

Ultracold atoms: spin orbit coupling and engineered interactions

I. B. Spielman

Current team

R. A. Williams, L. J. LeBlanc, K. Jimenez-Garcia, M. Beeler, and A. R. Perry

Alum

Y.-J. Lin, and R. Compton

Senior coworkers

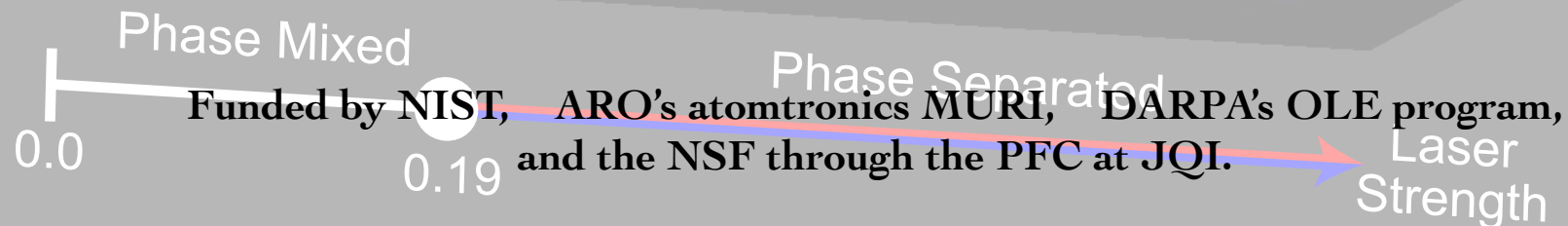
J. V. Porto, and W. D. Phillips



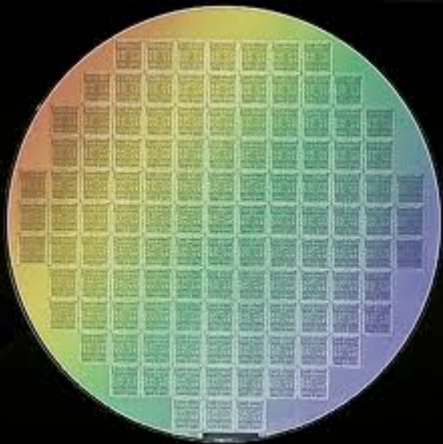
NIST

National Institute of Standards and Technology

Technology Administration, U.S. Department of Commerce

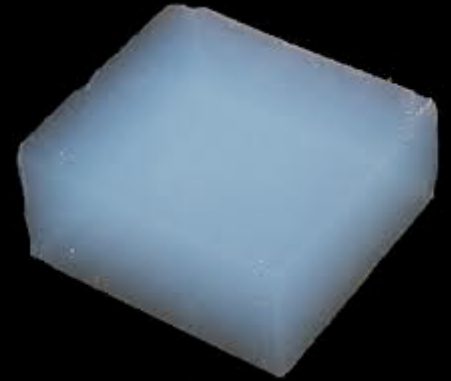


What are materials?



Si
 2.3 g/cm^3

Ian's answer I: "chunks of stuff."



Aerogel
 1 mg/cm^3

Liquid Helium
 125 mg/cm^3

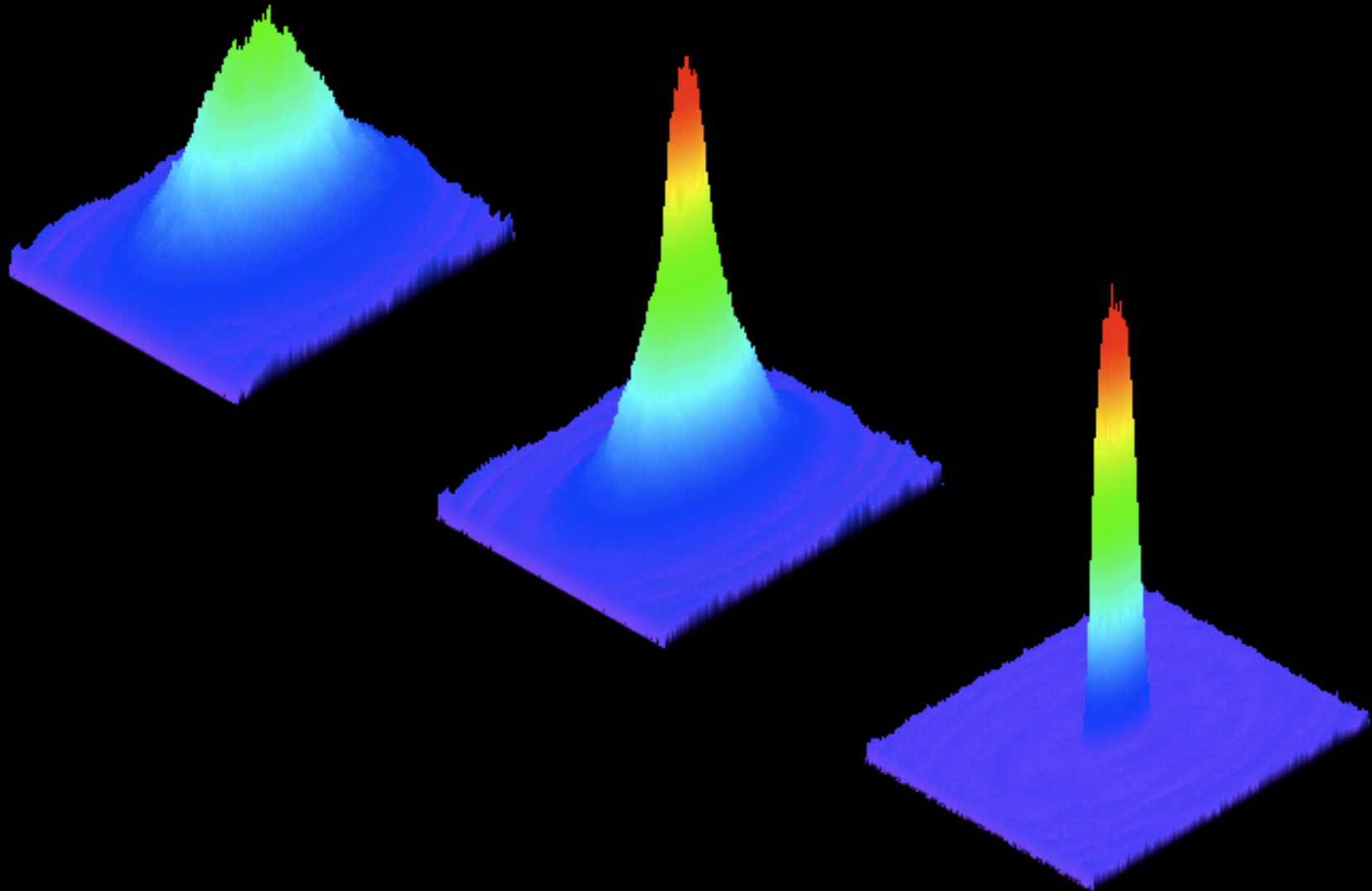


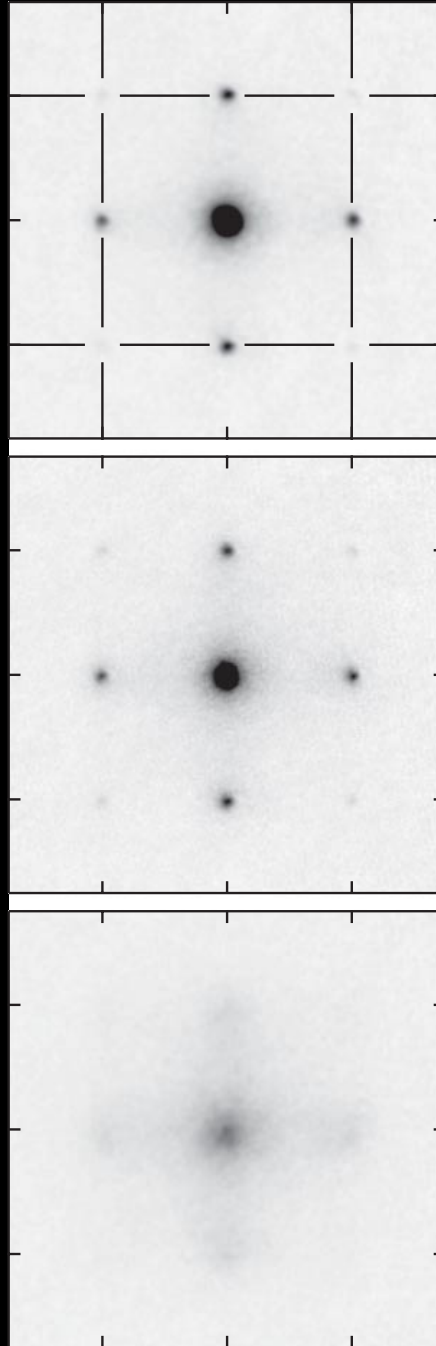
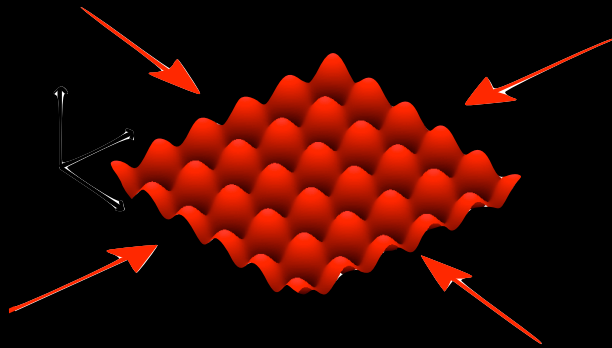
Ultracold neutral atoms

$\sim 10^{14} \text{ cm}^{-3}$ or 100 ng/cm^3

(air is $\sim 1 \text{ mg/cm}^3$)

Are these materials?



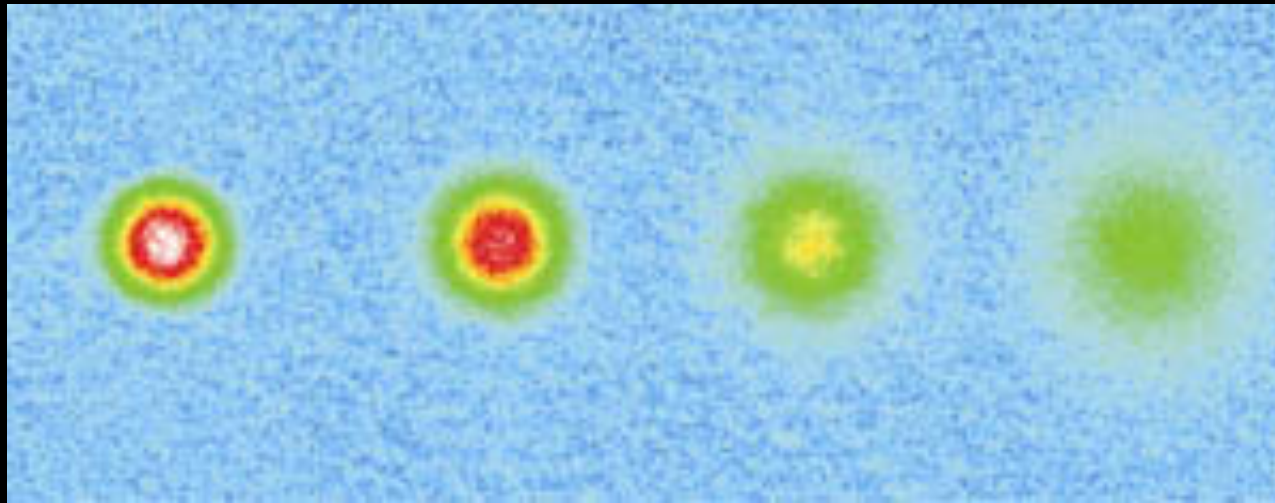


They can be fluids

They can be insulators

They can be bosons

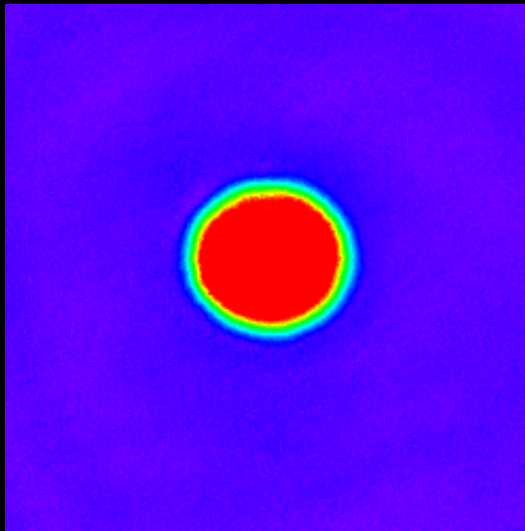
They can be fermions



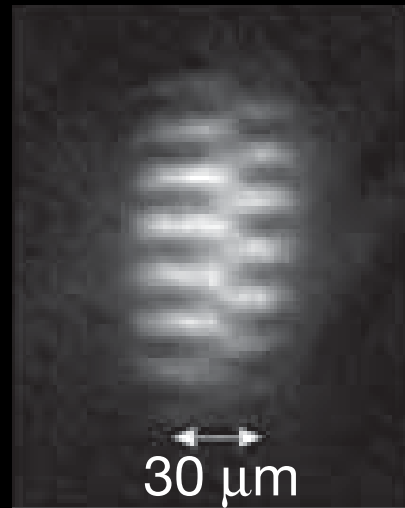
They can be molecules

They can be atoms

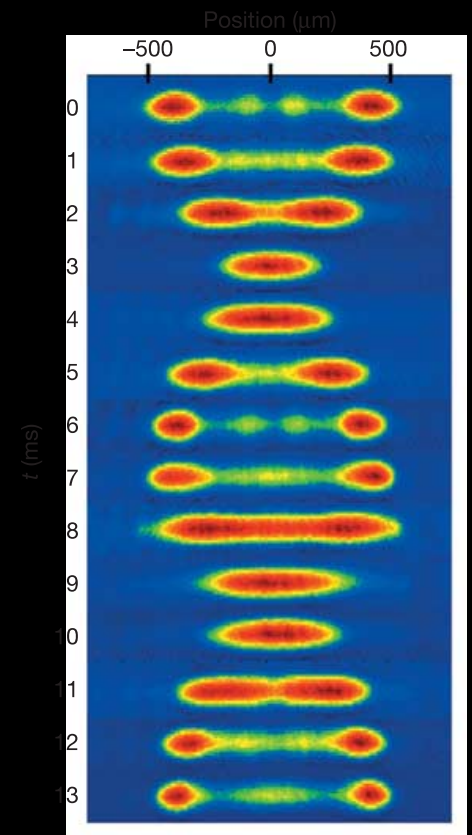
They can be 3D



They can be 2D



They can be 1D



e.g., Hadzibabic Nature (2006)

e.g., Kinoshita Nature (2006)

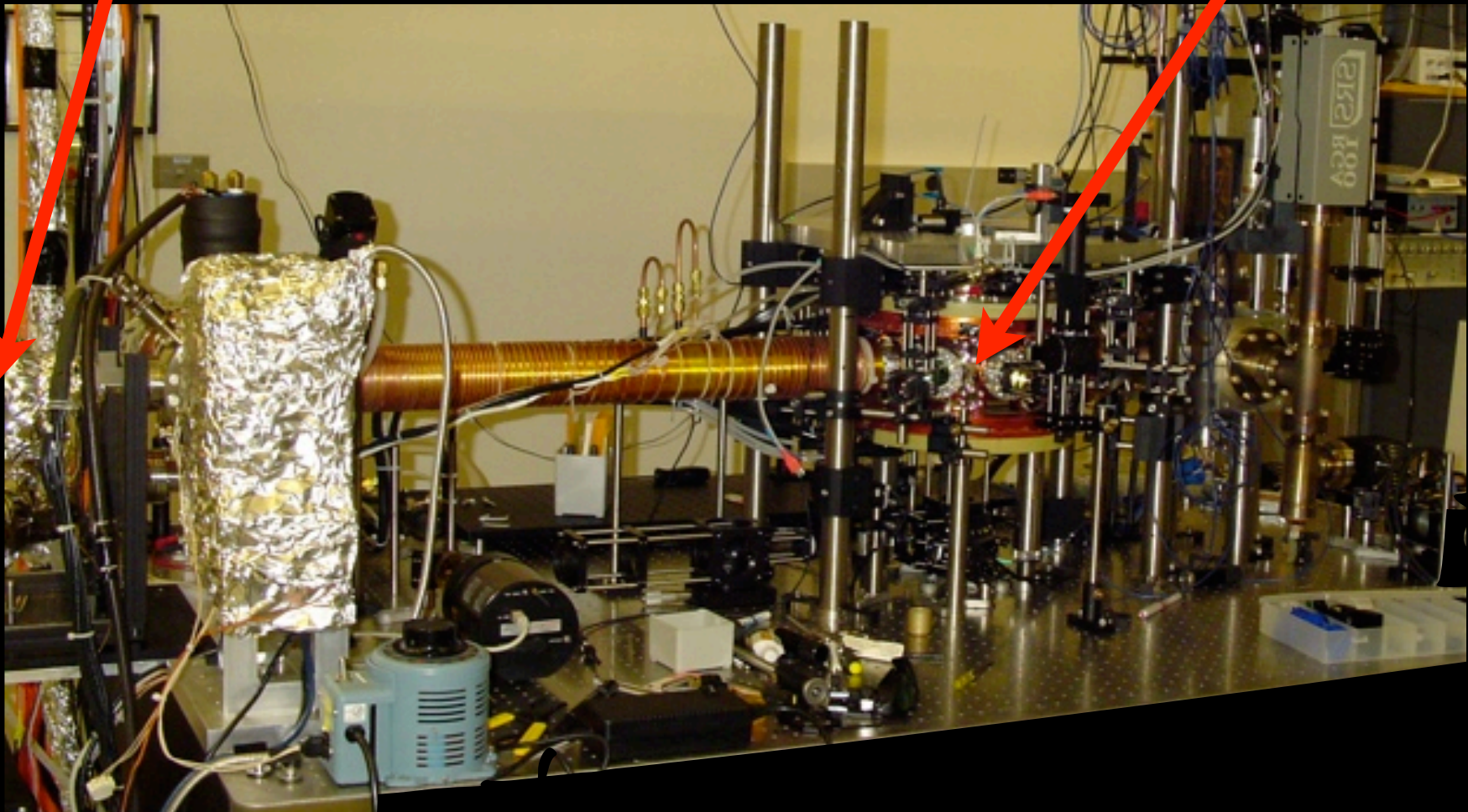
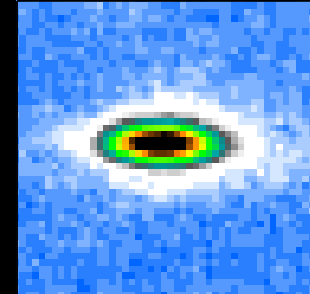
Starts like this

400 K

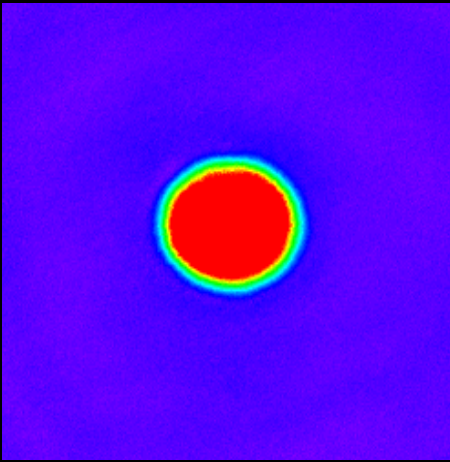


Ends here every 20 s

50 nK



What are materials?

