{\mathrm{F}} \\ &\approx 0.65 E_{\mathrm{F}}\end{aligned}$$

for $a \longrightarrow -\infty$

Length scales (in traps, @Feshbach res.)

- inter-atomic potential range, r_0: 2 nm
- thermal de Broglie wavelength: 100 nm
- average inter-particle spacing: 100 nm
  -same length scale as $1/k_F$
- scattering length, a
  -at Feshbach resonance: divergent
- ground state width:
  1μm @ 100Hz (typ. magnetic trap)
- cloud size: 1-100 μm

Length scales (in traps, @Feshbach, T=0)

- inter-atomic potential range, r_0: 2 nm
- average inter-particle spacing: 100 nm
  -same length scale as $1/k_F$
- scattering length, a
  -at Feshbach resonance: divergent
- ground state width:
  1μm @ 100Hz (typ. magnetic trap)
- cloud size: 1-100 μm

Only one length scale left in the problem! "Universal"

Tan's relations

Define the "contact", an unknown parameter (to be measured):

$$C \equiv \lim_{k \to \infty} k^4 n(k)$$

1. Adiabatic sweep theorem

$$2\pi \frac{dE}{d[-1/(k_F a)]} = C$$

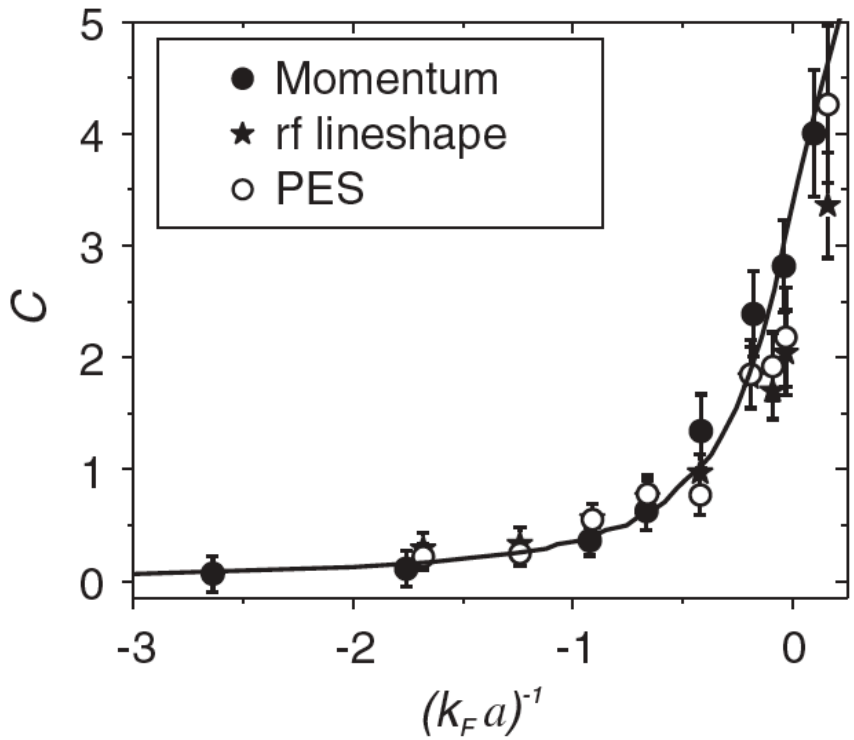
2. Generalized Virial theorem

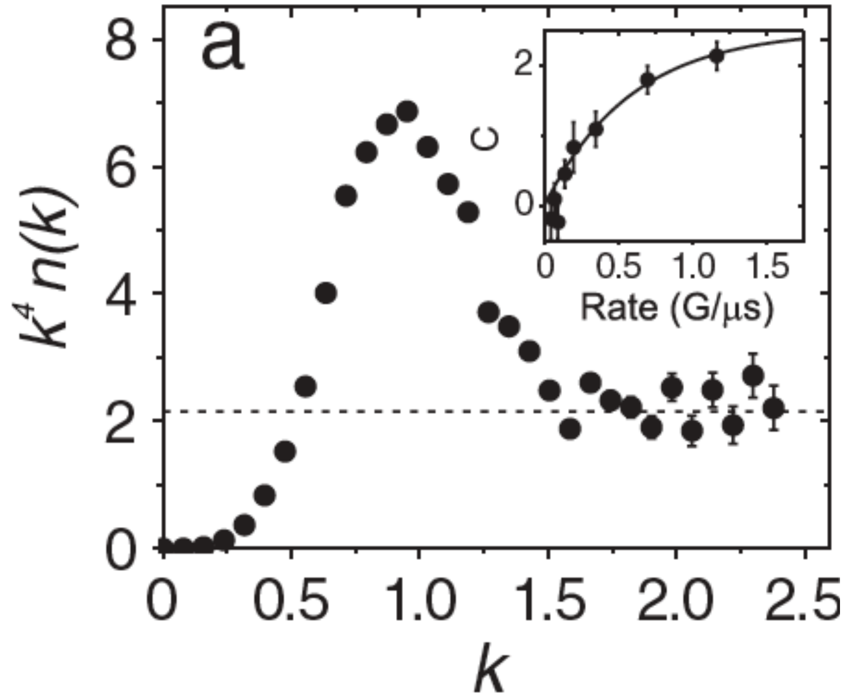
$$E - 2V = T + I - V = -\frac{C}{4\pi k_F a}$$

Tan's relations

Measure the contact:

$$C = \lim_{k\to\infty} k^4 n(k)$$





J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, *Phys. Rev. Lett.* **104**, 235301 (2010)

J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin,
Phys. Rev. Lett. **104**, 235301 (2010)

Tan's relations

Adiabatic sweep theorem:

$$2\pi \frac{dE}{d[-1/(k_F a)]} = C$$

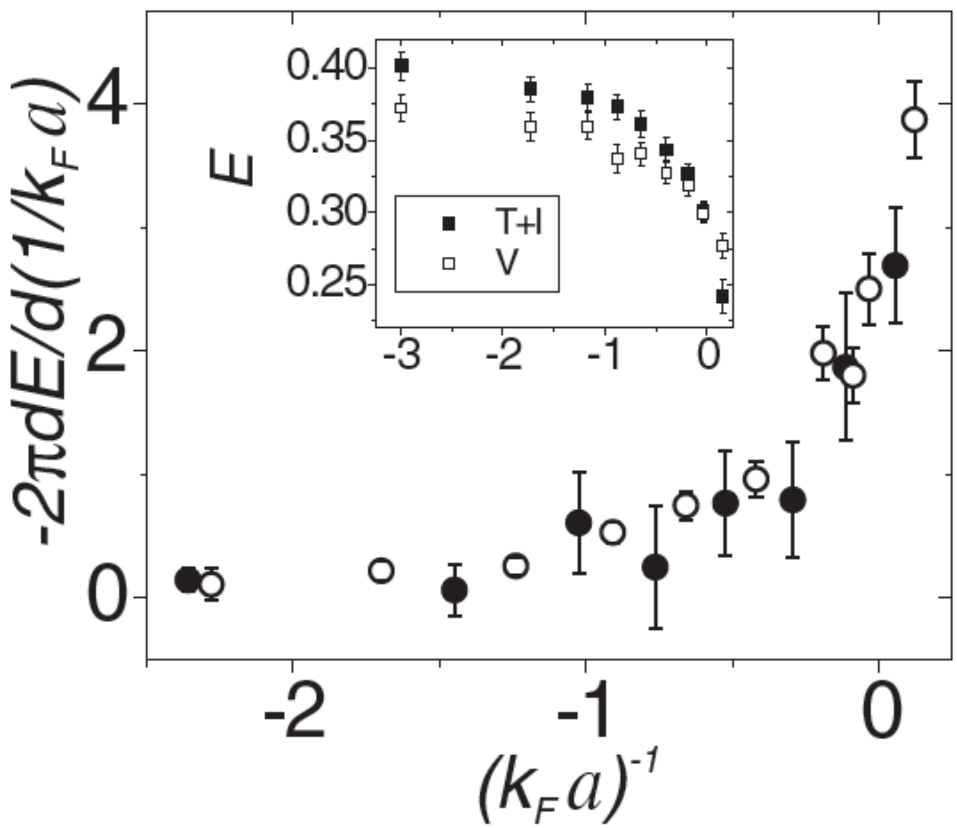


FIG. 3. *Testing the adiabatic sweep theorem.* (Inset) The measured potential energy, V, and release energy, $T + I$, per particle in units of E_F are shown as a function of $1/k_F a$. (Main) Taking a discrete derivative of the inset data, we find that $2\pi \frac{dE}{d[-1/(k_F a)]}$ (•) agrees well with the average value of C obtained from the measurements shown in Fig. 2 (∘).