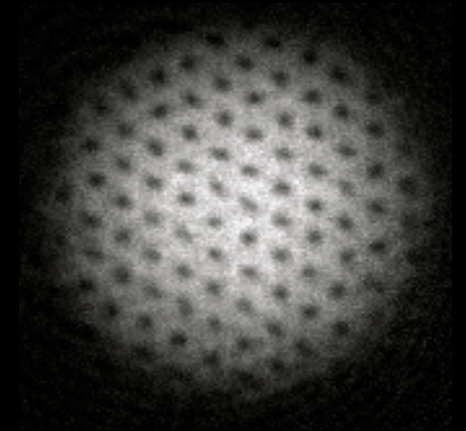


Ian's answer: "stuff"

In a finite volume of space

Have "mechanical properties"

Or "fluid properties"



Cold atoms are: good materials

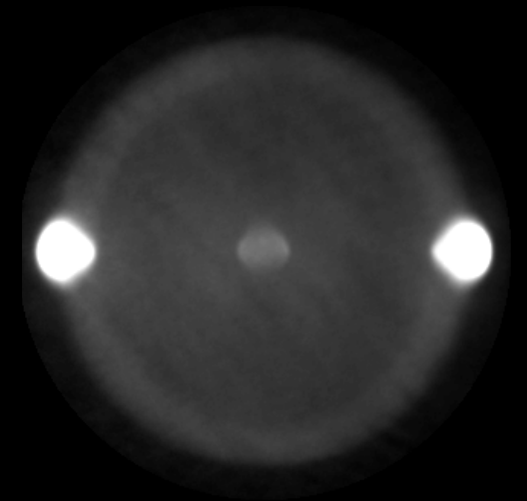
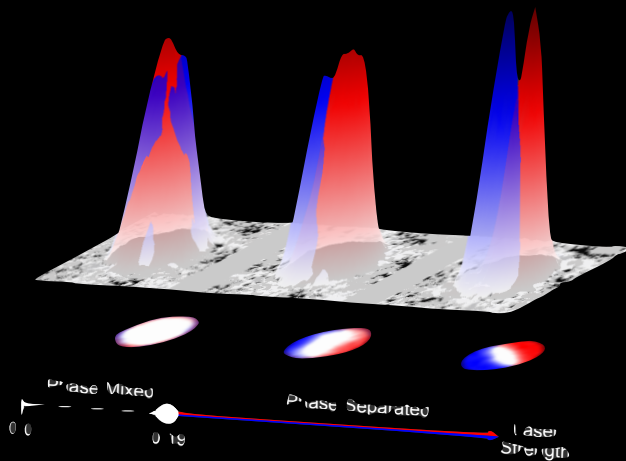
Numerous properties can
be controlled on many timescales

Very simple Hamiltonian

Cold atoms are: bad materials

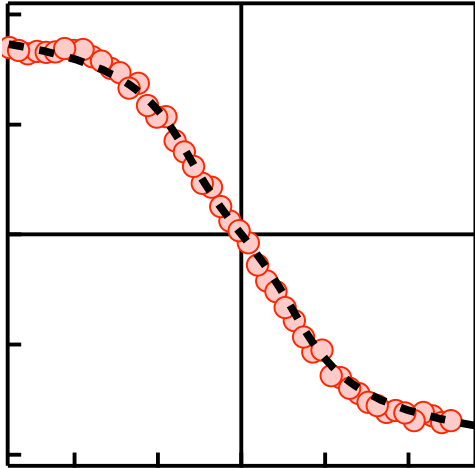
Short lived

Interesting features **all** added
by hand (complex experiments).

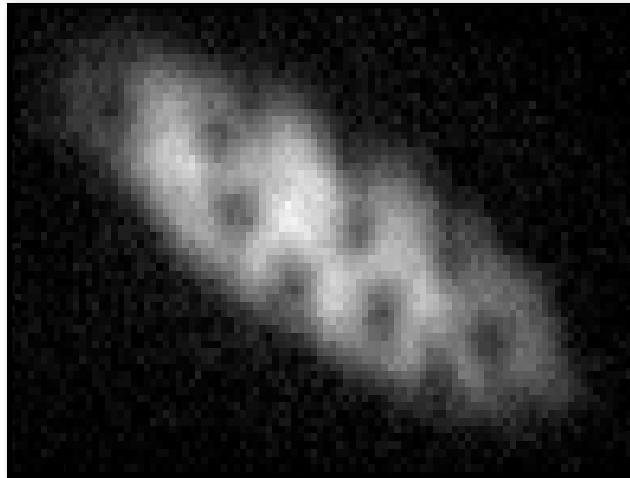


Artificial gauge fields

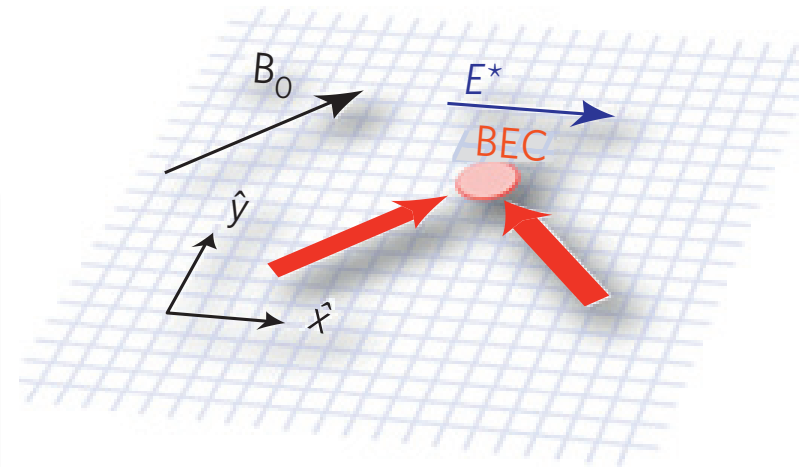
Vector potential



Magnetic field



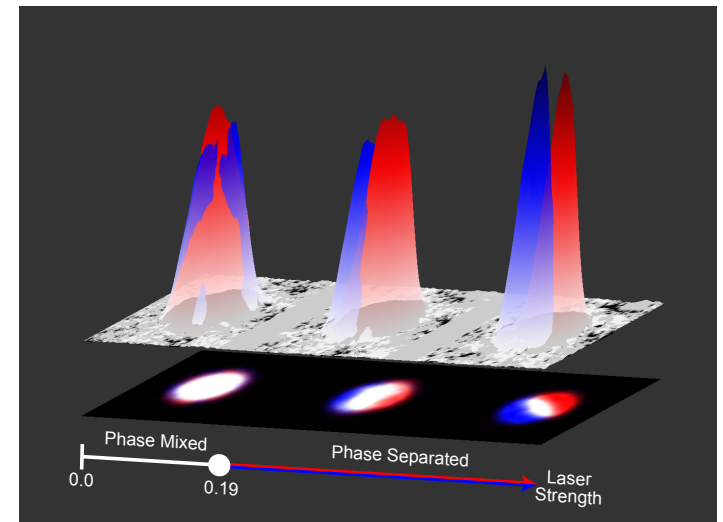
Electric field



$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

Can be a matrix

Spin Orbit coupling



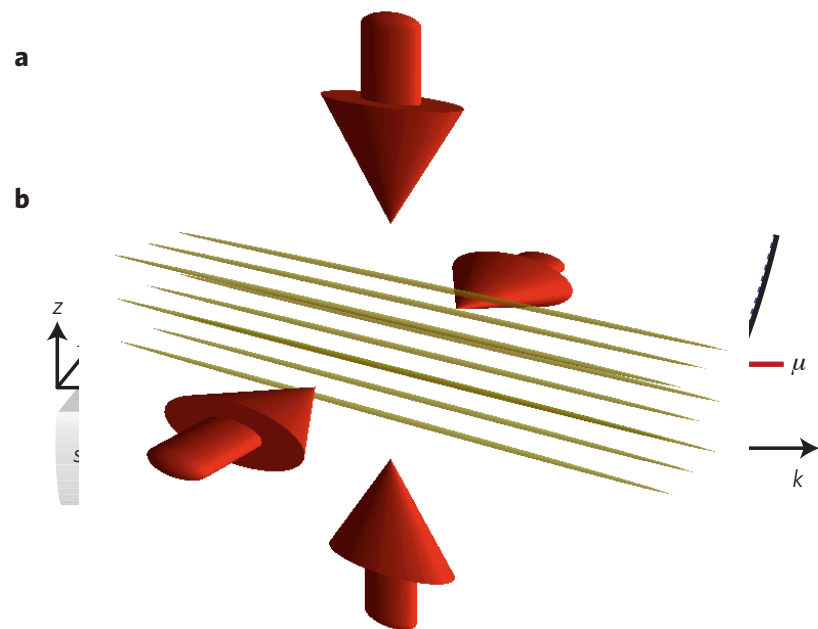
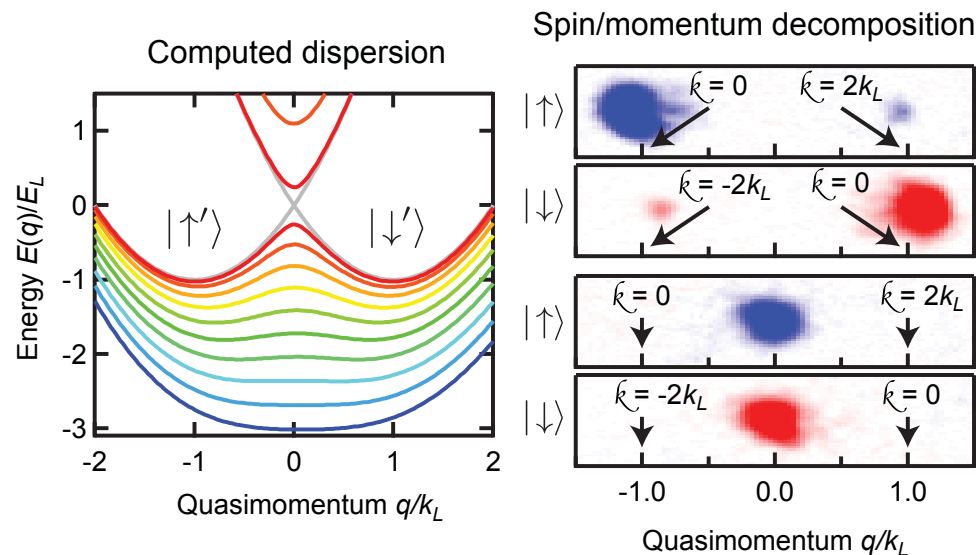
Spin orbit coupling of pseudo spin-1/2 atoms

Current experiments with Bosons

Spin orbit coupling for pseudo spin 1/2 **Bosons** (testbed platform)

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

$$\text{for } \Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$$



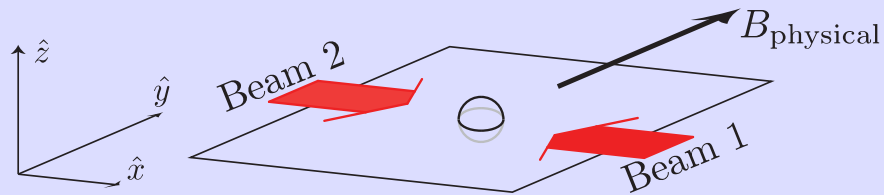
Refs.

Cold atom experiments: Y.-J. Lin et al Nature (2011), R. A. Williams Science (accepted, 2011)
Theory: C. Zhang et al, PRL (2008), J. D. Sau et al, PRL (2010), J. Alicea et al, N. Physics (2011)

Spin orbit coupling with ultracold atoms

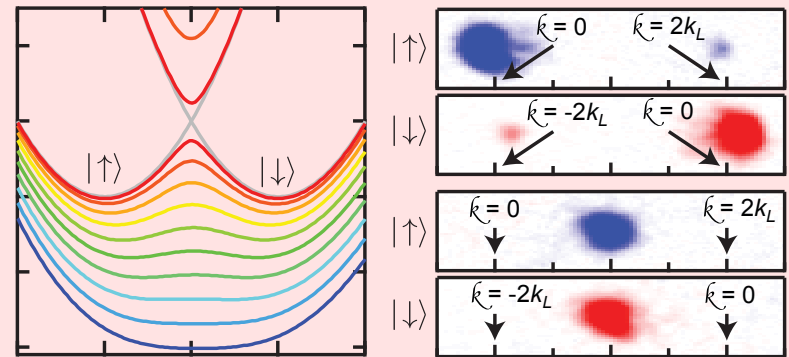
Raman dressed states

Concept



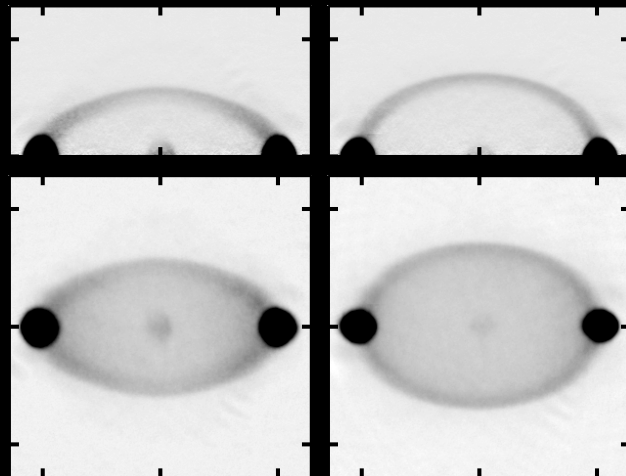
Spin orbit coupling, engineered

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{\mathbb{I}} - [\mathbf{B} + \mathbf{B}_{\text{SO}}(\hat{\mathbf{k}})] \cdot \hat{\boldsymbol{\mu}} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{\mathbb{I}} + \frac{\Omega}{2} \hat{\sigma}_z + \frac{\delta}{2} \hat{\sigma}_y + 2\alpha \hat{k}_x \hat{\sigma}_y$$

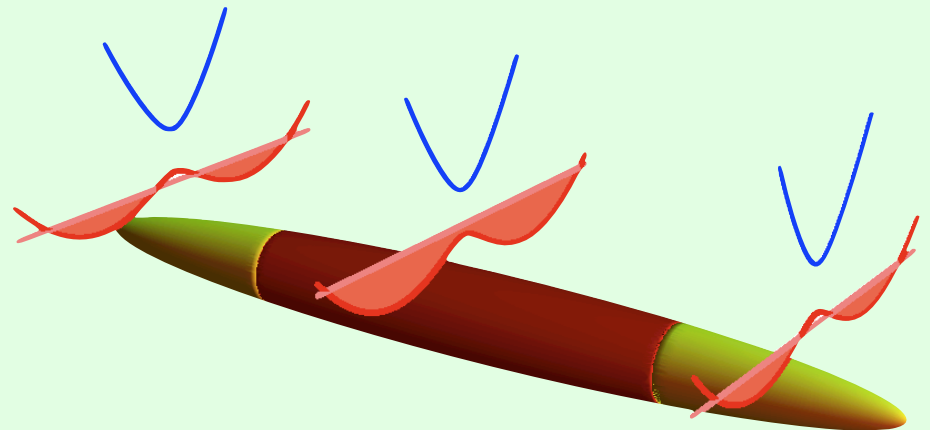


Interactions, rearranged

effective d- and g- wave interactions in bosons



Fermions, the future

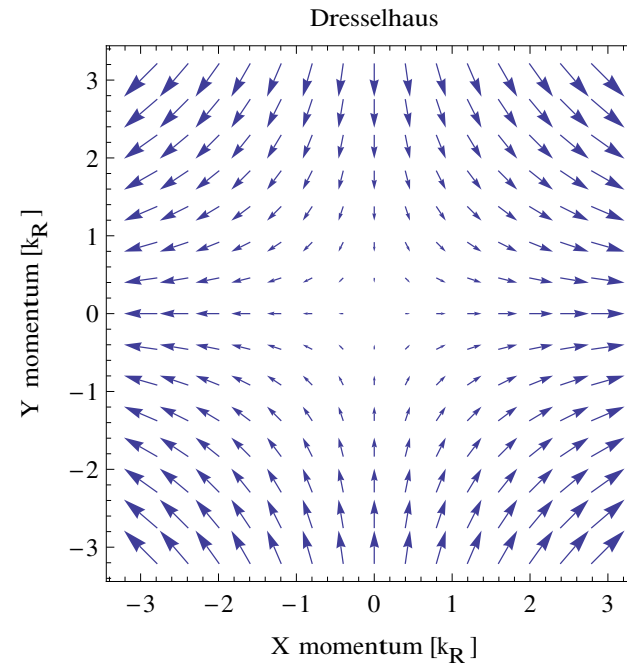
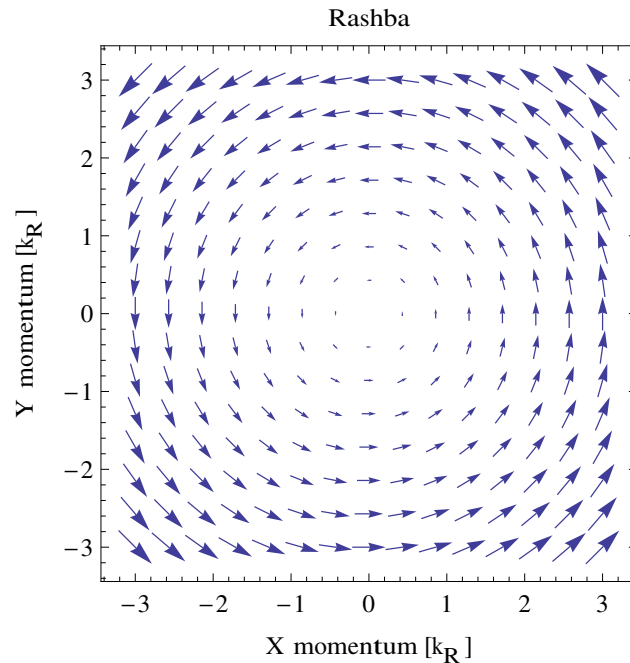


Spin orbit coupling: what do we desire?

Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \check{1} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

α gives the strength of the Rashba coupling; β yields the linear Dresselhaus coupling; and δ produces a Zeeman splitting

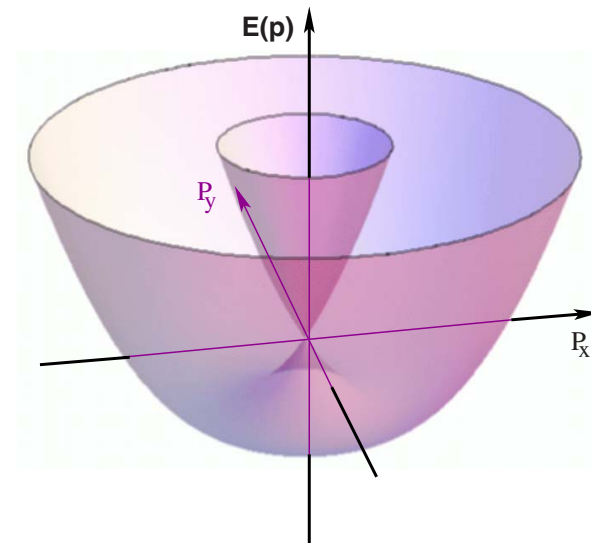
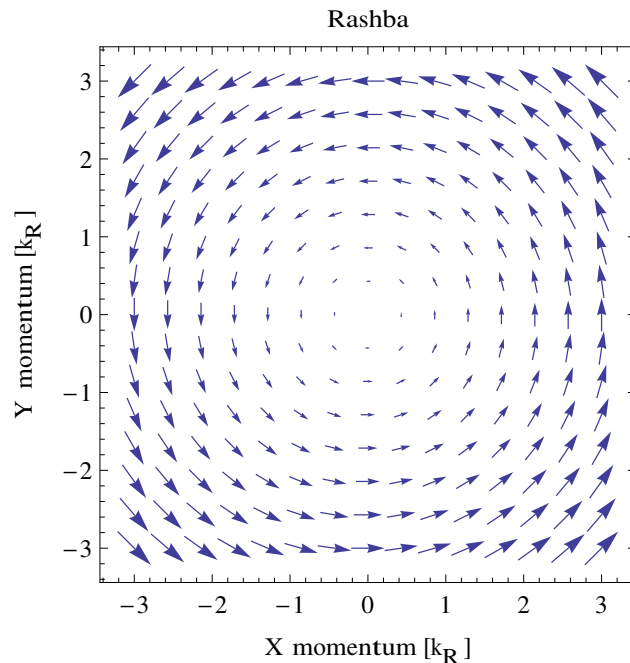


Spin-orbit coupling: Rashba

Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \check{1} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

Pure Rashba: $\beta = 0$



Cold atom Refs.

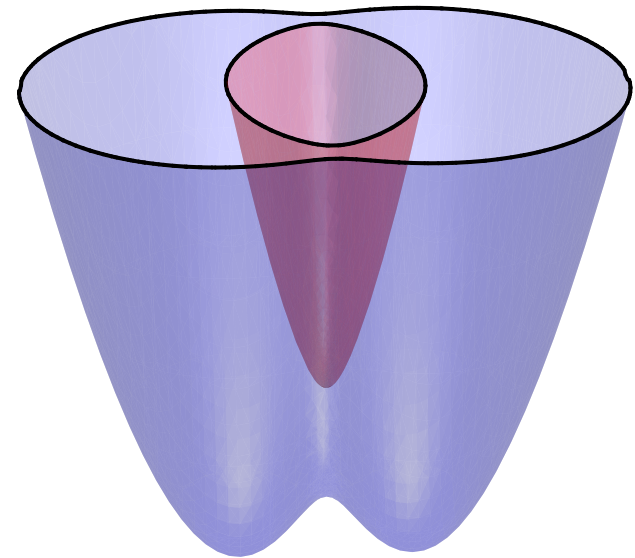
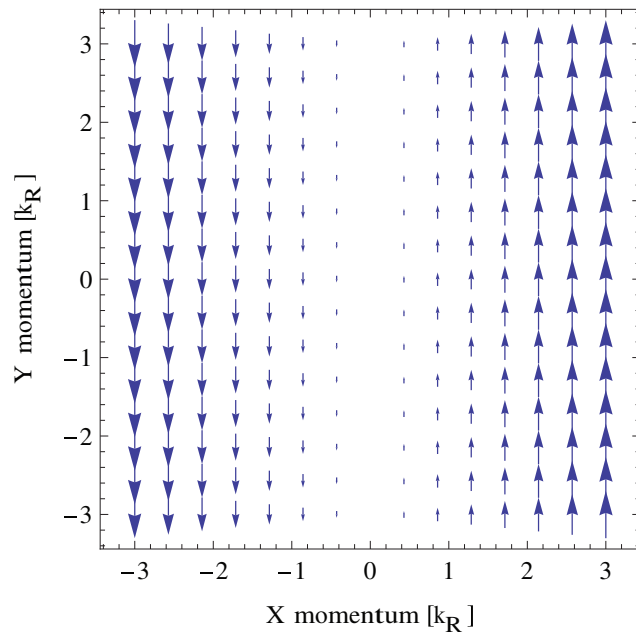
T. D. Stanescu and B. Anderson and V. Galitski PRA (2008)

Spin-orbit coupling: Rashba = Dresselhaus

Spin-orbit coupling

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \check{1} + 2\alpha k_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z$$

Equal Rashba and Dresselhaus: $\alpha = \beta$



GaAs Refs.

J. D. Koralek et al, Nature (2009); C. H. L. Quay et al, Nat. Phys. (2010)

Non-abelian gauge fields and Spin-Orbit coupling

Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \check{1} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

Uniform Non-abelian gauge field

$$\check{H} = \frac{\hbar^2}{2m} \left[\left(k_x \check{1} - \frac{q}{\hbar} \check{A}_x \right)^2 + \left(k_y \check{1} - \frac{q}{\hbar} \check{A}_y \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_z + E_0 \check{1}$$

Spin-orbit coupling is a (sometimes) non-abelian gauge field!

$$\check{H} = \frac{\hbar^2}{2m} \left\{ \left[\left(k_x \check{1} + \frac{1}{2} (\alpha \check{\sigma}_y + \beta \check{\sigma}_x) \right)^2 + \left(k_y \check{1} - \frac{1}{2} (\alpha \check{\sigma}_x + \beta \check{\sigma}_y) \right)^2 \right] \right\} + \frac{\delta}{2} \check{\sigma}_z + E_0 \check{1}$$

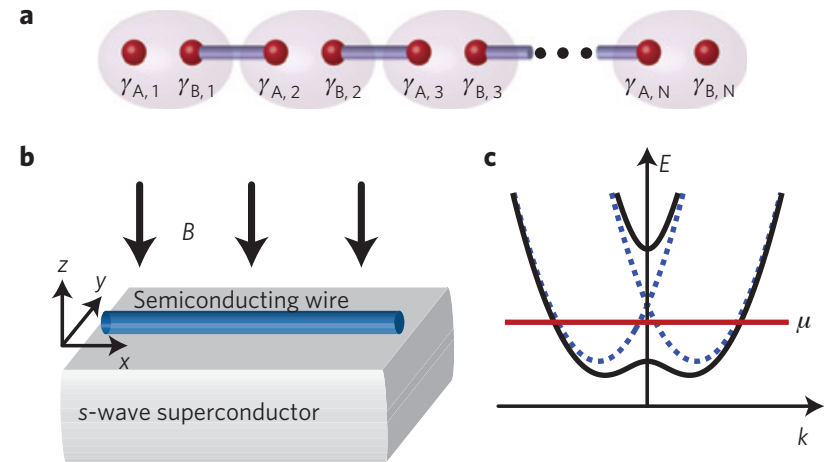
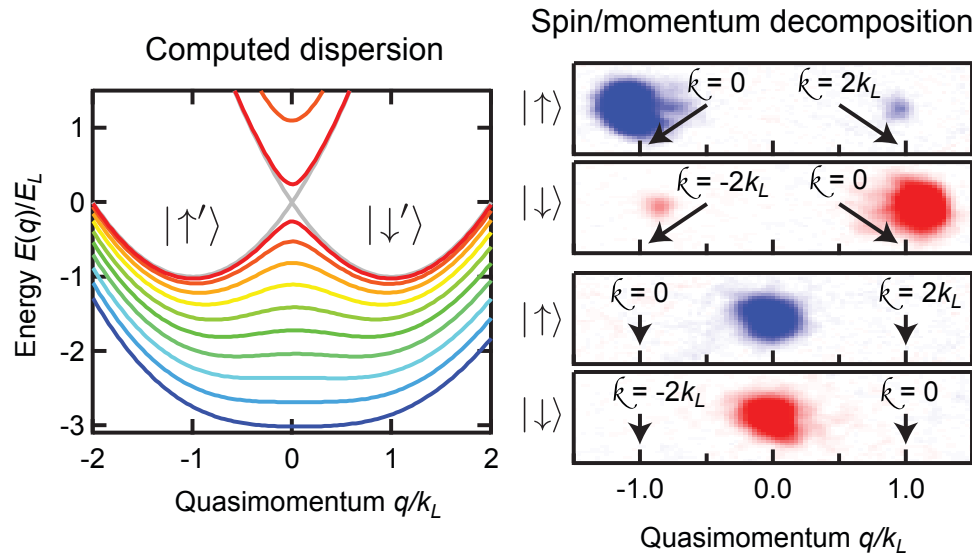
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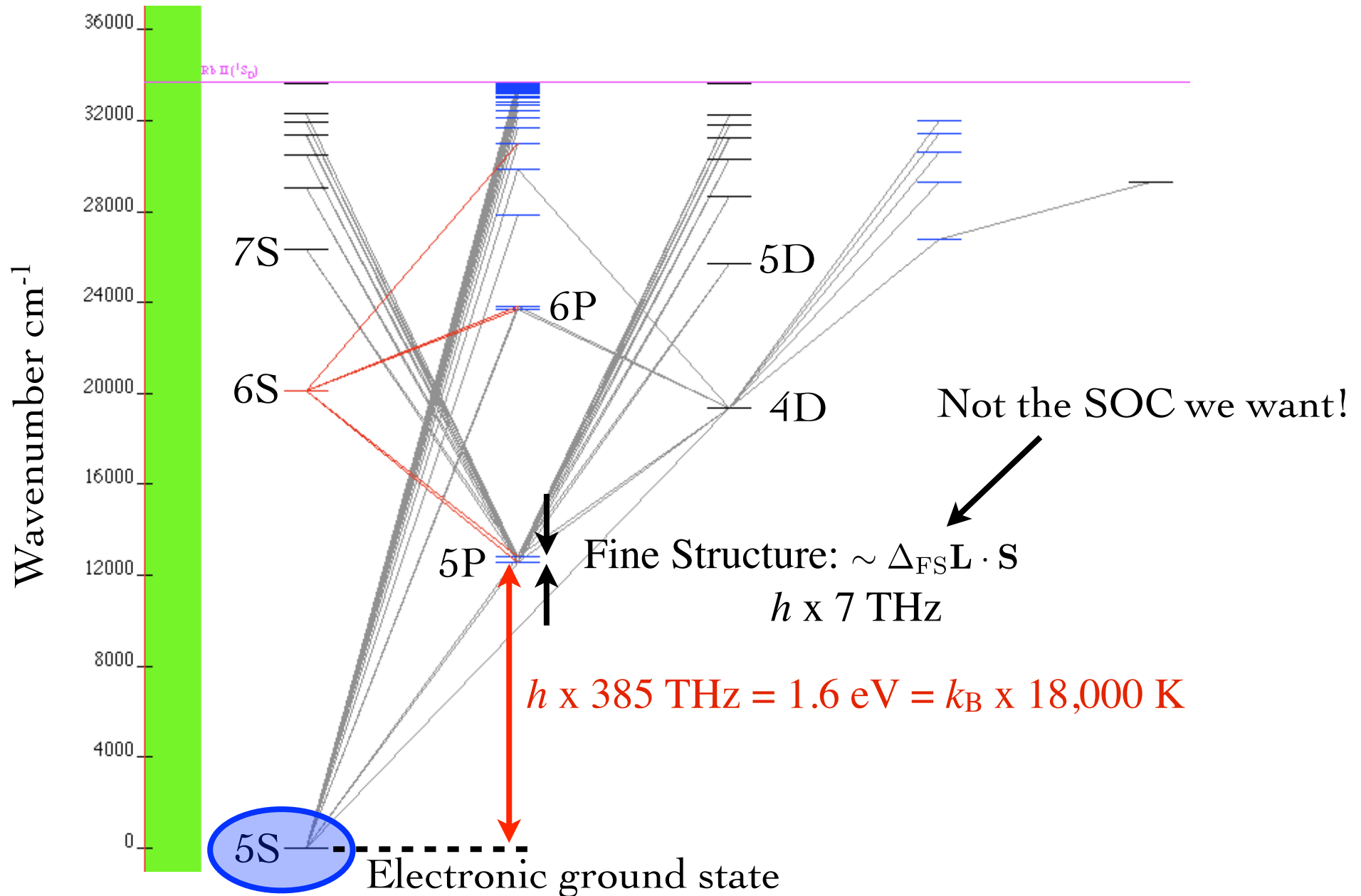
for $\Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$



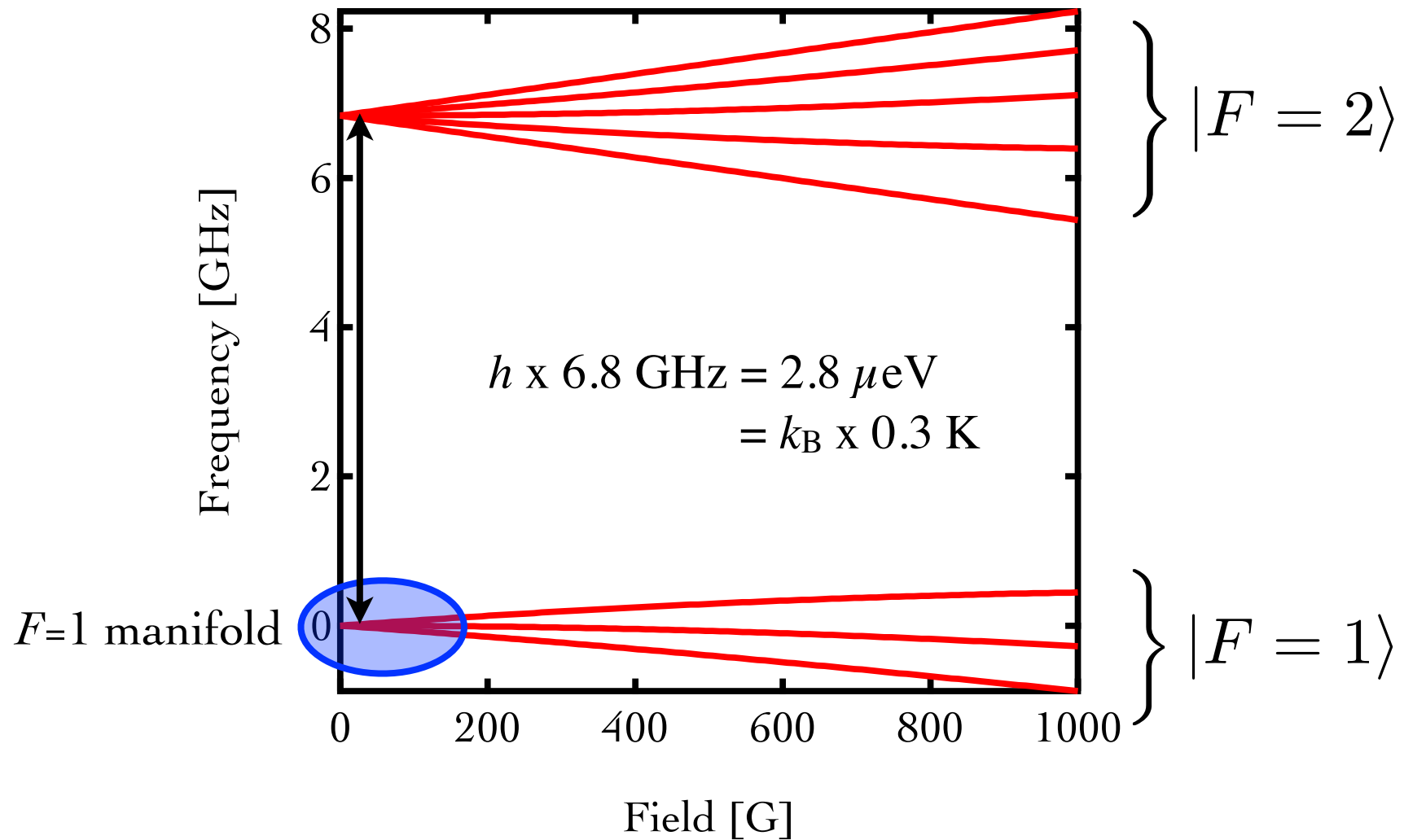
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Rubidium 87: “The GaAs of atoms”