Tan’s relations

Generalized Virial theorem

$$T + I - V = -\frac{C}{4\pi k_F a}$$

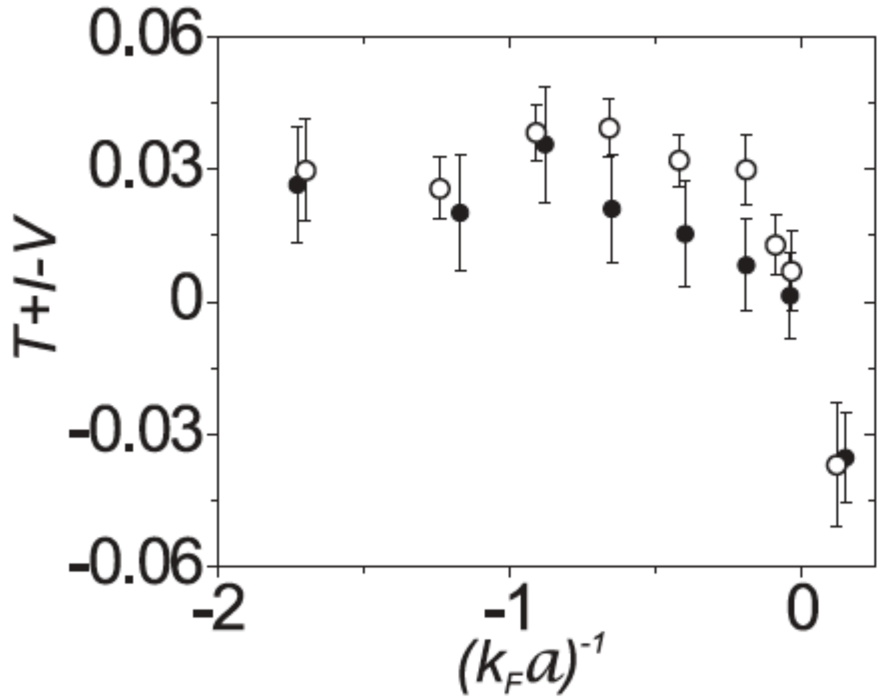


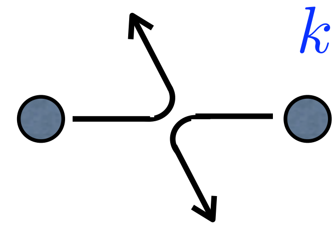
FIG. 4. *Testing the generalized virial theorem.* The difference between the measured release energy and potential energy per particle $T + I - V$ is shown as filled circles. This corresponds to the left-hand side of Eq. (3). Open circles show the right-hand side of Eq. (3) obtained from the average values of the contact shown in Fig. 2. The two quantities are equal to within the measurement uncertainty.

J.T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, *Phys. Rev. Lett.* **104**, 235301 (2010)

How could the Stoner model fail?

- At some point, shouldn't interactions be strong enough to make spin alignment energetically favourable?
- Unfortunately interactions can only be so strong.

Recall scattering theory:



$$k = 2\pi/\lambda_{\mathrm{dB}}$$

Cross-section

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0(k)$$

Contact potential:

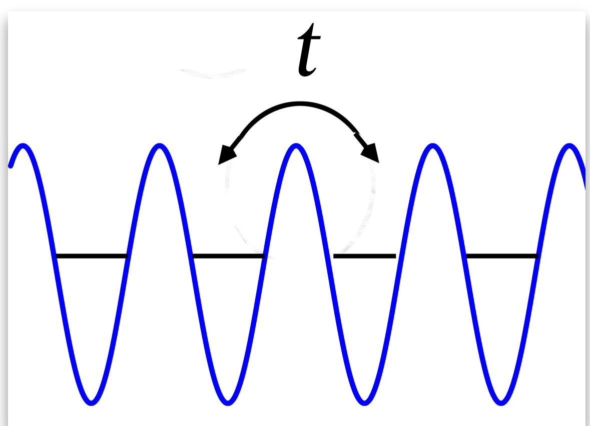
$$f_{\vec{k}} = -[1/a + ik]^{-1}$$

$$\boxed{\sigma_0 = \frac{4\pi a^2}{1 + a^2 k^2} \to \frac{4\pi}{k^2}}$$

"Unitarity limit:" Can't do more than reflect back. In fact, resonant scattering of a wave always has a cross-section of lambda squared!

Effect of a lattice

- Why would a lattice favour FM?
  -flattens bands, lowering EF
- Proofs of FM involving a lattice



See, for example: H. Tasaki, *Prog Theor. Phys.* **99**, 489 (1998)

For every problem, there is a simple, elegant solution....

...which is wrong.

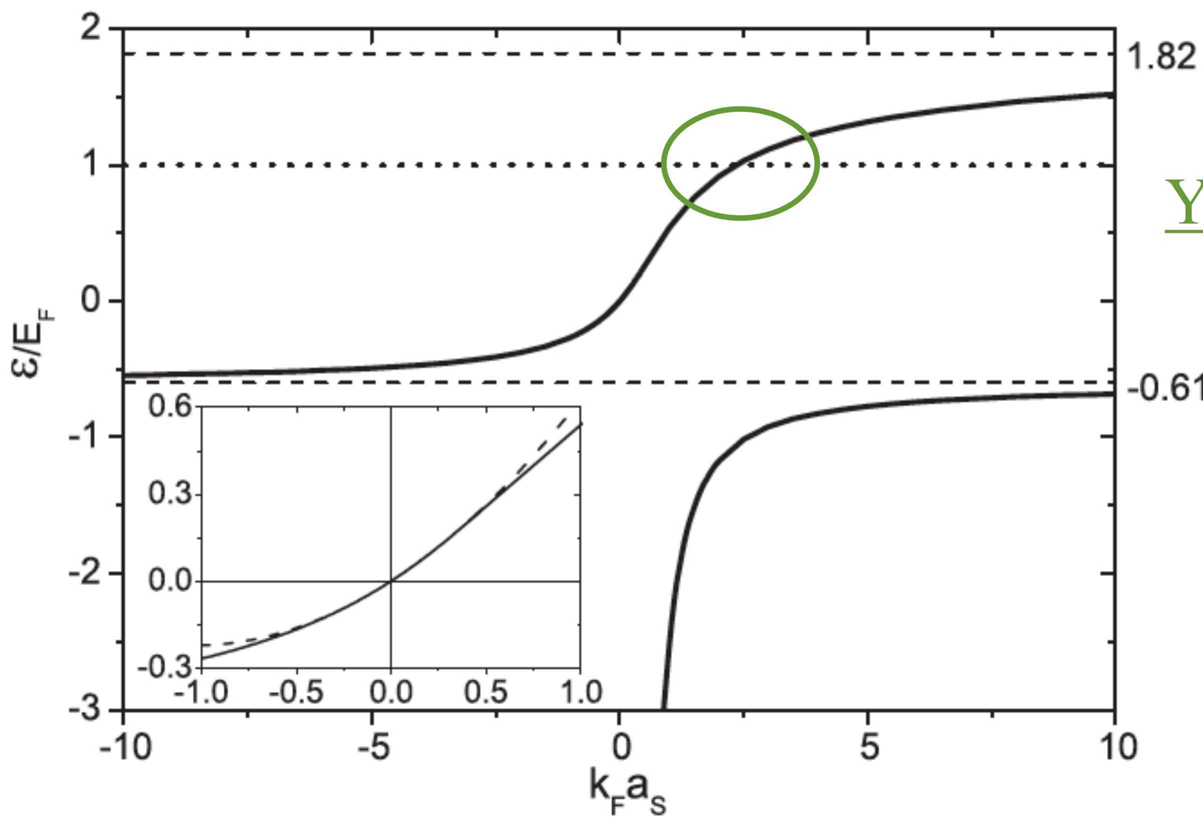
- Nowhere is this more true than in CM physics!
- Stoner model does *not* lead to FM in one dimension (1D) -Lieb (1962)
- No proof to date about 2D or 3D

Stability of a fully magnetized ferromagnetic state in repulsively interacting ultracold Fermi gases.
Xiaoling Cui and Hui Zhai, *Phys. Rev. A* **81** (4) 041602 (2010)

Polaron energy

- Wavefunction for a single flipped spin:

$$|\Psi\rangle = \left(\phi_0 c^\dagger_{\mathbf{q_0}\downarrow} + \sum_{\mathbf{k}>\mathbf{k}_F,\mathbf{q}<\mathbf{k}_F} \phi_{\mathbf{kq}} c^\dagger_{\mathbf{q_0}+\mathbf{q}-\mathbf{k}\uparrow} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\uparrow} + \cdots \right) |N\rangle$$