Rubidium 87: $5S_{1/2}$ ground state

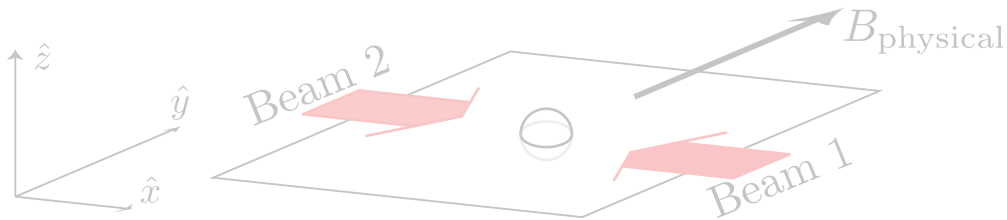


Engineered spin-orbit coupling

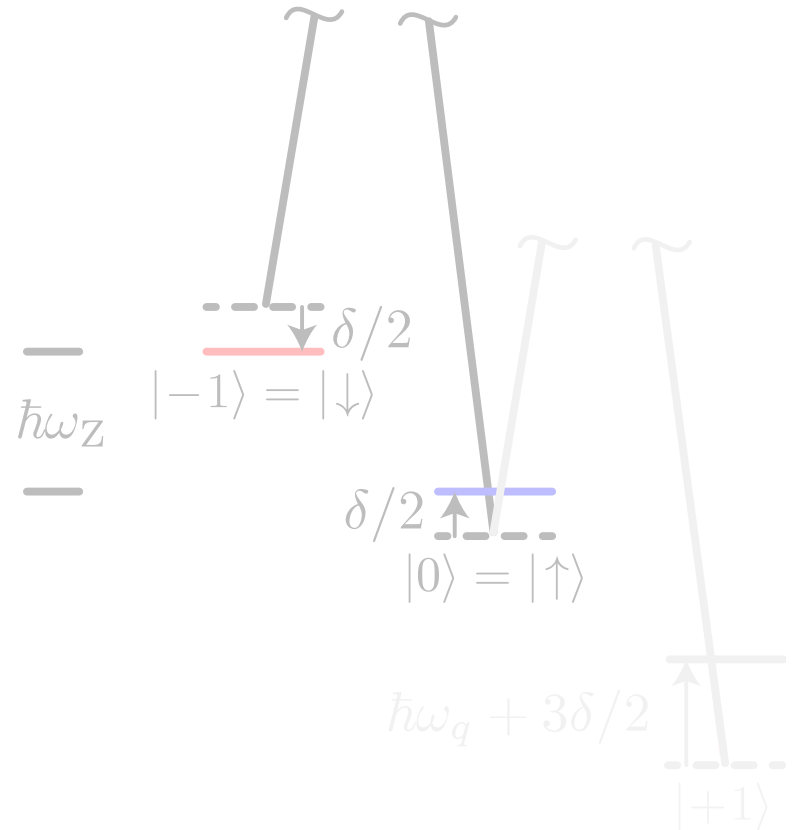
Momentum representation

$$H = \sum_k \left\{ \begin{pmatrix} \langle k-1, \uparrow | & \langle k+1, \downarrow | \end{pmatrix} \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1, \uparrow\rangle \\ |k+1, \downarrow\rangle \end{pmatrix} \right\}$$

Geometry



Levels



Natural dimensions

Length: $\lambda \approx 790 \text{ nm}$

Momentum: $\hbar k_r = 2\pi/\lambda$

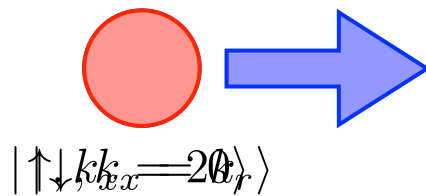
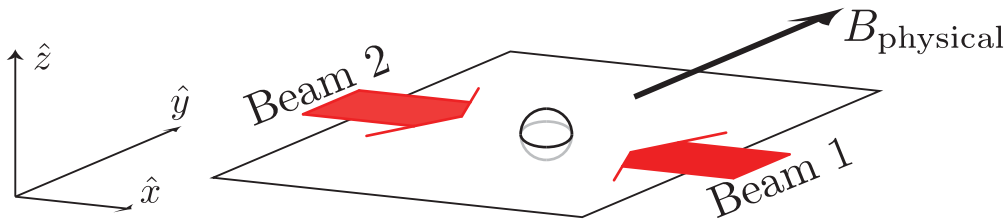
Energy: $E_r = \frac{\hbar^2 k_r^2}{2m} \approx h \times 3.4 \text{ kHz} = 14 \text{ peV}$

Engineered spin-orbit coupling

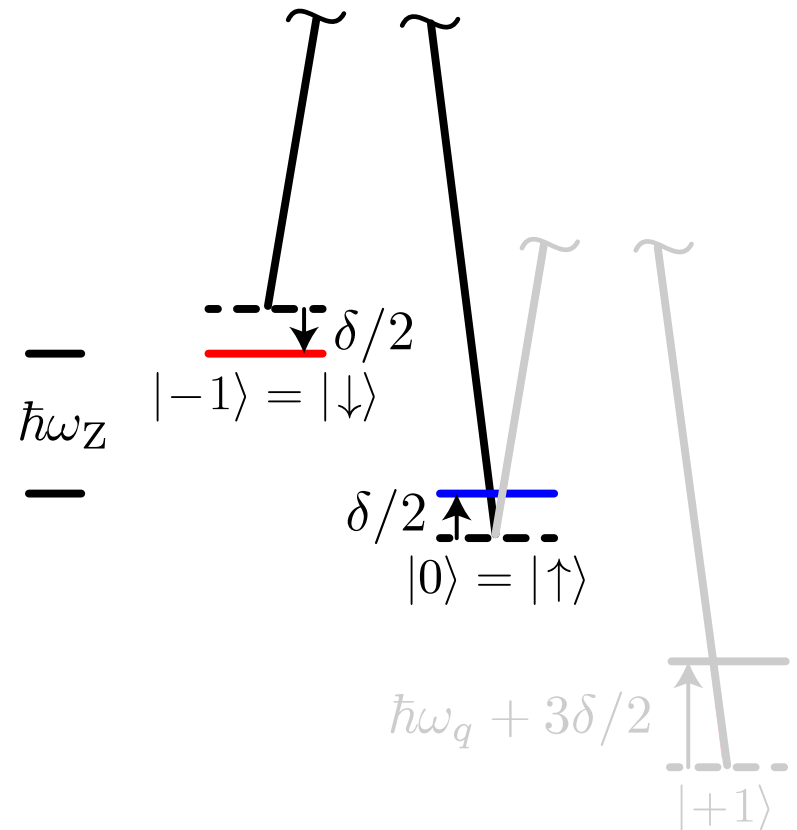
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Geometry



Levels

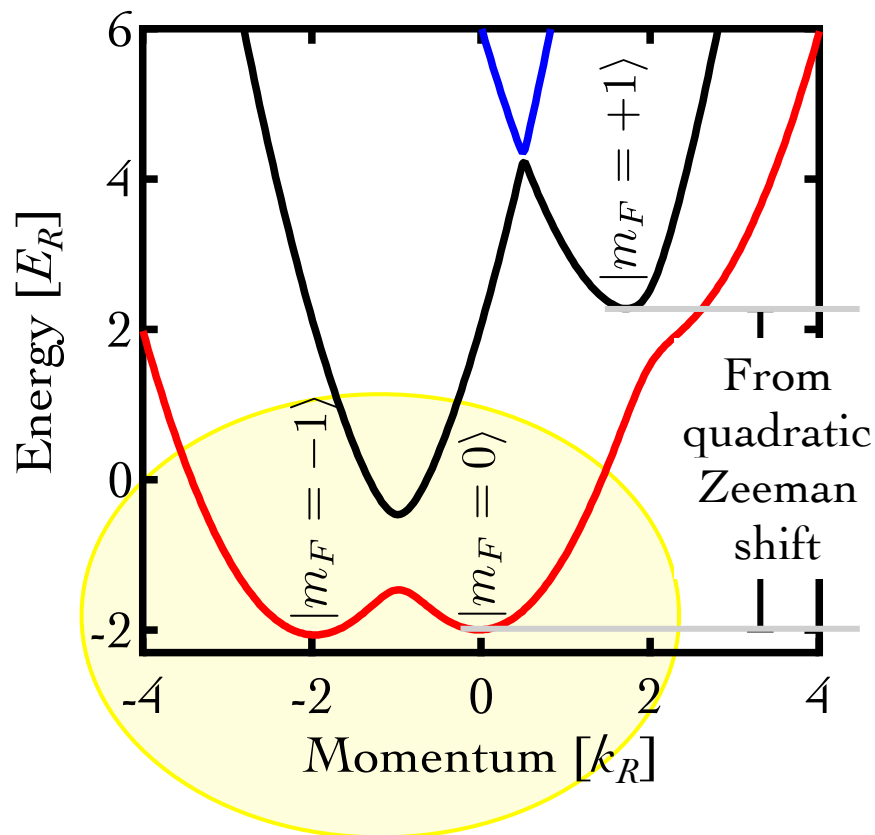


Spin orbit coupling: origin

Momentum representation

$$H = \sum_k \left\{ \left(\langle k-1, \uparrow | \quad \langle k+1, \downarrow | \right) \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1, \uparrow\rangle \\ |k+1, \downarrow\rangle \end{pmatrix} \right\}$$

Spin 1/2 bosons????



Transform to

$$\begin{aligned} \hat{H} &= \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \left(\frac{\delta}{2} + \frac{\hbar^2 k_R}{m} \hat{k}_x \right) \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z + \Delta E \hat{1} \\ &= \frac{\hbar^2}{2m} \left[\left(\hat{k}_x \hat{1} + k_R \check{\sigma}_y \right)^2 + \left(\hat{k}_y \hat{1} - 0 \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \end{aligned}$$

NOTICE

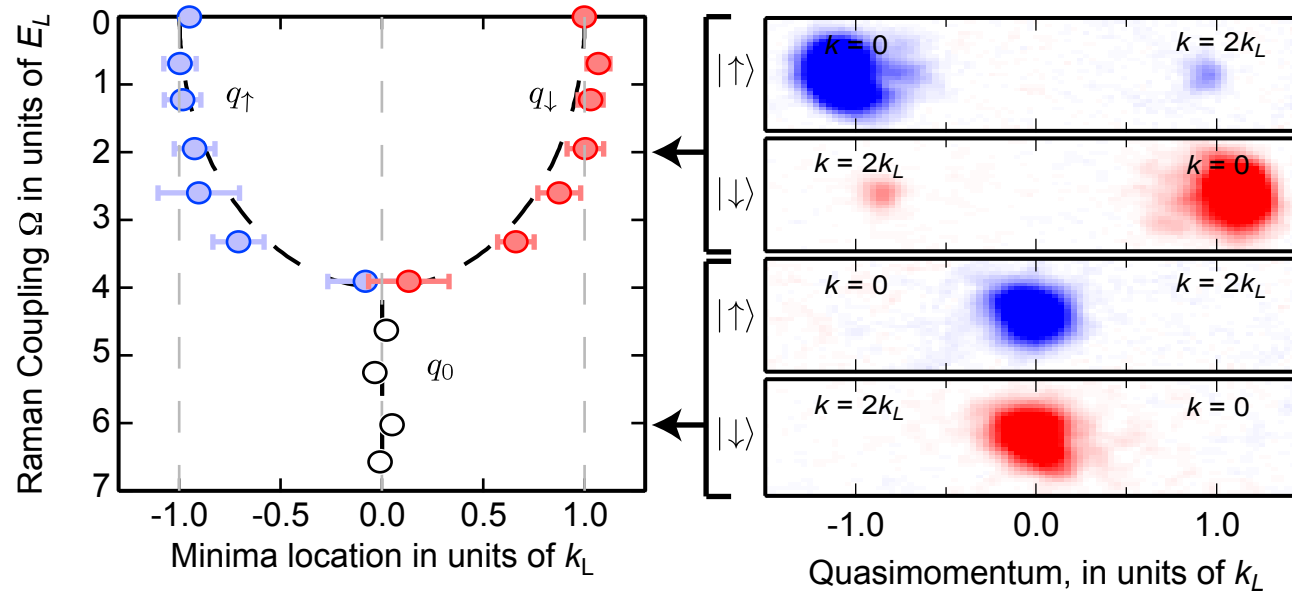
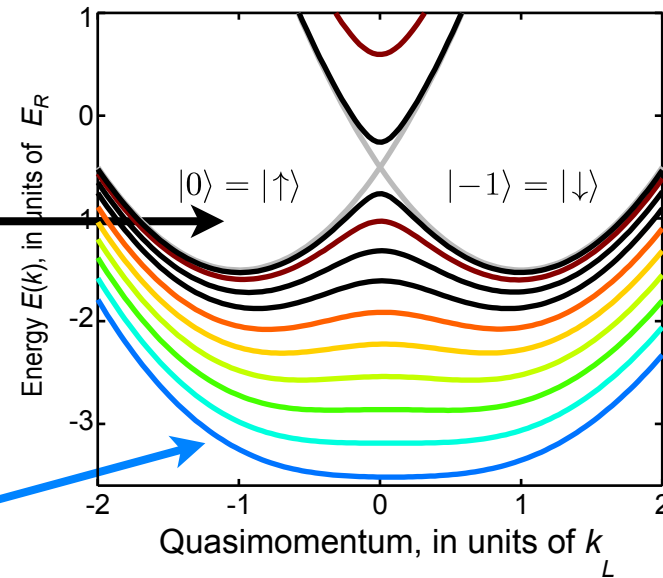
Written as a "2x2" vector potential, this S-O coupling is Abelian, $A_y = 0$: so $[A_x, A_y] = 0$.

However, the Hamiltonian is non-trivial owing to the Zeeman field along z .

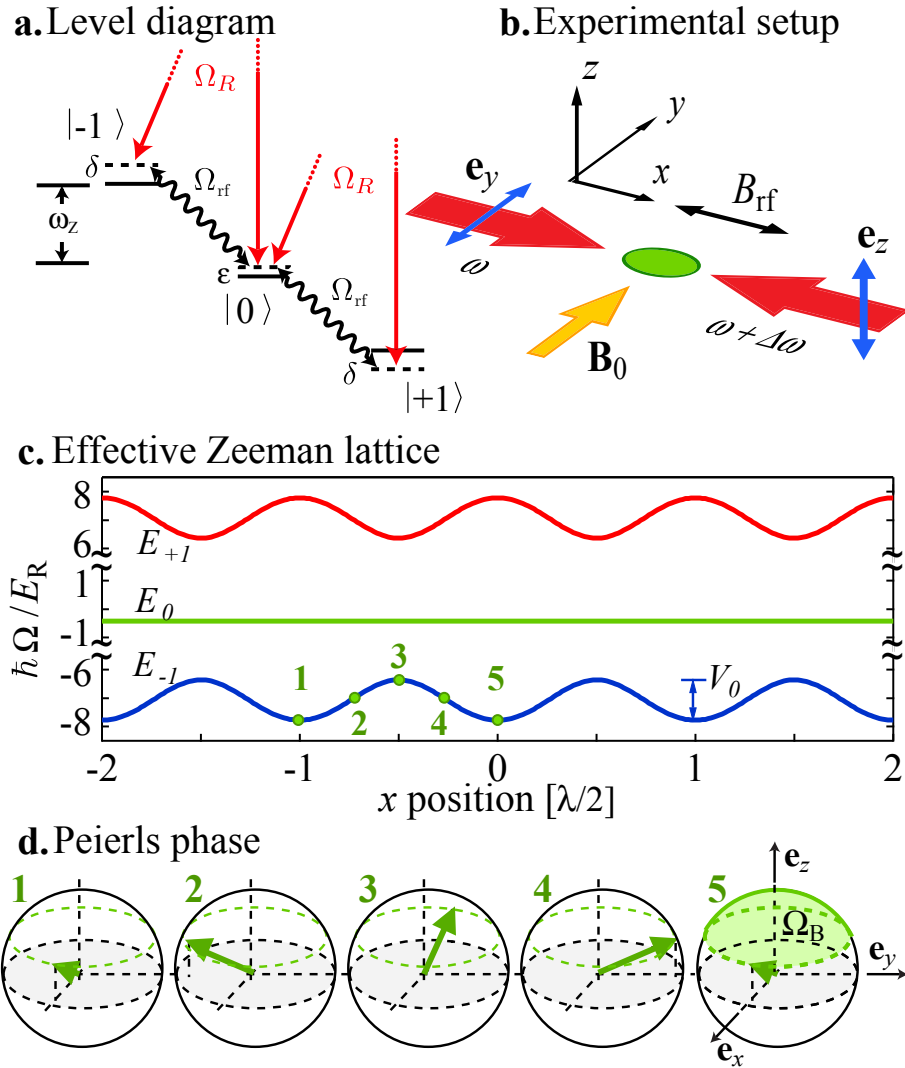
Typical data

Double Well
“Spin-orbit limit”