Yes, FM stable for $k_F a > 2.4$

Zwerger's proof

M. Barth and W. Zwerger, arXiv: 1101.5594 (2011)
"Tan Relations in One Dimension"

- Construct a variational wavefunction for a fully mixed state

$$\langle \{z_\ell\} | \Psi_{var} \rangle = |\det \phi_\ell(\{z_\ell\})| \, |S=0\rangle$$

- Want to compare its energy E_{var} with energy of ideal fermi gas E_F, whose wavefunction is:

$$\langle \{z_\ell\} | \Psi^{(F)} \rangle = \det \phi_\ell(\{z_\ell\}) |S=N/2\rangle$$

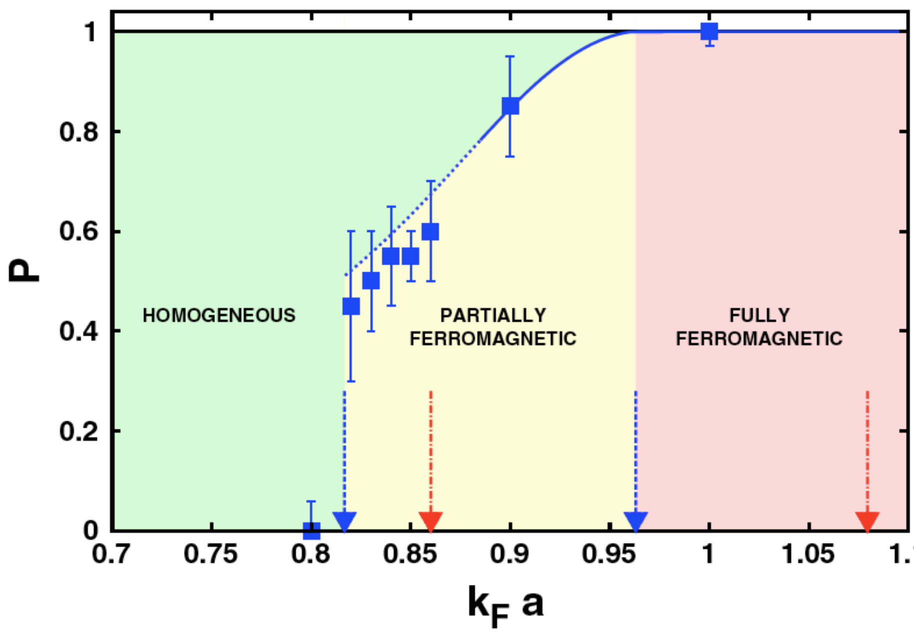
- However know that fermionized Bose gas, with same spatial wf, has energy E_{Tonks}.

1D: $E_{\rm mix} \leq E_{\rm var} = E_{\rm Tonks} = E_{\rm DFG} = E_{\rm pol}$ Lieb & Matthis

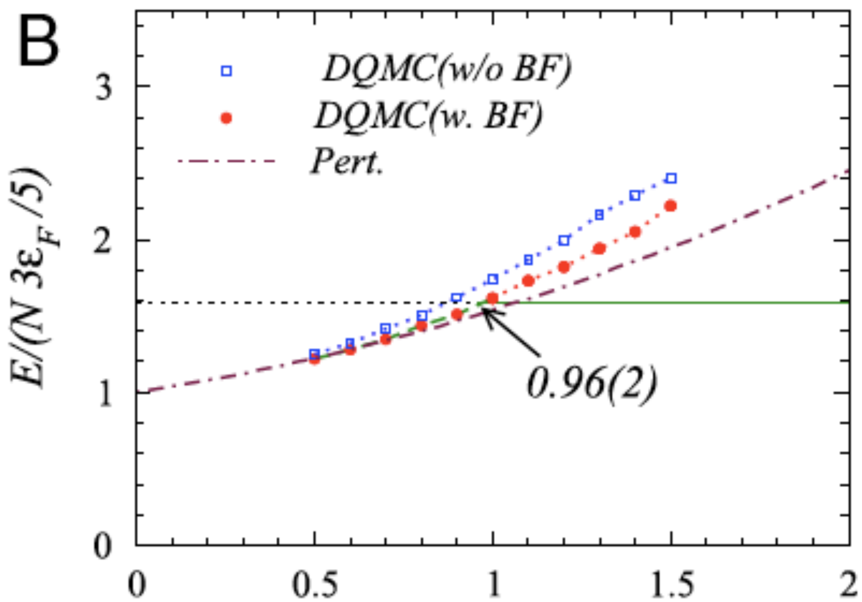
3D: $E_{\rm mix} \leq E_{\rm var} = E_{\rm Tonks} = 0.8 E_{\rm DFG} < E_{\rm pol}$ F. Zhou (UBC)

No FM in upper branch.

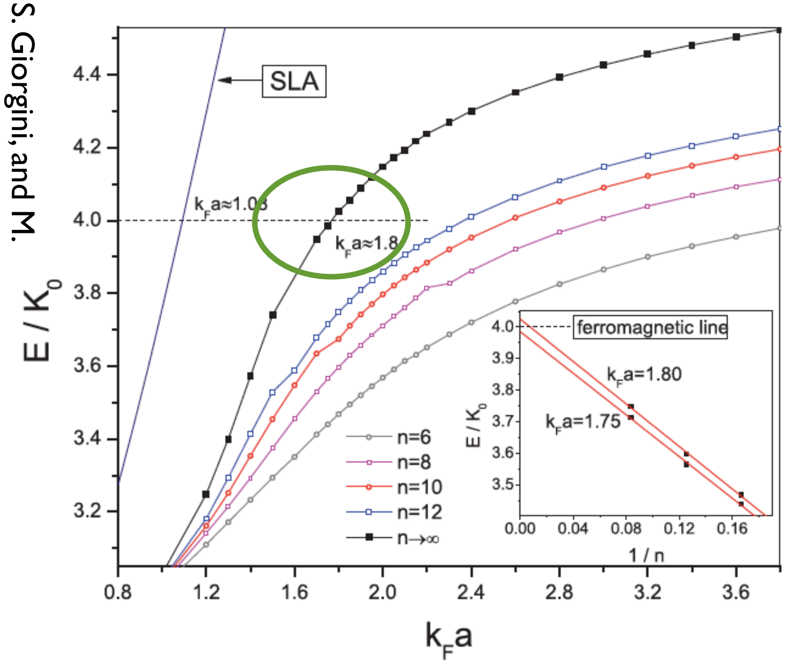
Quantum Monte Carlo



S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, *Phys. Rev. Lett.* **105**, 030405 (2010)



S.-Y. Chang, M. Randeria, N. Trivedi, PNAS **108**, 51 (2011)

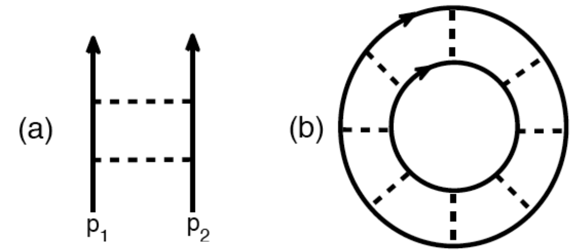


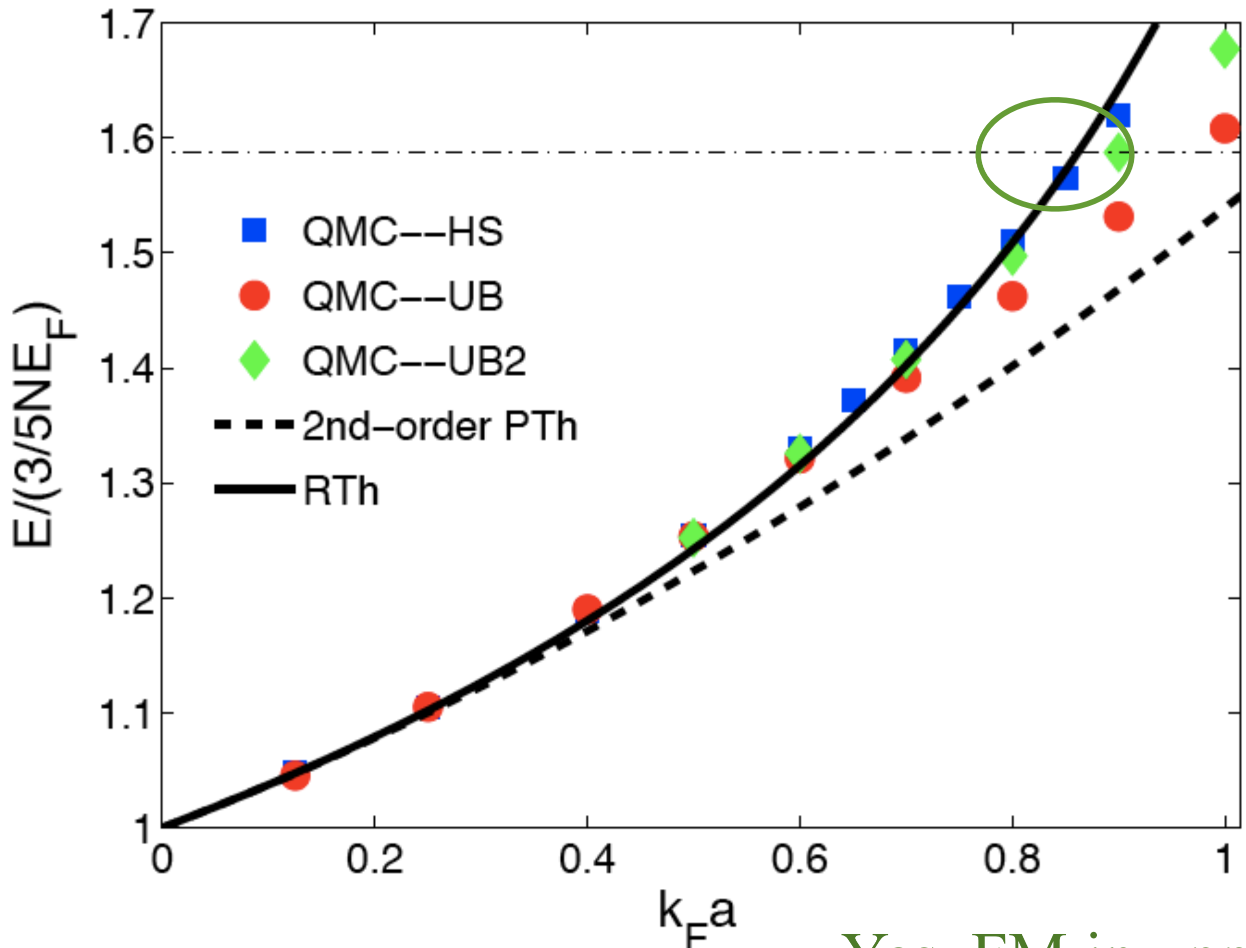
S. Q. Zhou, D. M. Ceperley, and S. Zhang, Phys. Rev. A **84**, 013625 (2011)

Yes, FM in upper branch.

Non-perturbative field theory

Lianyi He and Xu-Guang Huang, "Non-Perturbative Prediction of the Ferromagnetic Transition in Repulsive Fermi Gases." [arXiv:1106.1345v2]



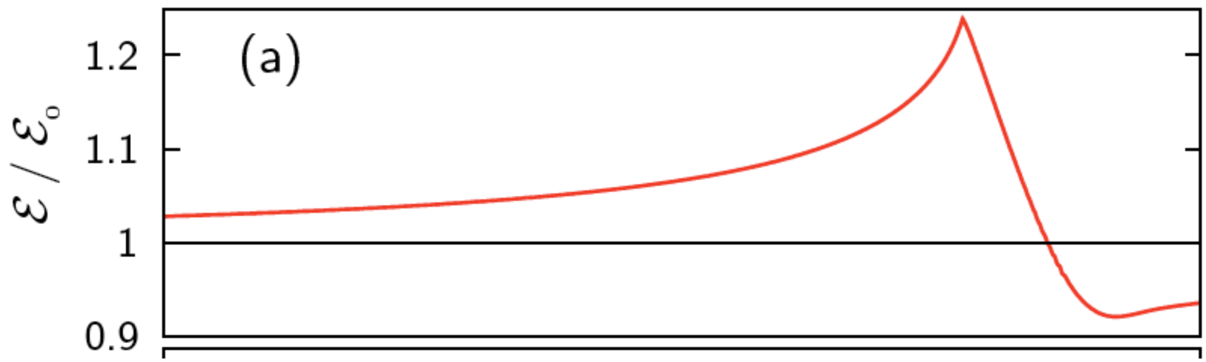


Yes, FM in upper branch (for $k_F a$>0.9)

Nozieres-Schmidt-Rink

The Nature and Properties of a Repulsive Fermi Gas in the "Upper Branch"
Vijay B. Shenoy, Tin-Lun Ho, *Phys. Rev. Lett.* **107**, 210401 (2011)

No FM in upper branch.



How to explain a deviation from Tan's relations?

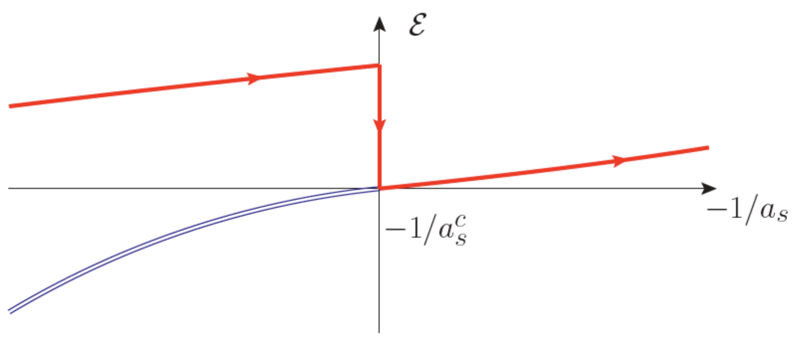
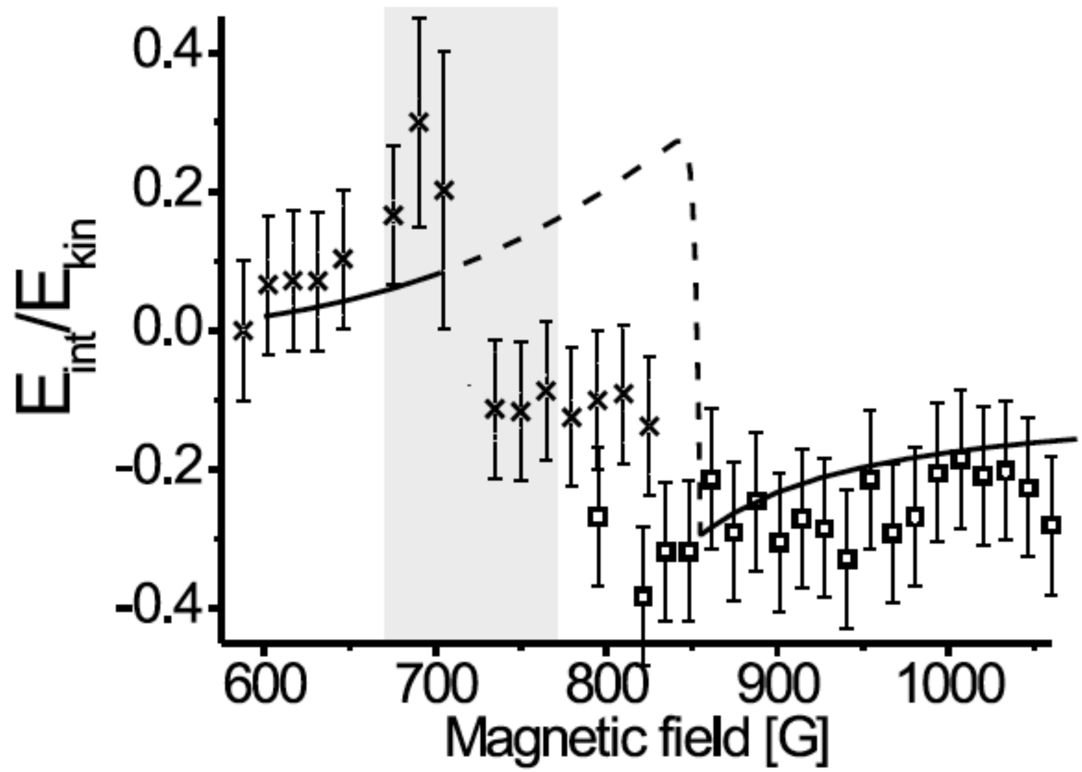


FIG. 4. (Color online) The discontinuous change of the energy of the scattering state (solid line) of a two body system up on the disappearance of the molecular bound state (double line). [10] Similar phenomena occur in a q-dependent fashion in the many body setting.