Single minimum
“vector potential limit”

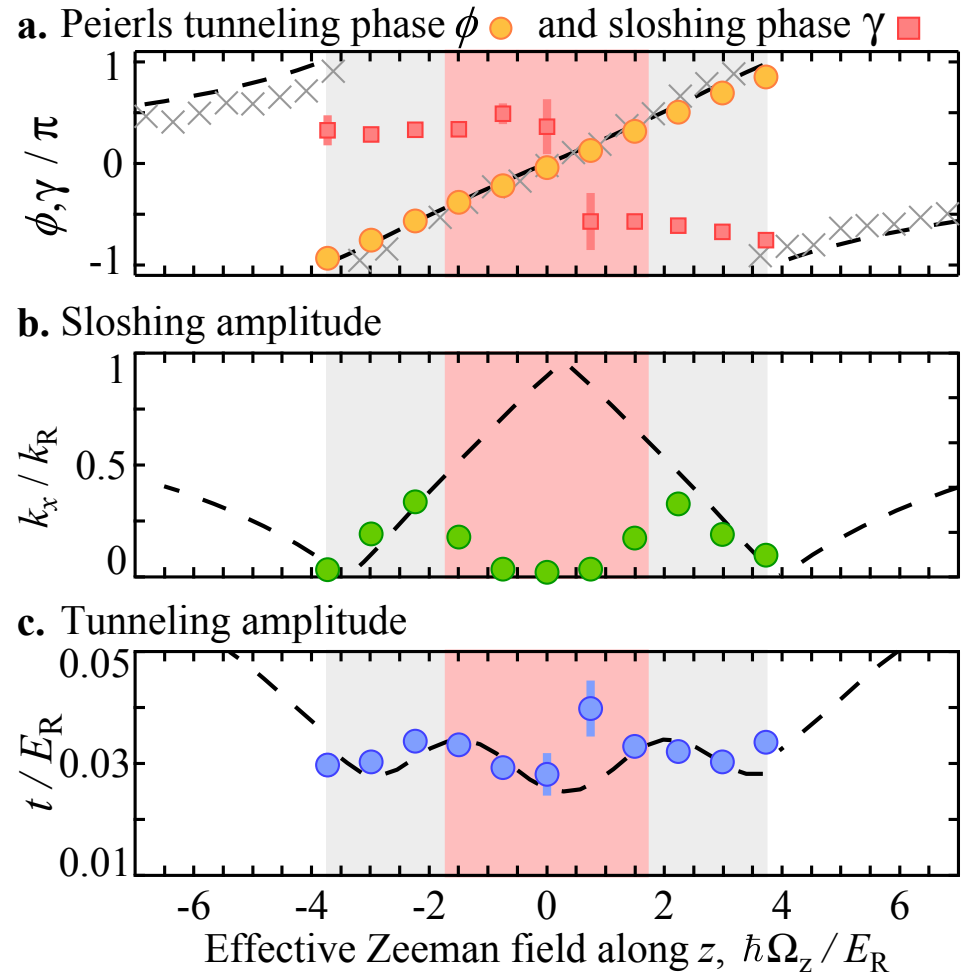


Peierls lattice



$$H(x) = \mathbf{\Omega} \cdot \mathbf{F} + H_Q$$

$$\mathbf{\Omega} = \{ \Omega_{\text{rf}} + \Omega_{\text{R}} \cos(2k_{\text{R}}x), -\Omega_{\text{R}} \sin(2k_{\text{R}}x), \sqrt{2}\delta \}$$



$$H = \sum_j [t \exp(i\phi) \hat{a}_{j+1}^\dagger \hat{a}_j + \text{h.c.}]$$

Refs.

K. Jimenez-Garcia *et al* (in preparation)

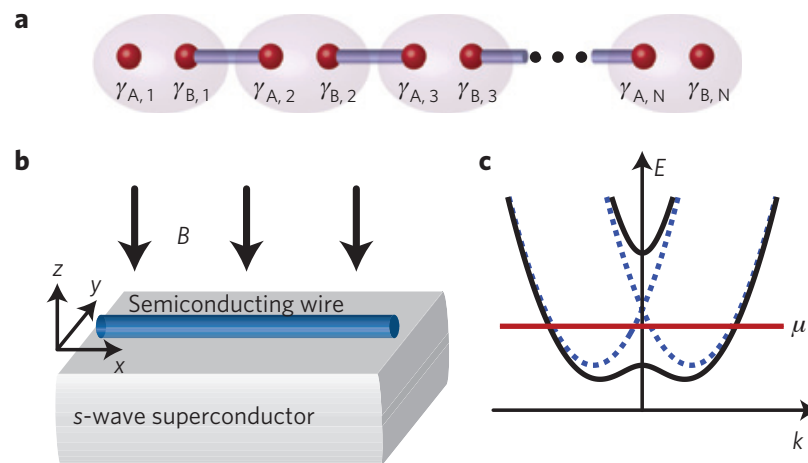
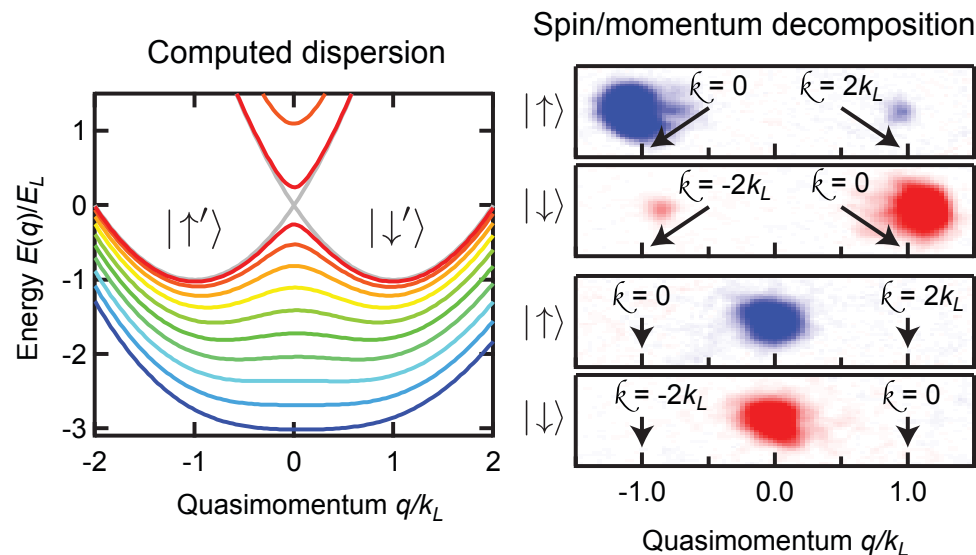
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$$\text{for } \Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$$



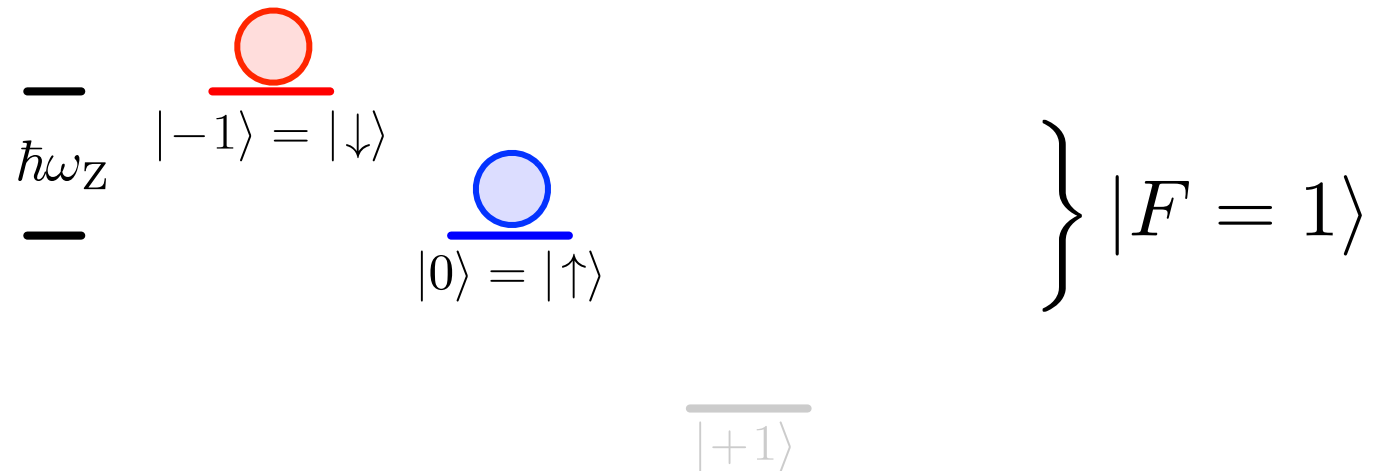
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Effective Hamiltonian for dressed spins

Two pseudo-spin contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow} + \hat{\rho}_{\uparrow})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow}^2 - \hat{\rho}_{\uparrow}^2) + c_2 \hat{\rho}_{\downarrow} \hat{\rho}_{\uparrow} \right] :$$



Effective Hamiltonian for dressed spins

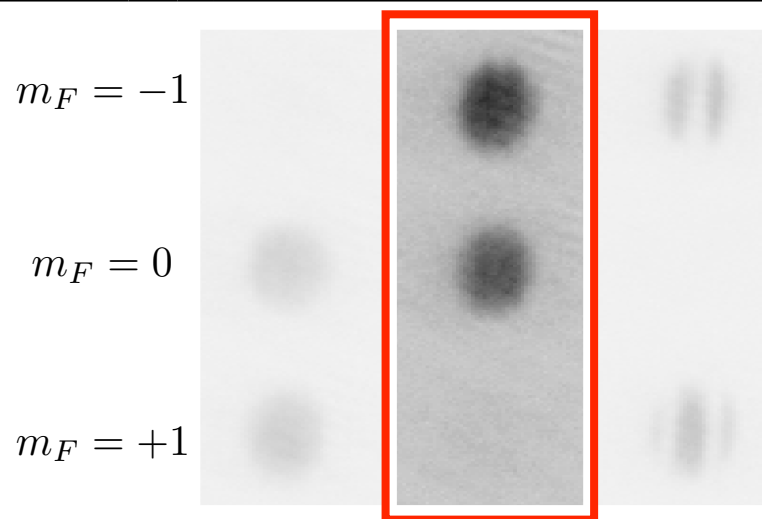
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$m_F = -1, m_F = 0$ mixture: miscible for ^{87}Rb

$$c_0 = 7.8 \times 10^{-12} \text{ Hz} \cdot \text{cm}^3$$

$$c_2 = -3.6 \times 10^{-14} \text{ Hz} \cdot \text{cm}^3$$

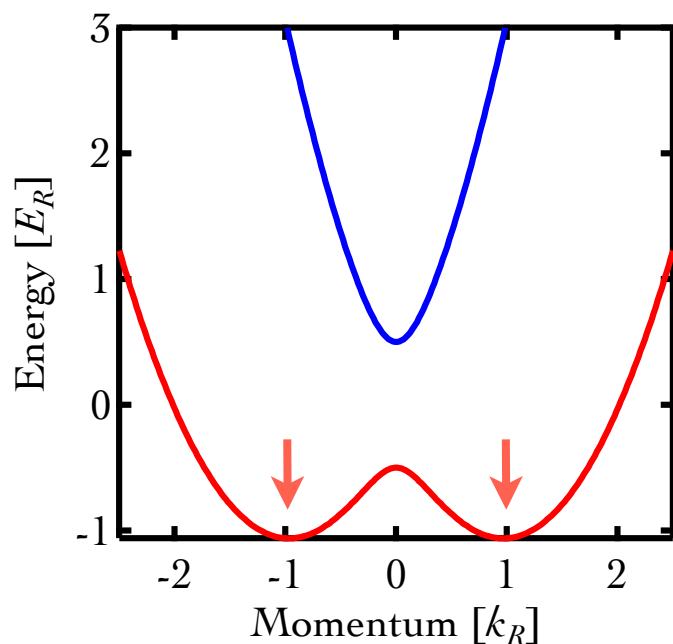


Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

Effective Hamiltonian for dressed spins

Two pseudo-spin contact interactions

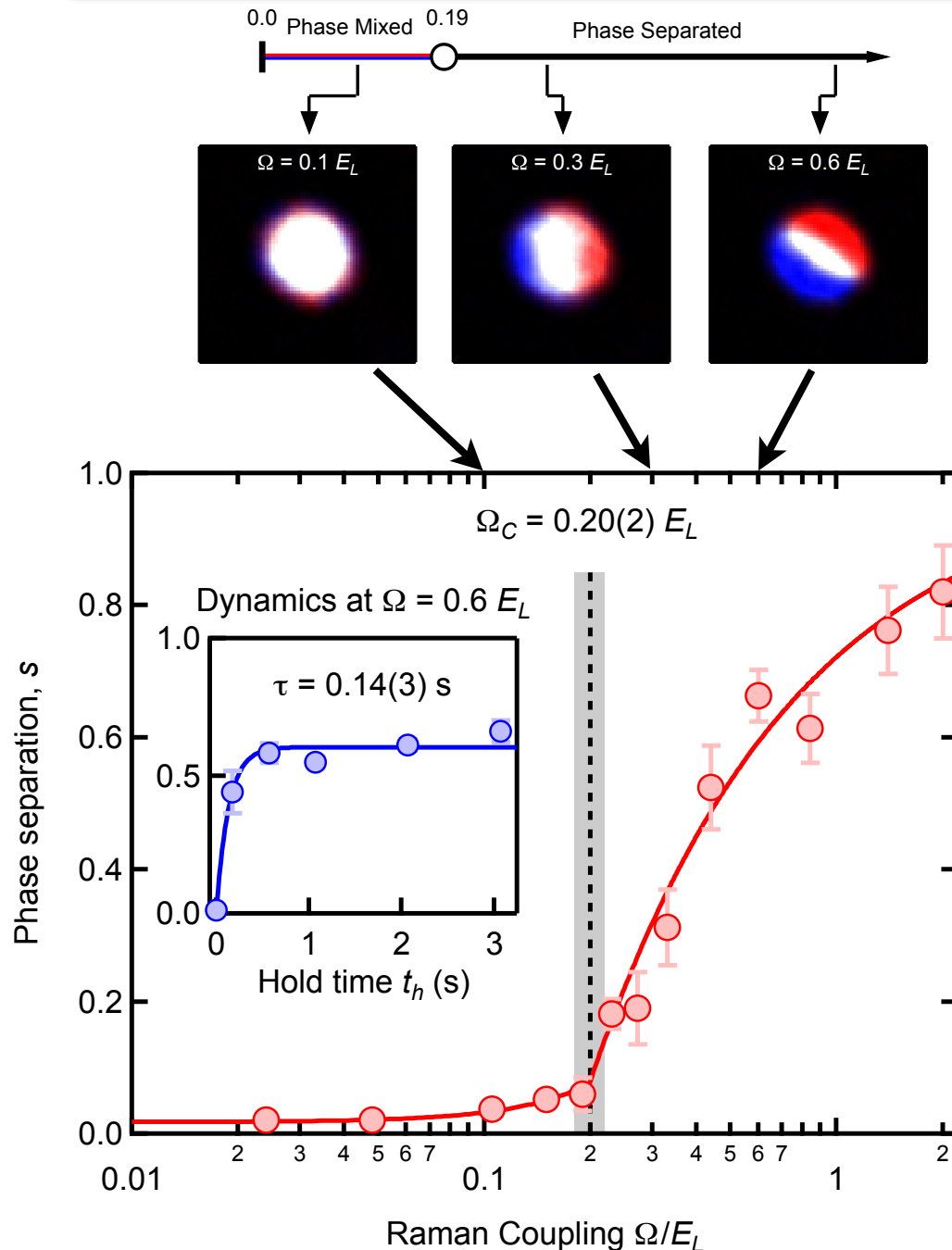
$$\begin{aligned}\hat{H}_{\text{int}} &= \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow} + \hat{\rho}_{\uparrow})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow}^2 - \hat{\rho}_{\uparrow}^2) + c_2 \hat{\rho}_{\downarrow} \hat{\rho}_{\uparrow} \right] : \\ &\rightarrow \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow'} + \hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow'}^2 - \hat{\rho}_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \hat{\rho}_{\downarrow'} \hat{\rho}_{\uparrow'} \right] :\end{aligned}$$



Spin-orbit term

$$c'_{\uparrow,\downarrow} \approx c_0 \frac{\Omega_R^2}{8}$$

Transition from miscible to immiscible



A quantum phase transition
Previously unexpected

Our MFT prediction
Phase separation at $\Omega = 0.19 E_L$

Ref.
Y.-J. Lin et al Nature (2011),

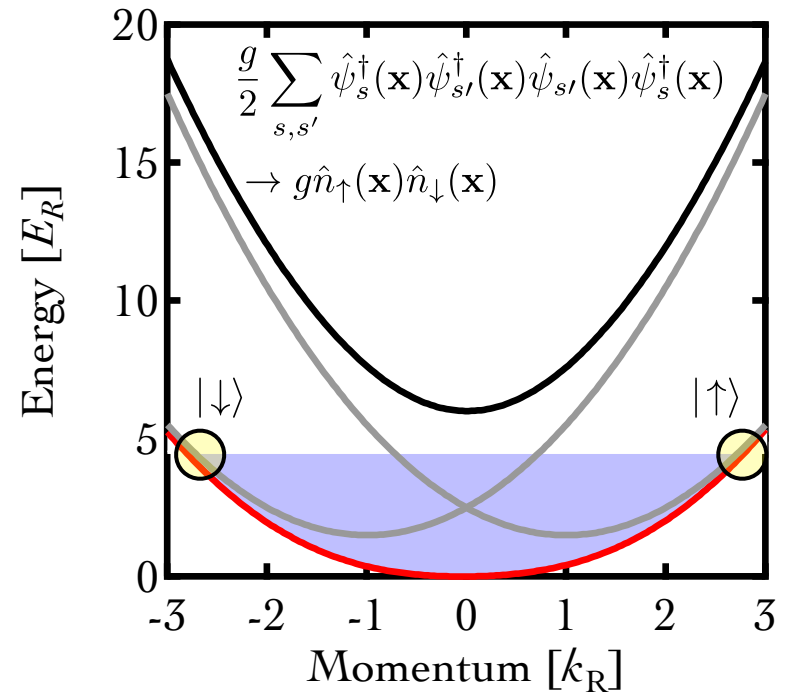
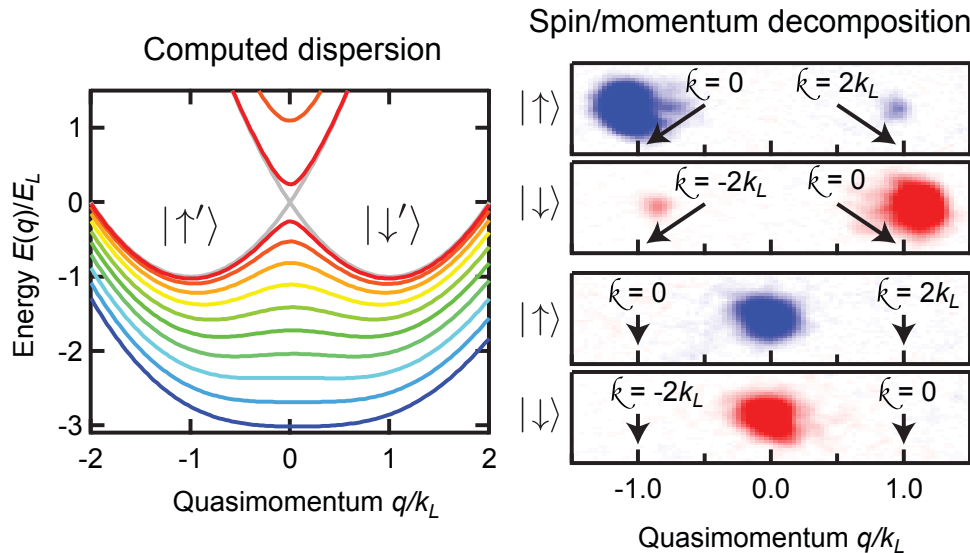
Spin orbit coupling of pseudo spin-1/2 atoms

Current experiments with Bosons

Spin orbit coupling for pseudo spin 1/2 **Bosons** (testbed platform)

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

$$\text{for } \Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$$



Refs.