Dynamics & instabilities

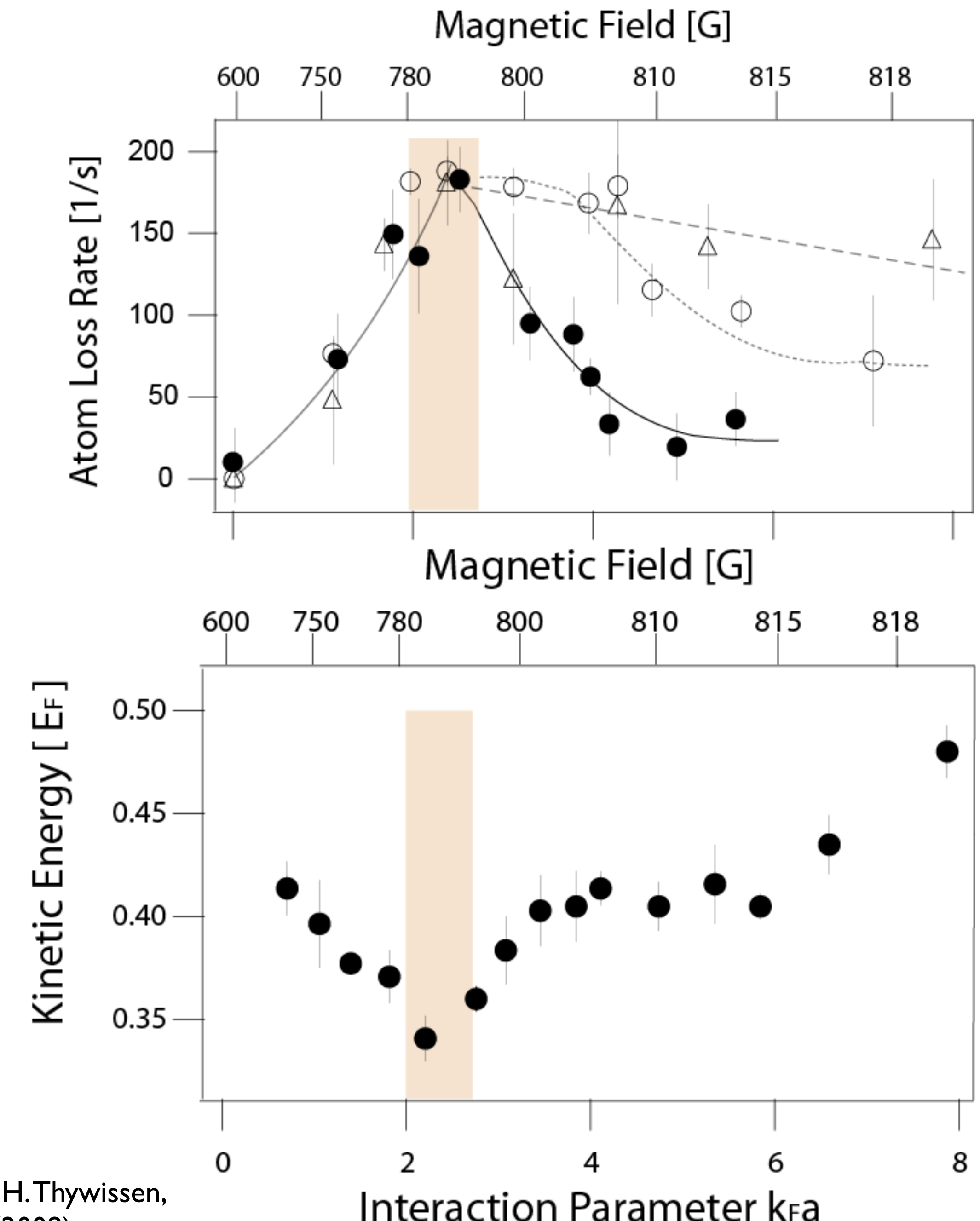
Experiment?

- ENS Paris 2003:
  $T=0.6T_F$
  ^{6}Li



- Generally: problem is 3-body loss near Feshbach resonance, on repulsive side.
  -short lifetime
  -adiabaticity difficult
  -situation complicated by molecules

Experimental Observations



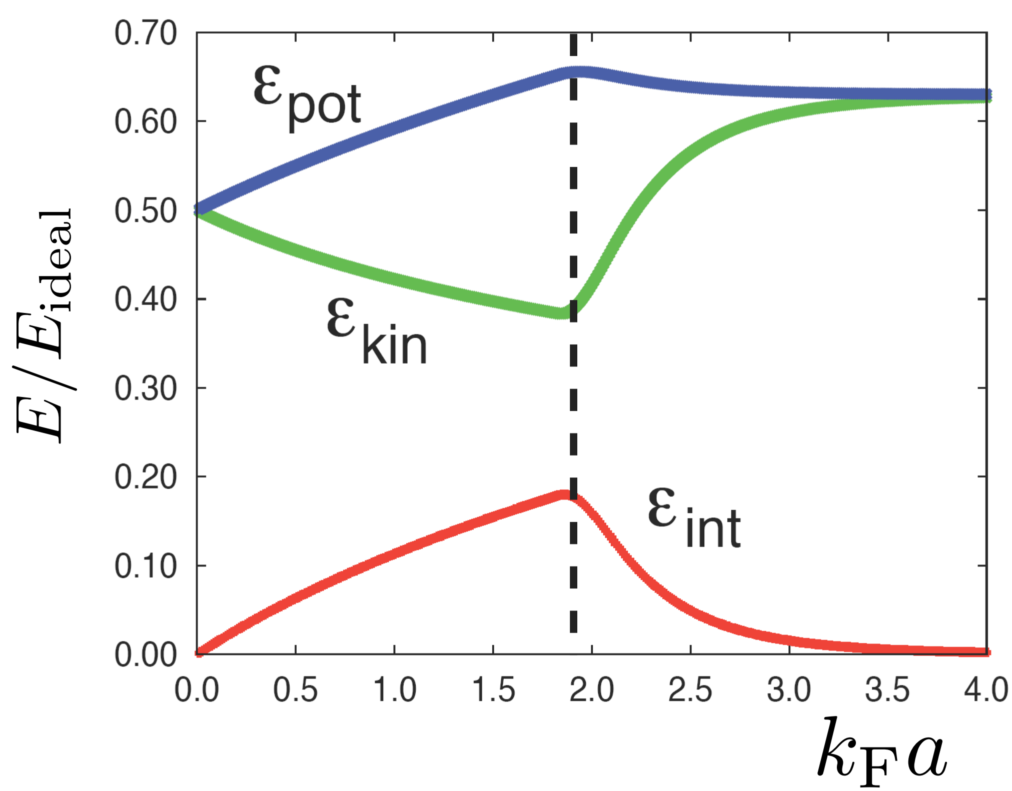
ferromagnetism!

G B Jo, Y. R. Lee, J. H. Choi, C. Christiensen, H. Kim, J. H. Thywissen, D. Pritchard, W. Ketterle, Science **325**, 1521 (2009)

Mean field energetics

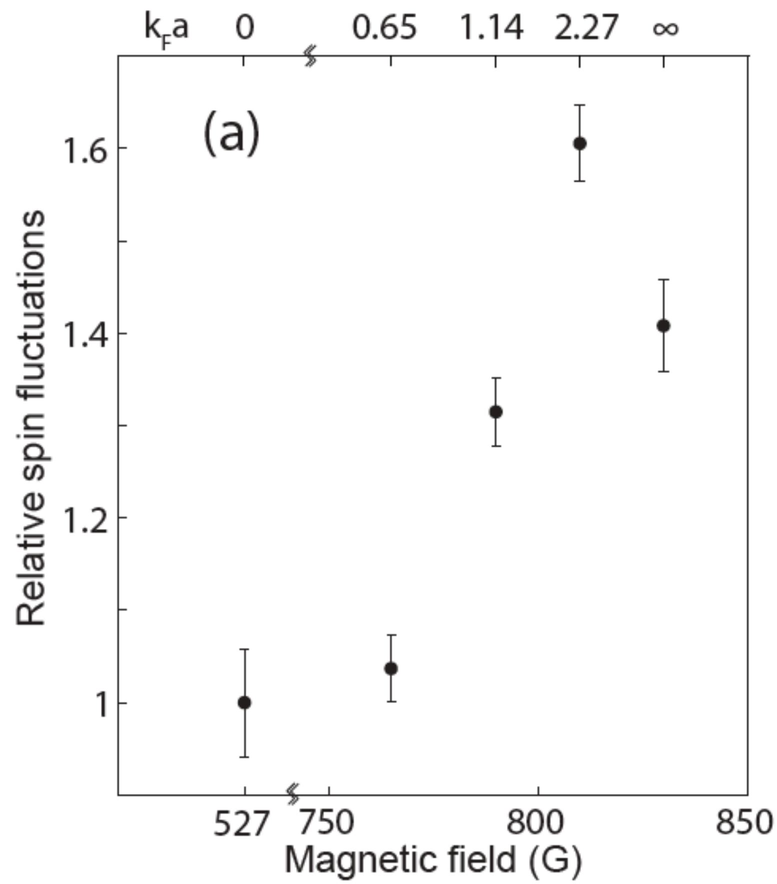
$$E[\{\rho_\sigma(\mathbf{r})\}] = \int d^3\mathbf{r} \left[\underbrace{\frac{3}{5}\sum_\sigma \frac{\hbar^2(6\pi^2\rho_\sigma)^{2/3}}{2m}\rho_\sigma(\mathbf{r})}_{\text{kinetic energy, like } \frac{\hbar^2 k_F^2(\mathbf{r})}{2m}} + \underbrace{V(\mathbf{r})\sum_\sigma \rho_\sigma(\mathbf{r})}_{\text{potential energy}} + \underbrace{g\rho_\uparrow(\mathbf{r})\rho_\downarrow(\mathbf{r})}_{\text{interaction energy } g=\frac{4\pi a\hbar^2}{m}} \right]$$

numerical minimization



No divergence in magnetic susceptibility.

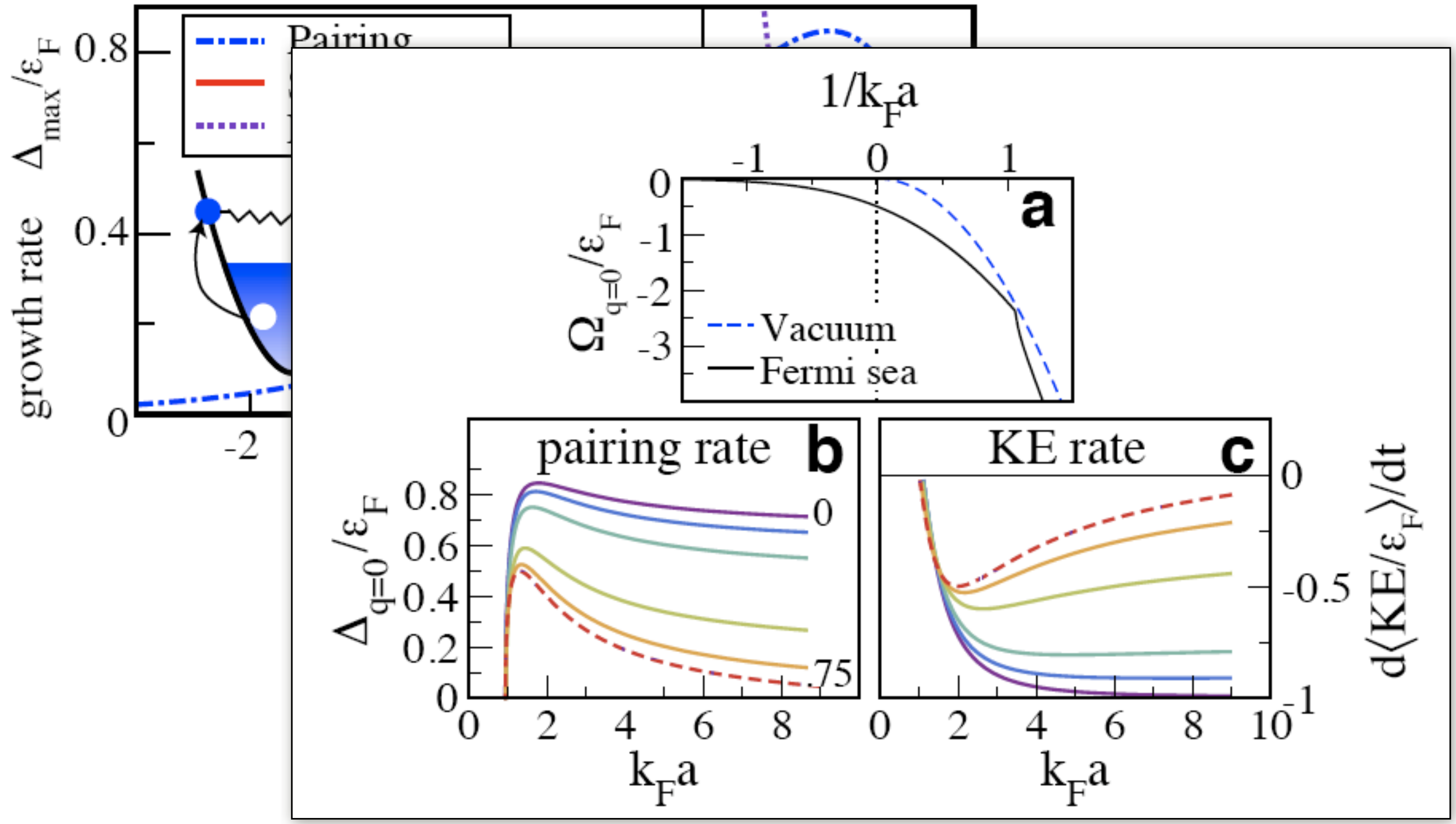
-> No FM.



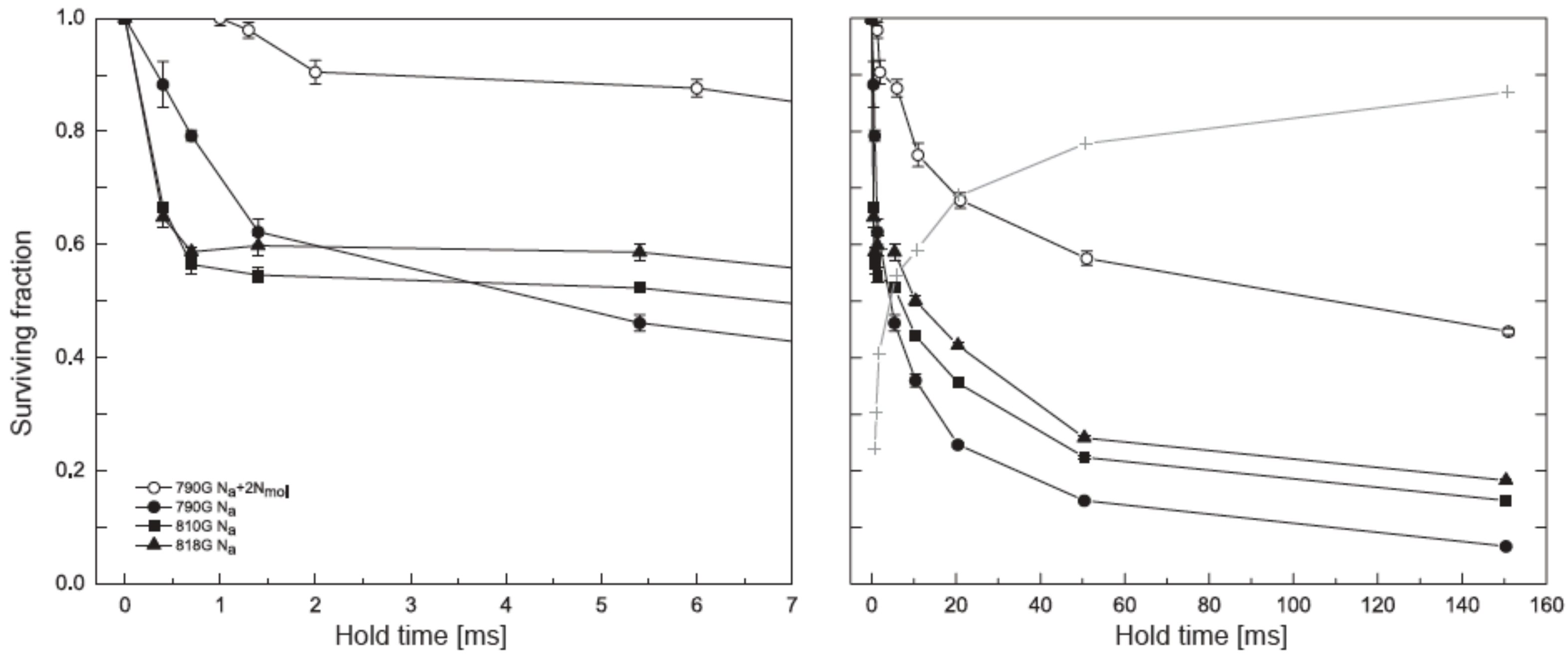
C. Sanner, E. J. Su, W. Huang, A. Keshet, J. Gillen, and W. Ketterle, *arXiv: 1108.2017*

Competitive instabilities

Pekker...Demler: "Competition between pairing and ferromagnetic instabilities in ultracold Fermi gases near Feshbach resonances" Phys. Rev. Lett. **106**, 050402 (2011)