Cold atom experiments: Y.-J. Lin et al Nature (2011), R. A. Williams Science (accepted, 2011)
Theory: C. Zhang et al, PRL (2008), J. D. Sau et al, PRL (2010), J. Alicea et al, N. Physics (2011)

Modified interactions: optical screening

Interacting fermions in a **single component** gas
Effective p -wave interactions!!

Test with Bosons, look for d- and g- wave interactions

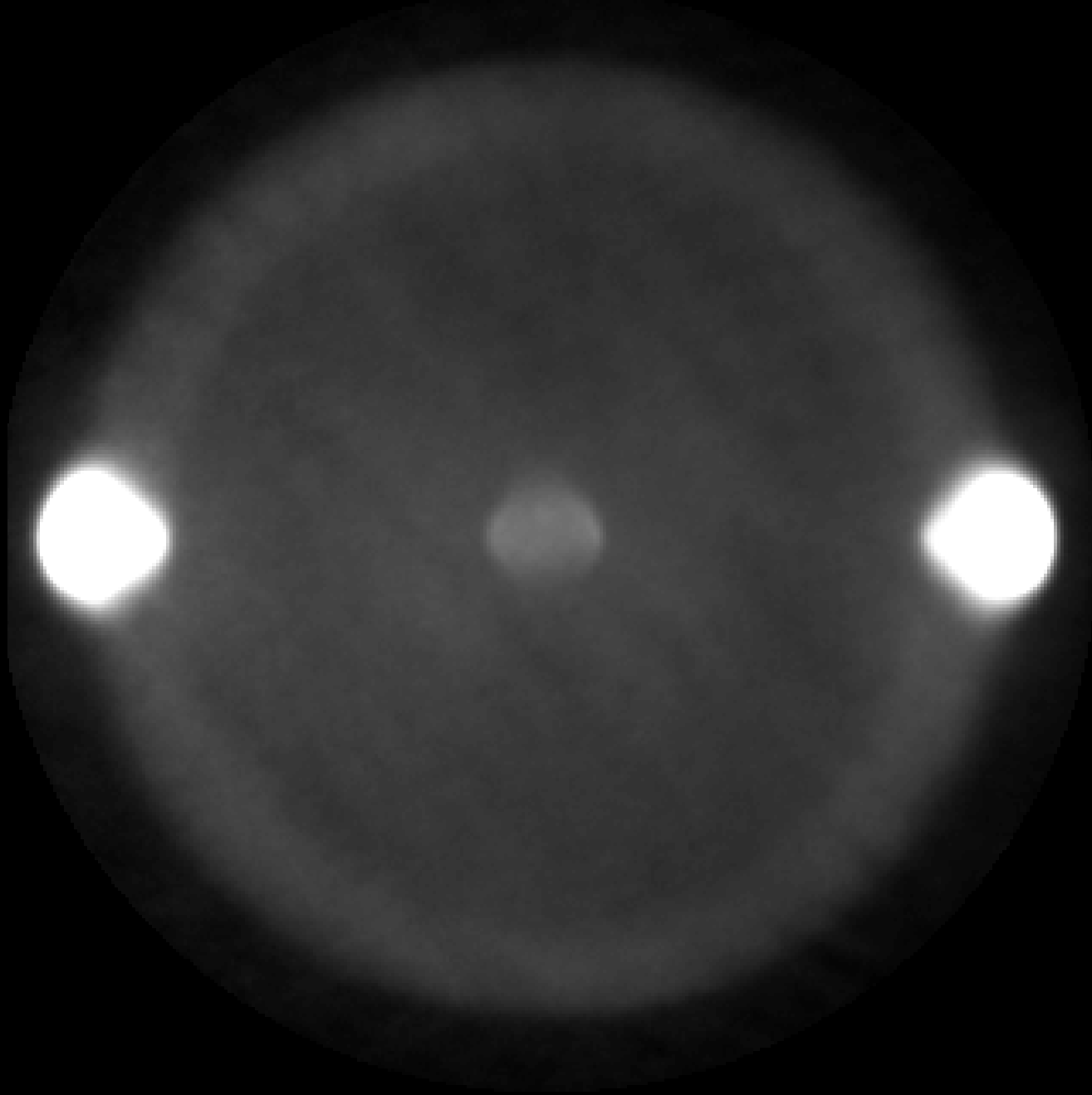
$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \check{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

A dielectric function

$$V_{\text{eff}} = g \sum_{\sigma_1, \sigma_2} U_{-, \sigma_1}(\mathbf{k}_1) U_{\sigma_1, -}^\dagger(\mathbf{k}_3) U_{-, \sigma_2}(\mathbf{k}_2) U_{\sigma_2, -}^\dagger(\mathbf{k}_4) \\ = V(\mathbf{k}_1 - \mathbf{k}_3) \chi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

$$V_{\text{eff}}(\mathbf{x}) \approx g \frac{\Omega|x| + 2}{16} \left(e^{-\Omega|x|/2} \right) \delta(y) \delta(z)$$

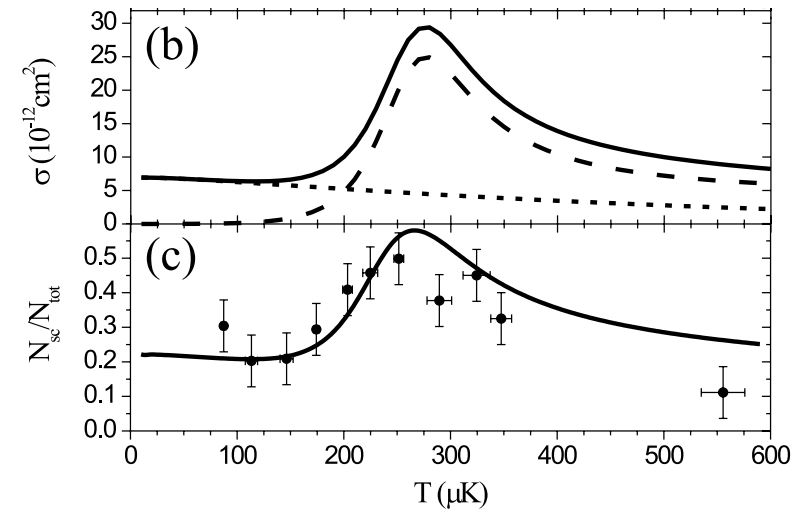
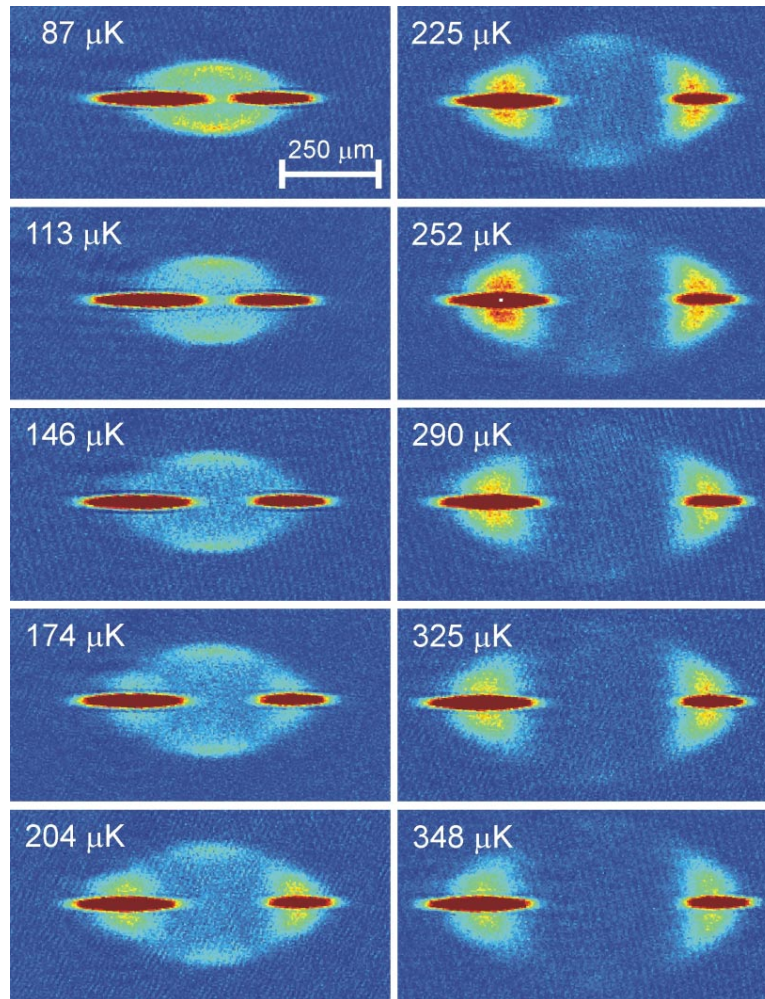
Colliding BEC's