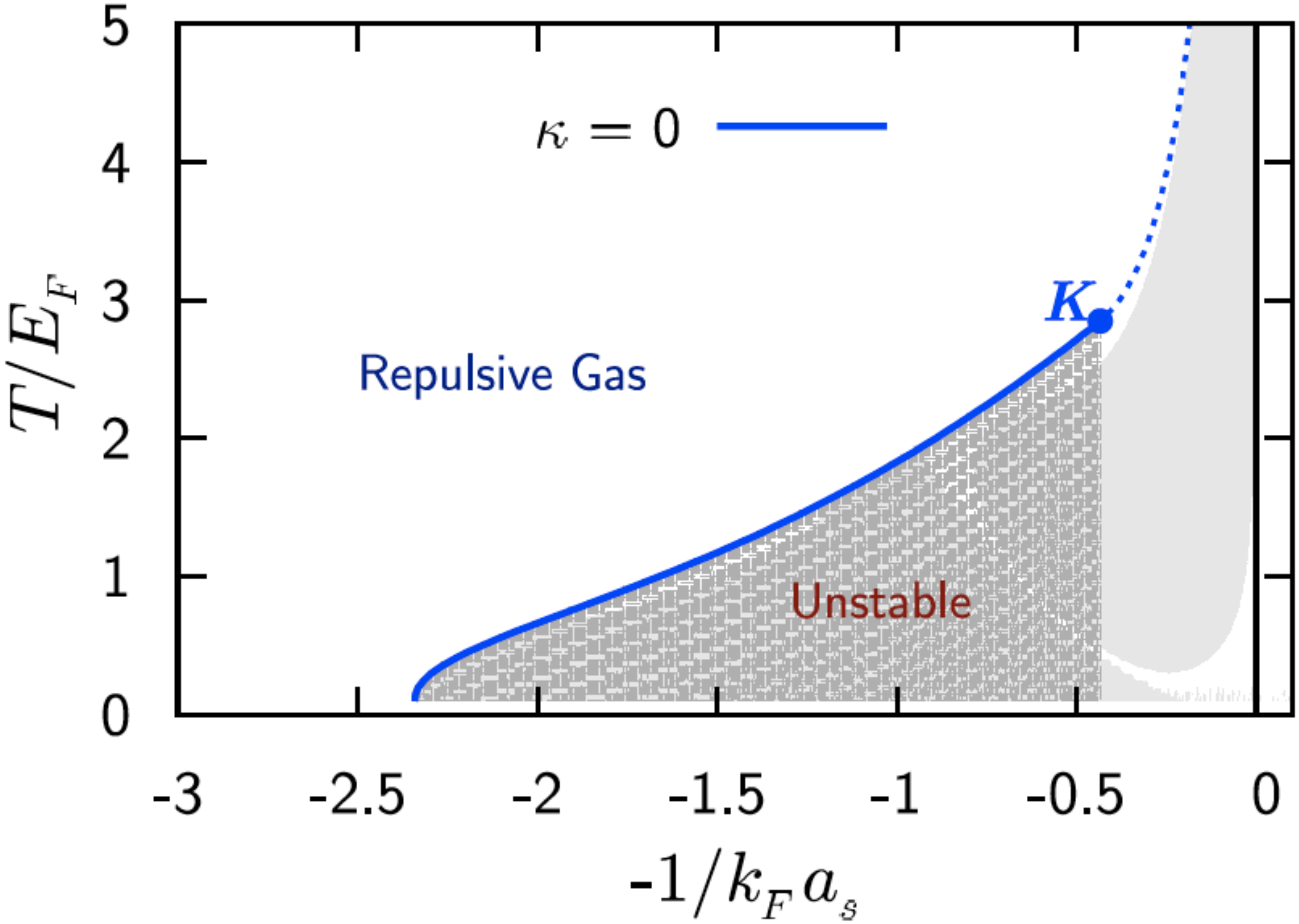
Fast molecular loss observed.



"The fast formation of molecules and the accompanying heating makes it impossible to study such a gas in equilibrium [...] Therefore, Nature does not realize a stronglyrepulsive Fermi gas with short range interaction, and the widely used Stoner model is unphysical."

C. Sanner, E. J. Su, W. Huang, A. Keshet, J. Gillen, and W. Ketterle, *arXiv: 1108.2017*

The Nature and Properties of a Repulsive Fermi Gas in the "Upper Branch"
Vijay B. Shenoy, Tin-Lun Ho, *Phys. Rev. Lett.* **107**, 210401 (2011)



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Experiments discussed in these lectures:

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•G B Jo, Y. R. Lee, J. H. Choi, C. Christiensen, H. Kim, J. H. Thywissen, D. Pritchard, W. Ketterle, Science **325**, 1521 (2009).
•see also 'news&views' by W Zwerger, *Science* **325**, 1508 (2009)
•J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, *Phys. Rev. Lett.* **104**, 235301 (2010)
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Theoretical work on (& related to) putative ferromagnetism in cold atoms

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Theoretical work on (& related to) putative ferromagnetism in cold atoms (green=yes, FM; red=no FM)

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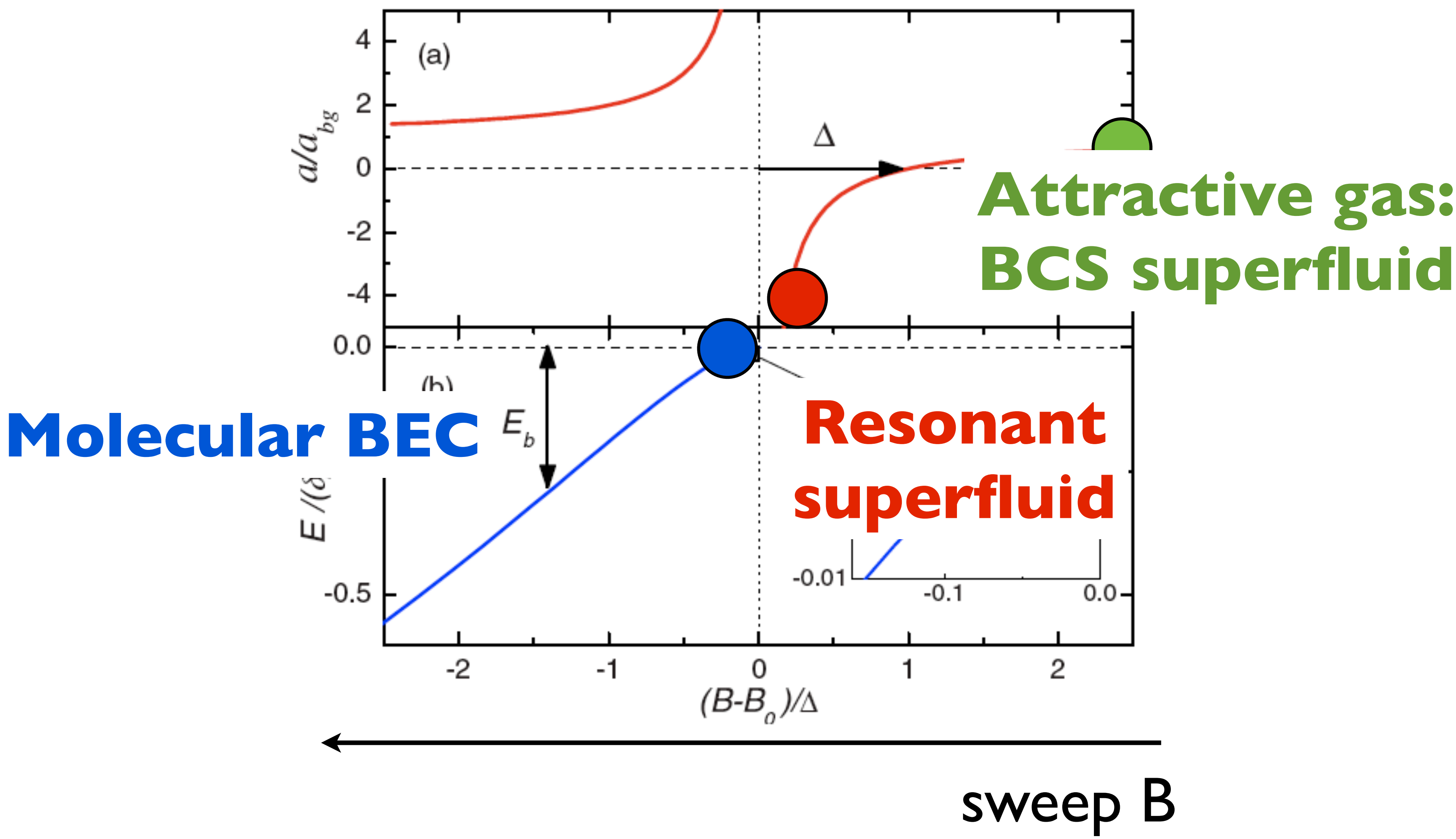
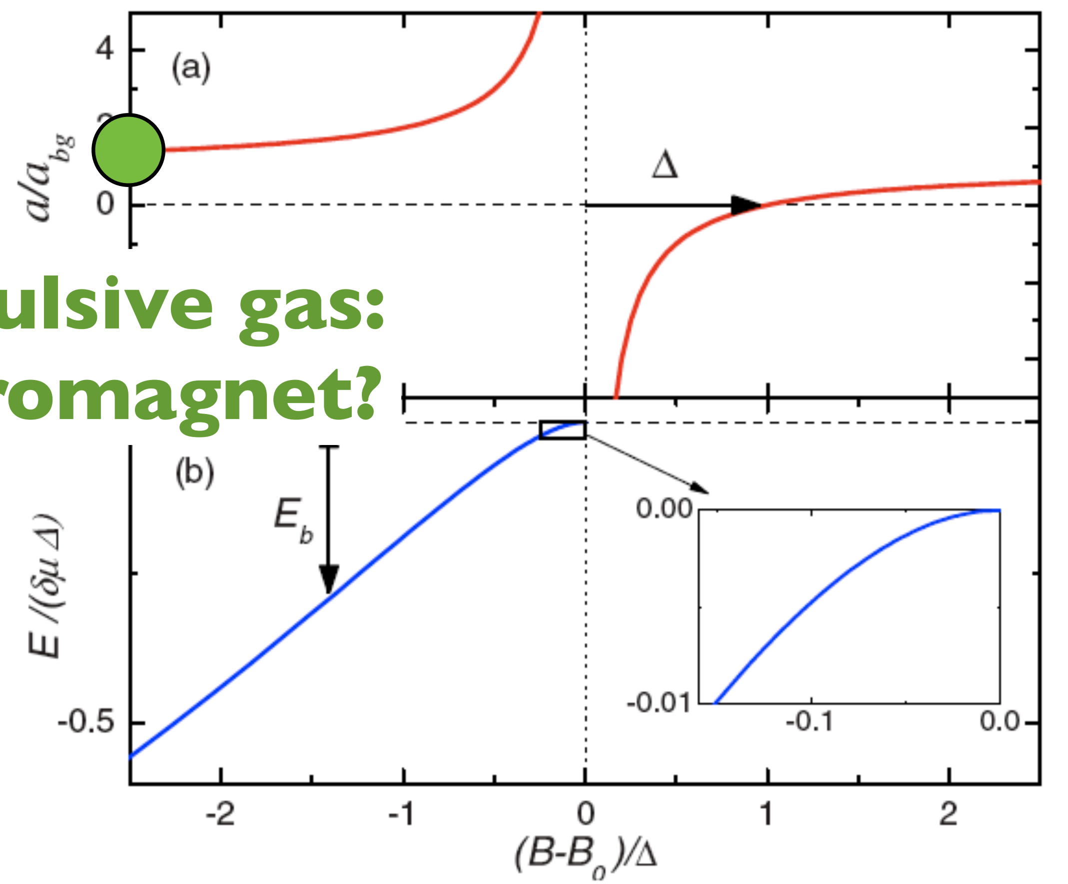
BEC-BCS crossover



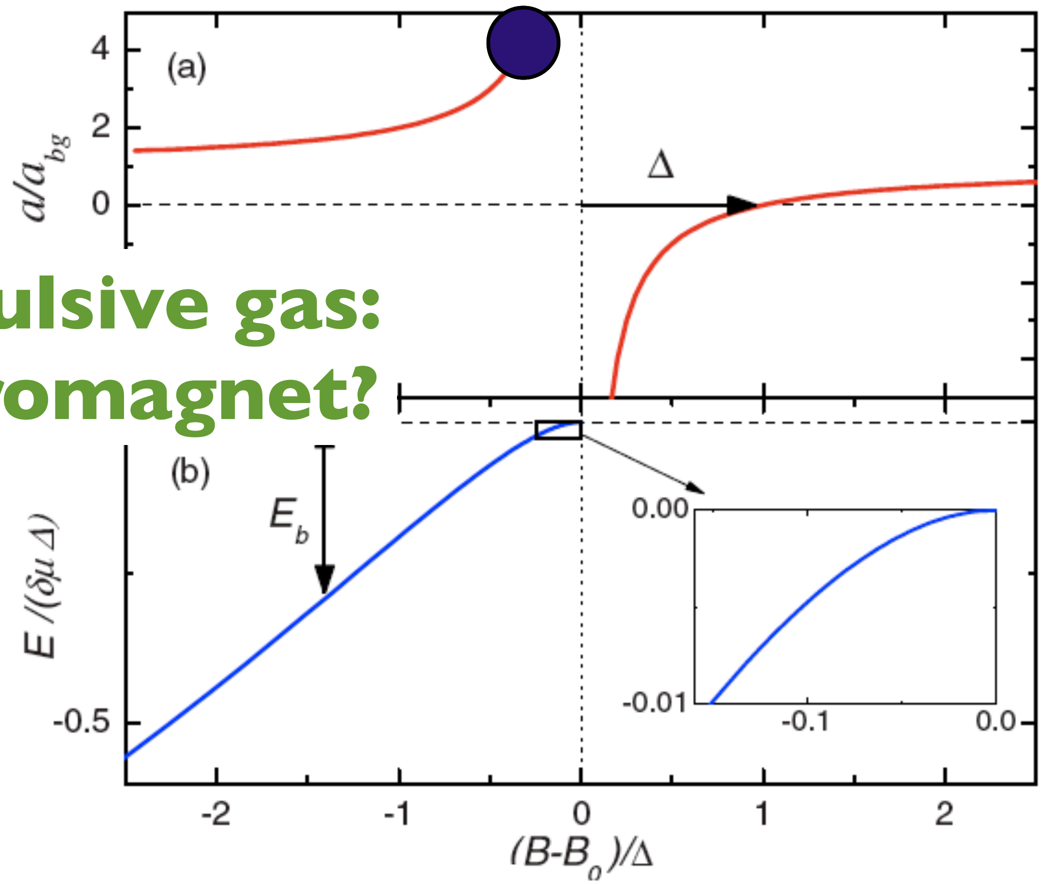
Strongly repulsive Fermi gas



Repulsive gas: Ferromagnet?

sweep B

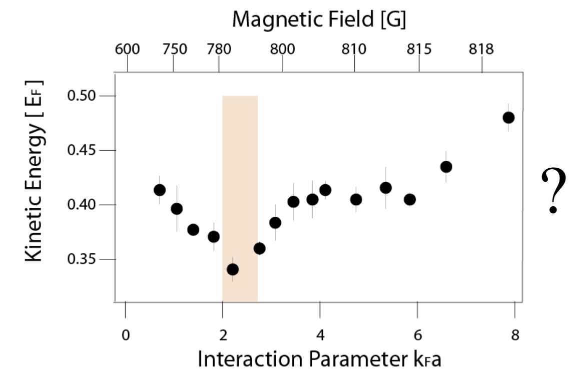
Strongly repulsive Fermi gas



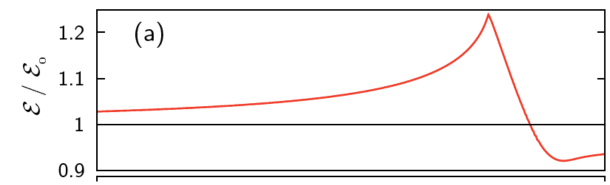
Repulsive gas: Ferromagnet?

or just molecular decay?

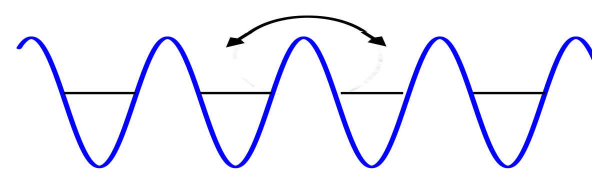
Conclusions



Dynamics essential to understand
-Metastability of ferromagnetic state is weak
-might dynamics give us a clue to equilibrium physics?
-breakdown of "effective Hamiltonian"
-connections to spintronics & transport



Controversy in theoretical analysis of ferromagnetism
-controversy continues in 2011
-few meaningful experimental results



Routes to increased stability?
-lower dimensionality
-weak lattice
-spin imbalance

More questions than answers! Oldest discussion in condensed matter physics continues.

Group members & collaborators

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Thank you!

<u>Collaborators:</u>
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OLE collaboration
Optical Lattice MURI consortium