Collisions as a probe of interatomic potentials

Colliding BEC's

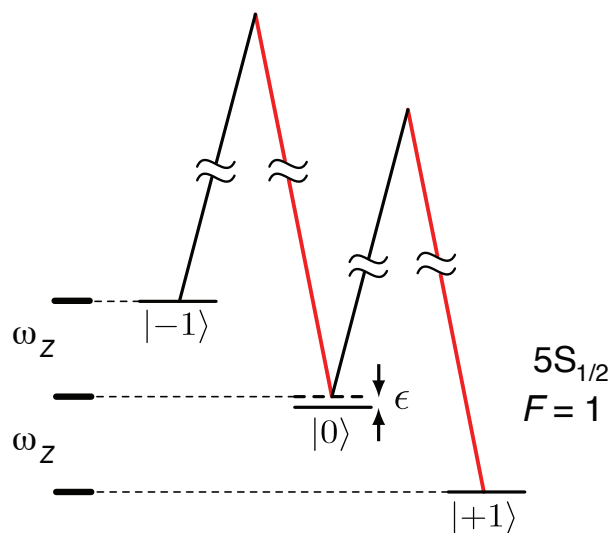
All s-wave



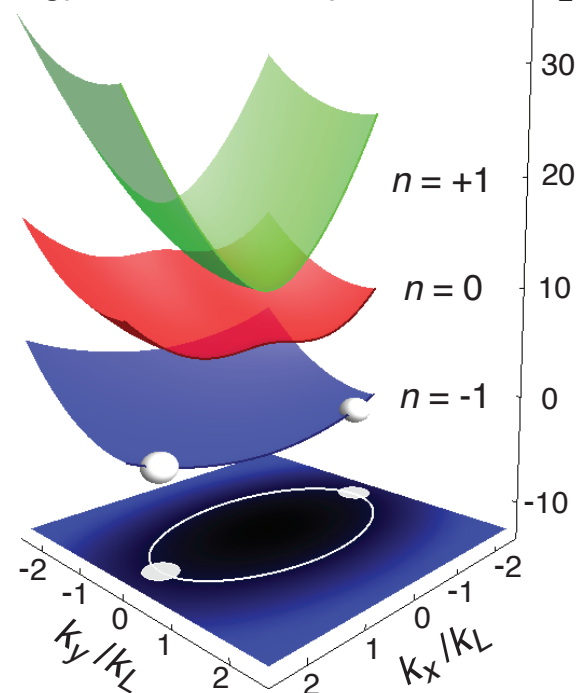
lots of d-wave

Schematic

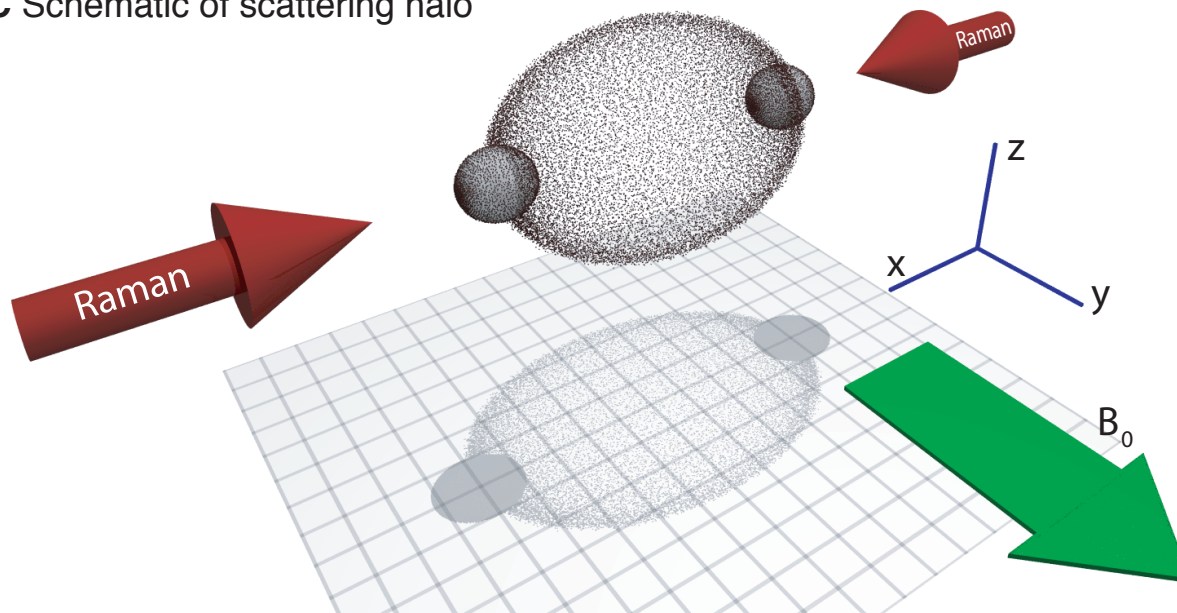
A Three-level coupling scheme



B Energy-momentum dispersion E/E_L

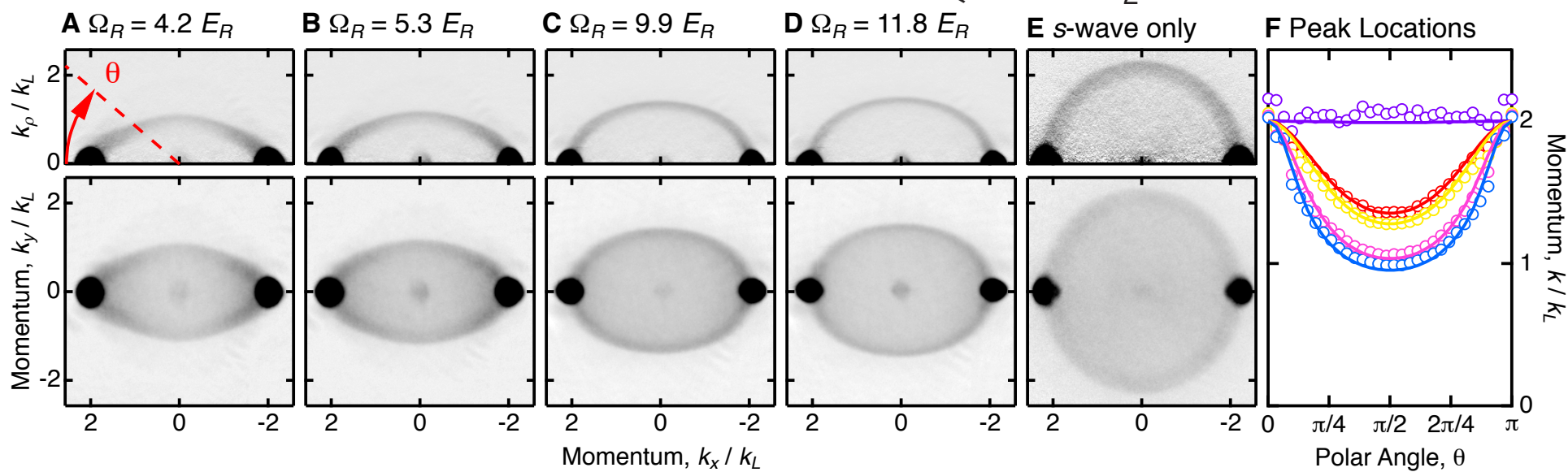
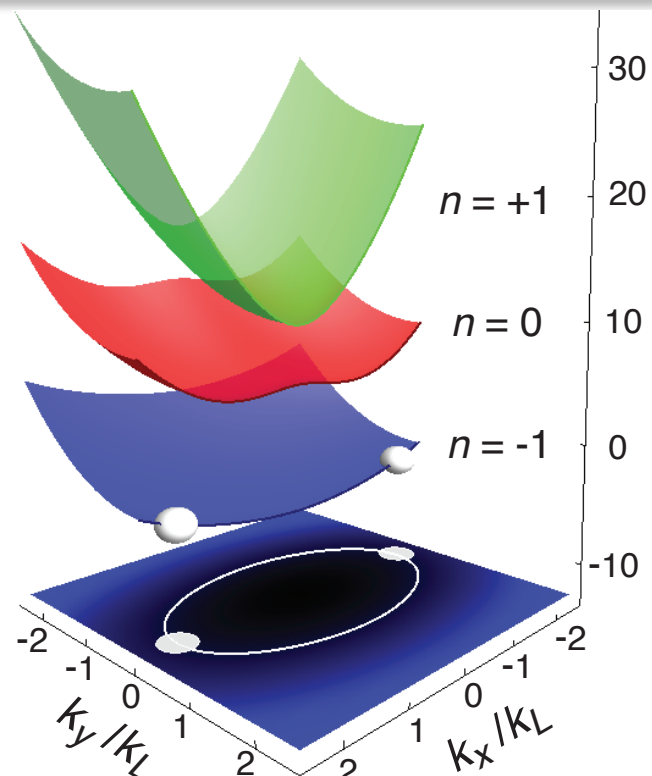
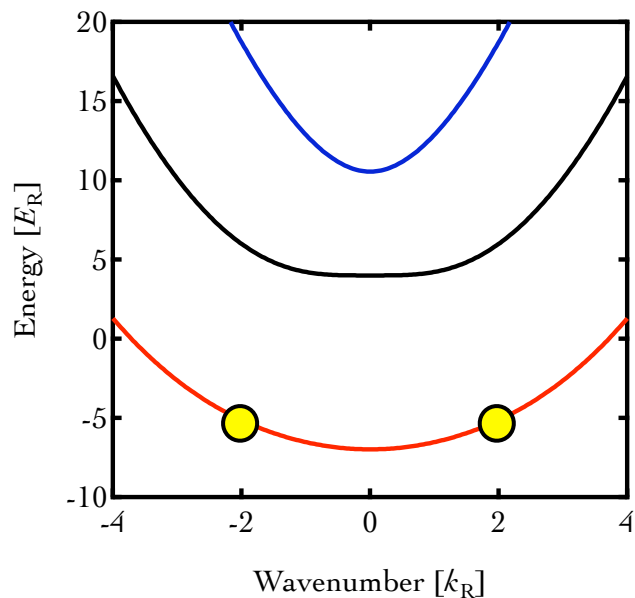


C Schematic of scattering halo



Effective mass effect

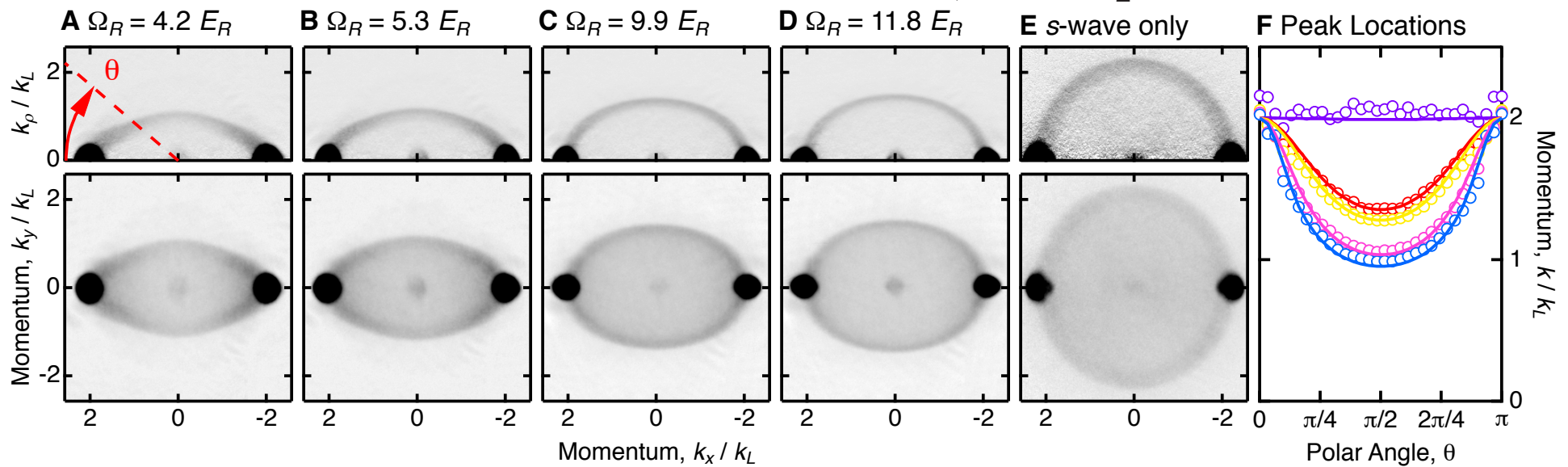
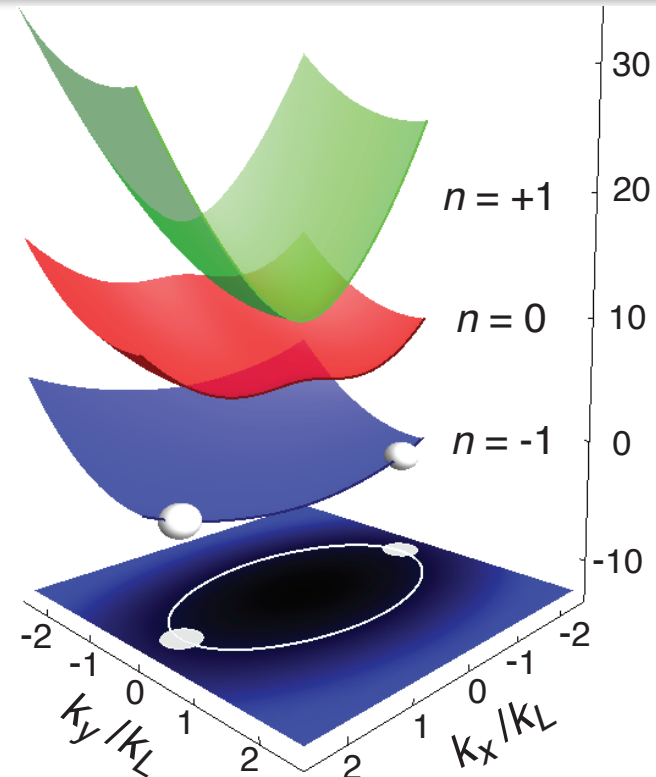
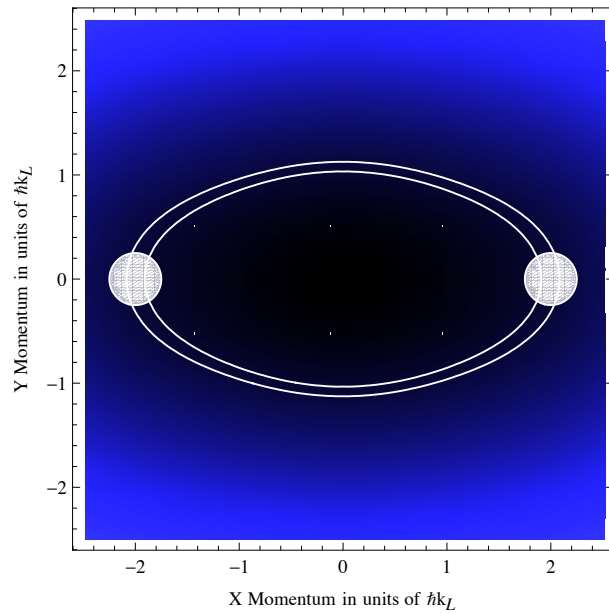
"Band" structure



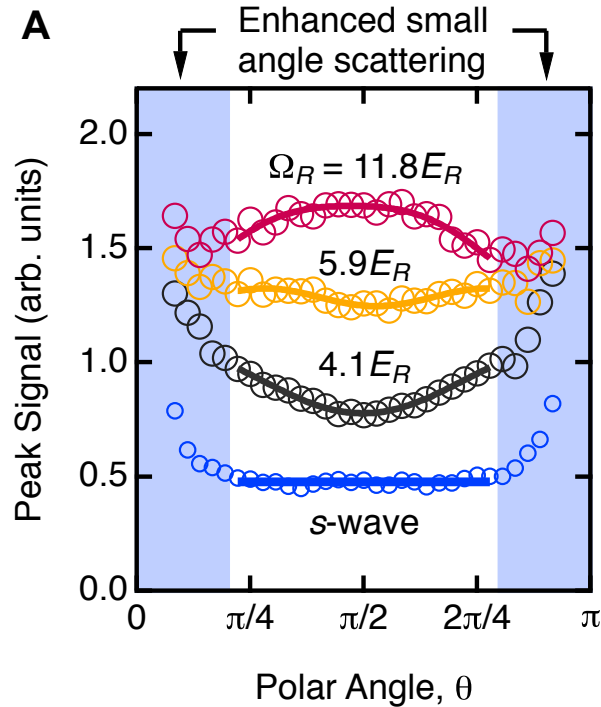
Density of states effect

$$g(E)\delta E = N(E + \delta E) - N(E)$$

$$\Omega_R = 6 E_L$$



Modified collisions

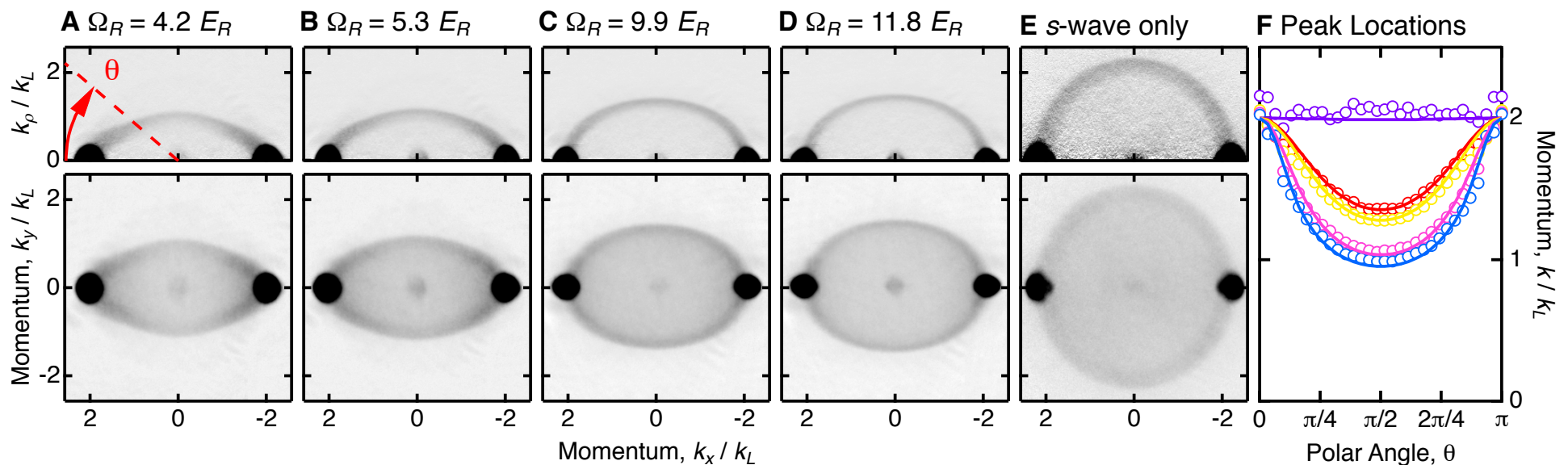


Remove density of states effect

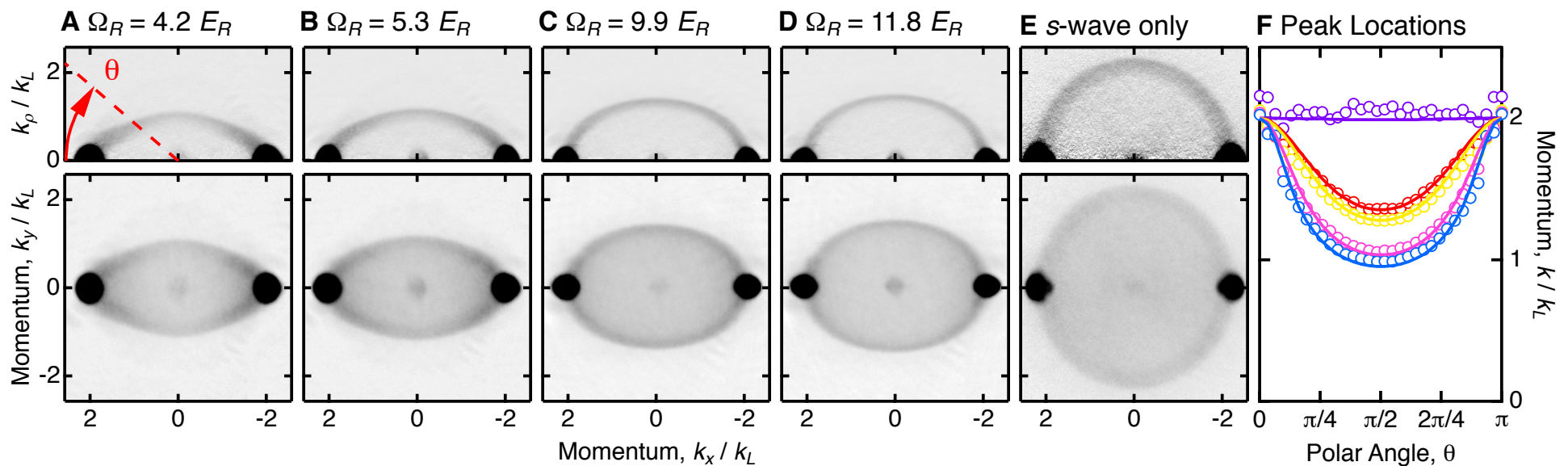
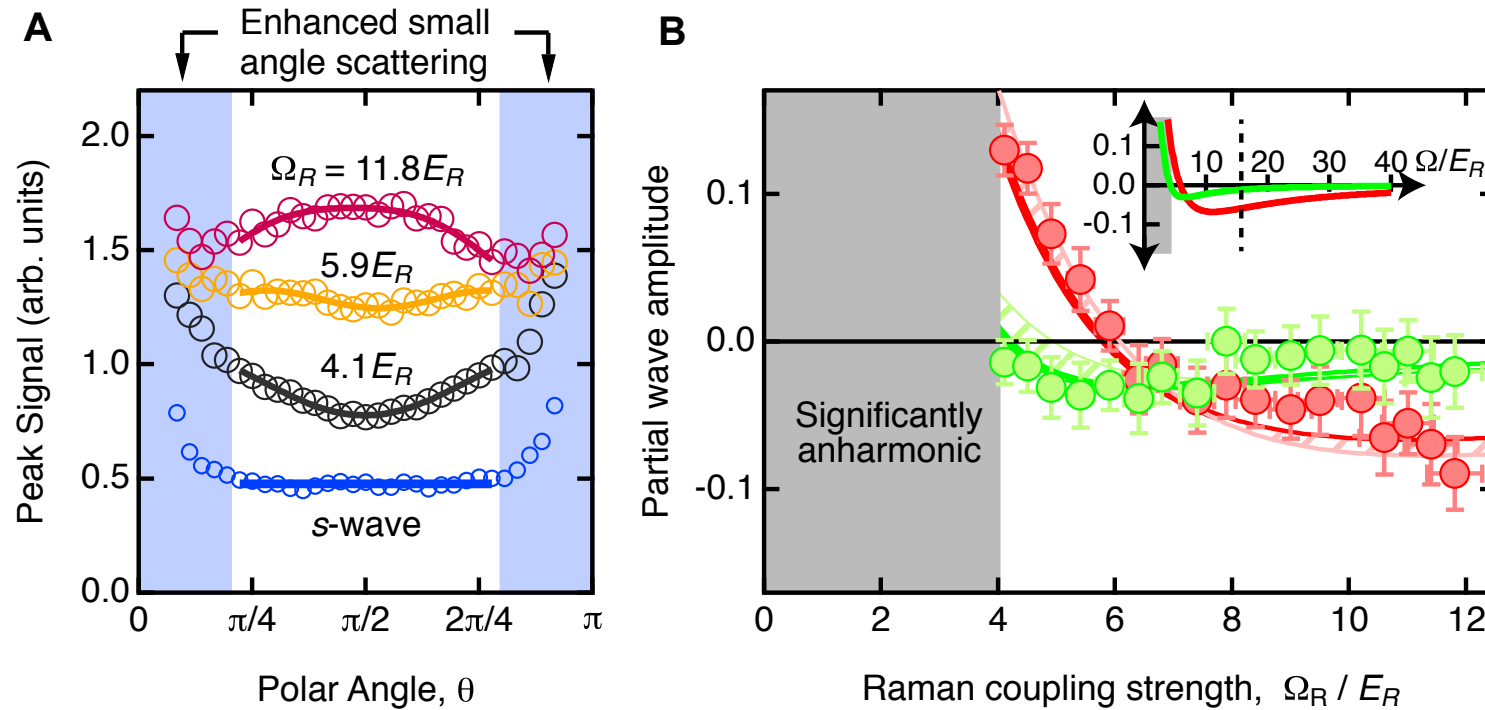
$$\Gamma_i = \frac{2\pi}{\hbar} \int_{S_i} \frac{dS_k}{(2\pi)^2} \frac{|\langle i | \hat{V} | S_k \rangle|^2}{|\nabla_k E(S_k)|}$$

Then fit to partial wave expansion

$$\left| \sum_l (\exp 2i\eta_l - 1)(2l + 1)P_l(\cos(\theta)) \right|^2$$



Modified matrix element effect

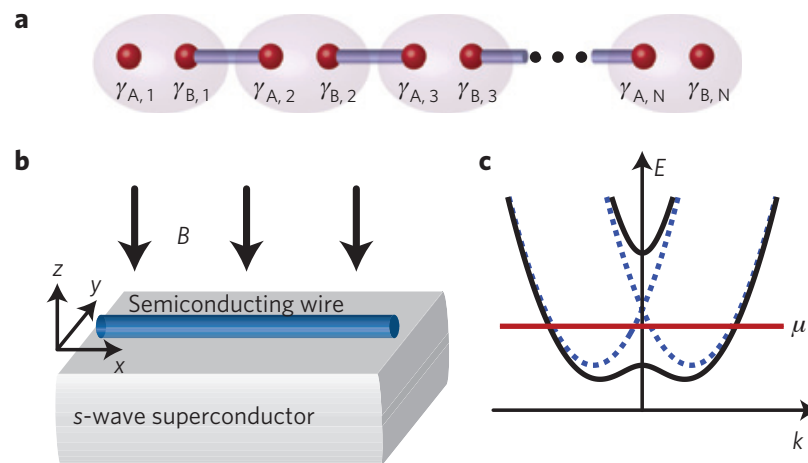
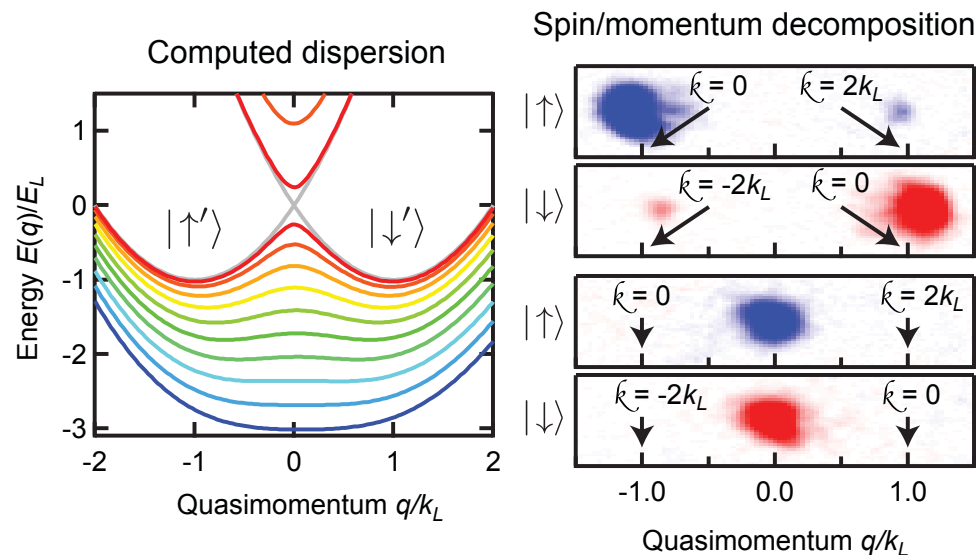


Spin orbit coupling of pseudo spin-1/2 atoms

Moving forward to Fermions

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

for $\Psi^\dagger(\mathbf{x}) = \{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \}$



Refs.

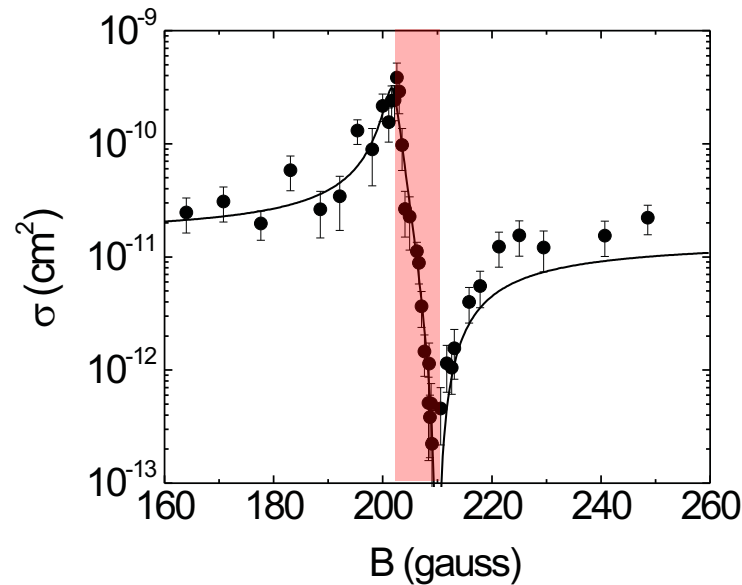
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Pairing: ^{40}K

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \hat{\sigma}_y + \frac{\Omega}{2} \hat{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \left[\Delta \hat{\Psi}_\uparrow(\mathbf{x}) \hat{\Psi}_\downarrow(\mathbf{x}) + \text{h.c.} \right]$$

$E_{B,1D} \propto g^2$
 $E_{B,3D} \propto e^{-\pi/k_F|a|}$, with $g = 2\pi\hbar^2 a/m$

In 1D and 2D robust pairing (at single particle level)
at all attractive coupling strengths



Refs.

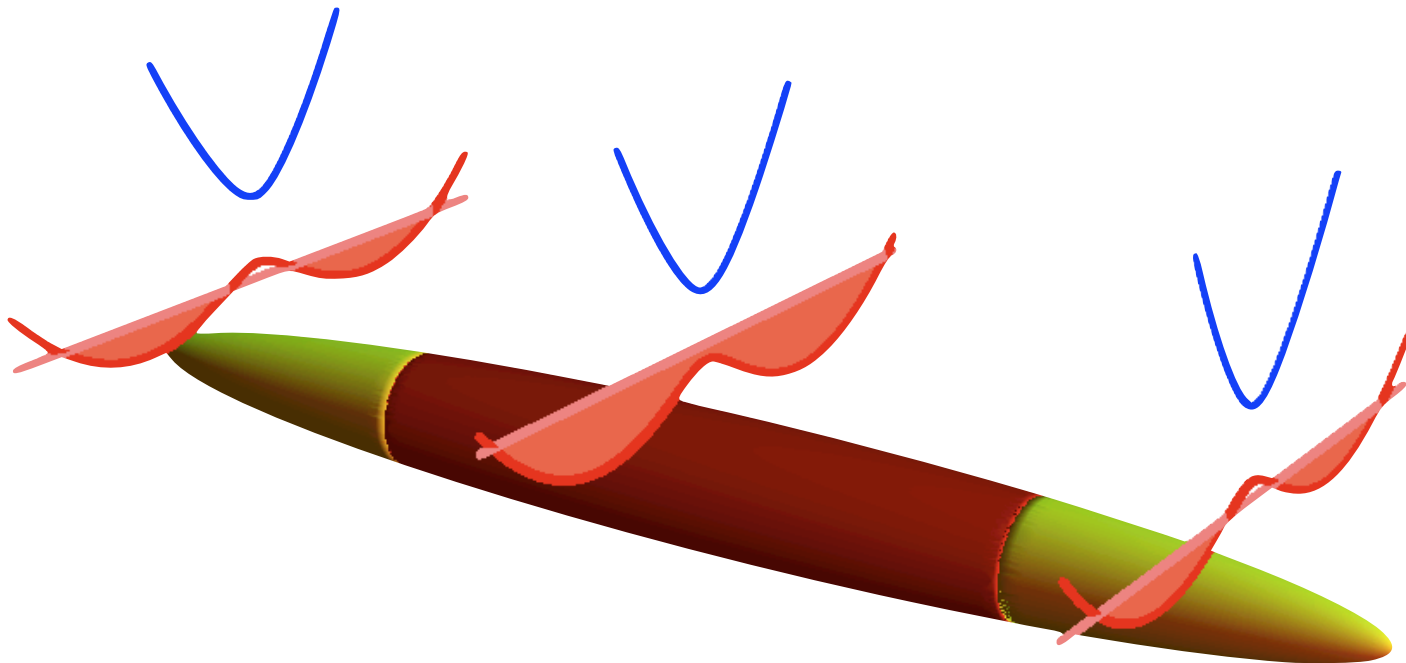
C. Regal (Ph.D. thesis); Bloch, I., Dalibard, J. & Zwierger, Rev. Mod. Phys. 80, 885–964 (2008).

Theory questions

Pairing gap is *intrinsic* not from proximity effect

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left(\frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \hat{\sigma}_y + \frac{\Omega}{2} \hat{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

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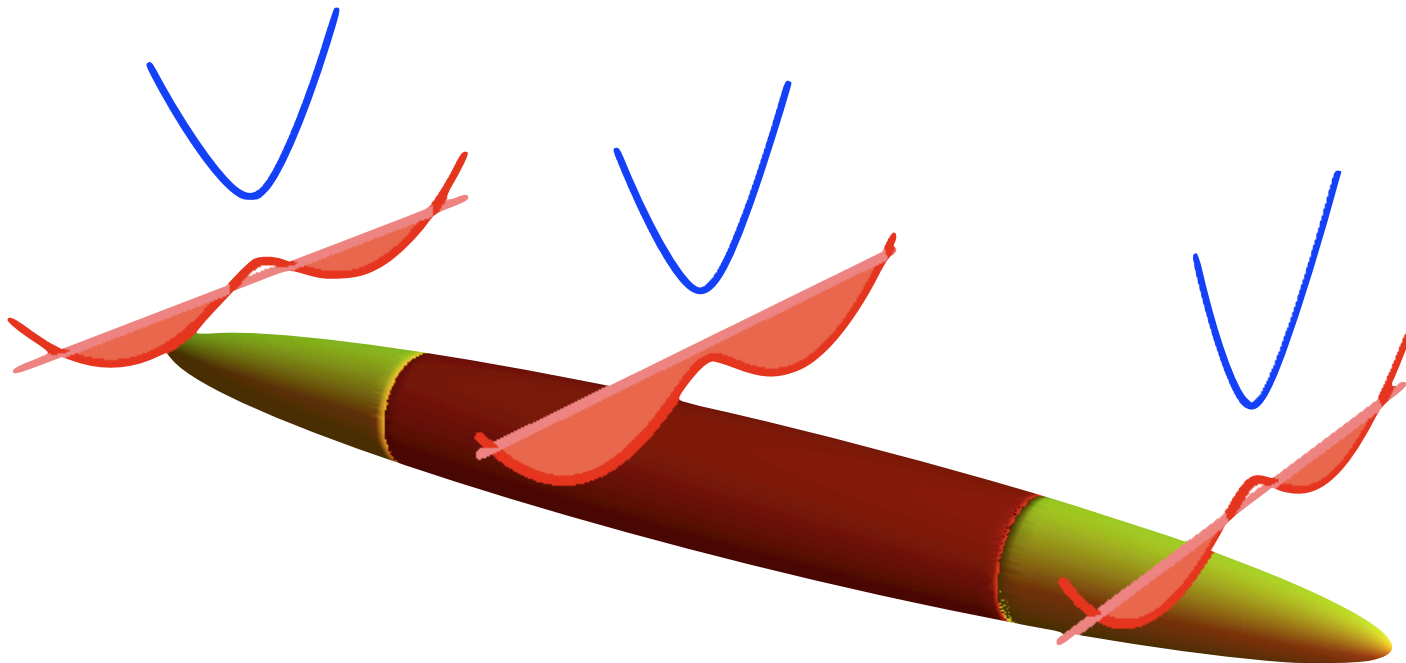


Theory questions

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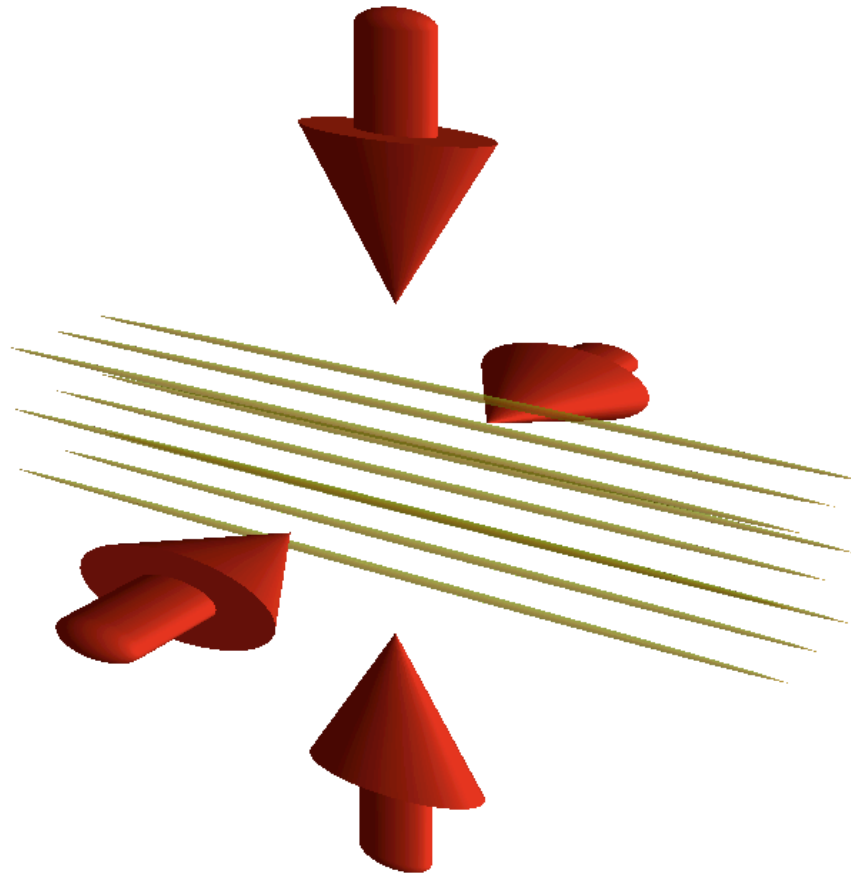


Theory questions

Pairing gap is *intrinsic* not from proximity effect
Coupling between needles important? Finite size important?

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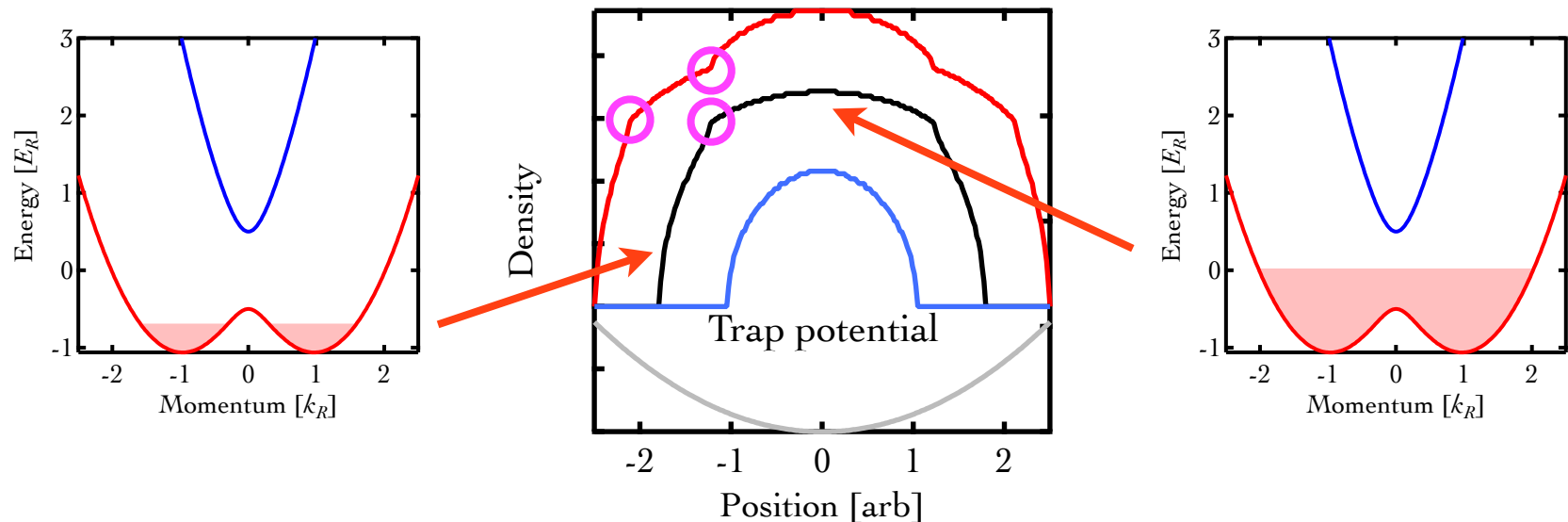
$$\text{for } \Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$$



Theory questions

Pairing gap is *intrinsic* not from proximity effect
Coupling between needles important? Finite size important?

What happens at μ smoothly changes and crosses from
TSC to SC phases?



If Majorana fermions's do form at these interfaces, how to detect?

The ugly: ^{40}K

Lifetime, lifetime, stability

Spontaneous emission
(lights scattering)
 $t \sim 0.5 - 1 \text{ s}$ (double well)
 $t \sim 0.1 \text{ s}$ (single well)

Feshbach losses
(molecule formation)
 $t \sim 1 \text{ s}$ ($1/k_F a \sim -1$)
 $t \sim 0.1 \text{ s}$ ($1/k_F a \sim 0$)

$\sim 200 \text{ } \mu\text{G}$ absolute
stability at 200 G
(1 ppm)

Gauge fields



Mixtures

