

# Ultracold atoms: spin orbit coupling and engineered interactions

I. B. Spielman

Current team

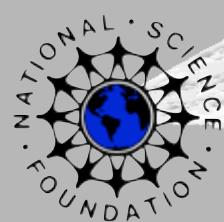
R. A. Williams, L. J. LeBlanc, K. Jimenez-Garcia, M. Beeler, and A. R. Perry

Alum

Y.-J. Lin, and R. Compton

Senior coworkers

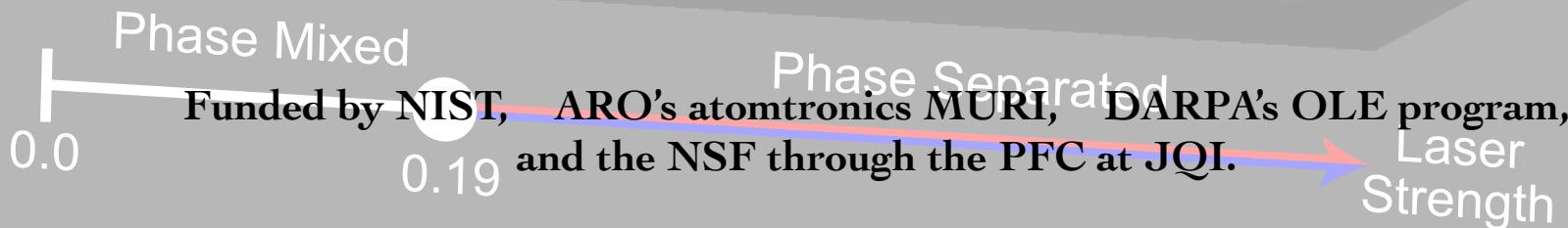
J. V. Porto, and W. D. Phillips



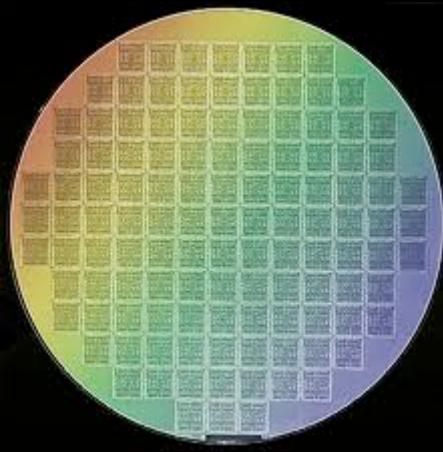
**NIST**

**National Institute of Standards and Technology**  
Technology Administration, U.S. Department of Commerce

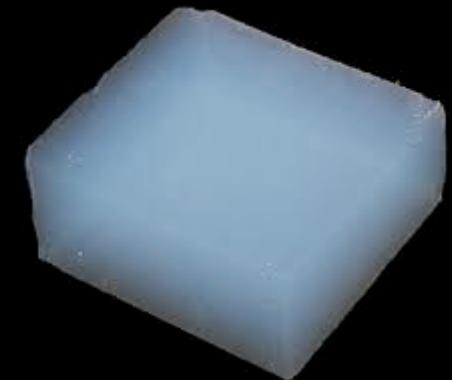
**jqi**



# What are materials?



Si  
2.3 g/cm<sup>3</sup>



Aerogel  
1 mg/cm<sup>3</sup>

**Ian's answer I: "chunks of stuff."**

Liquid Helium  
125 mg/cm<sup>3</sup>

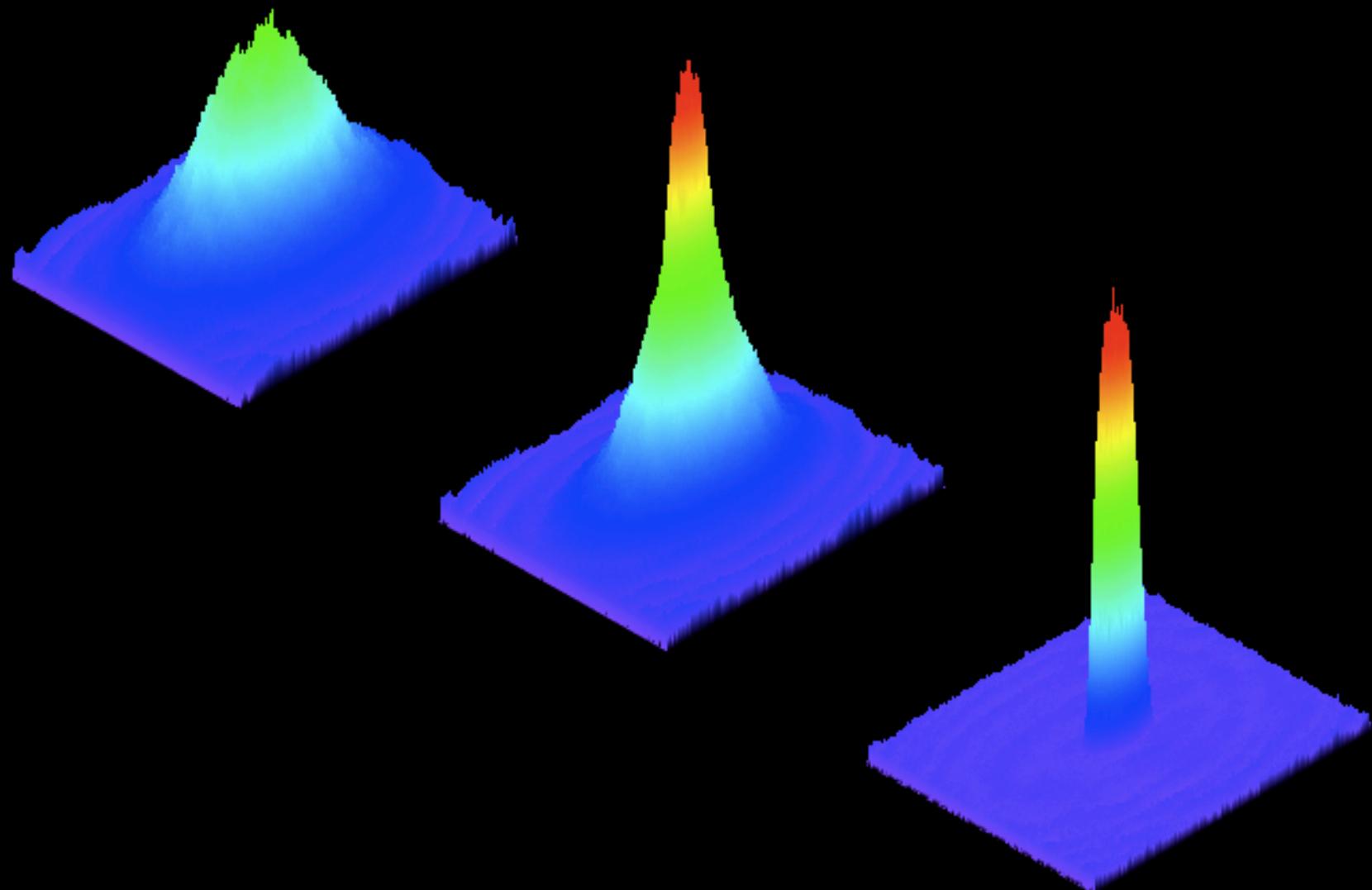


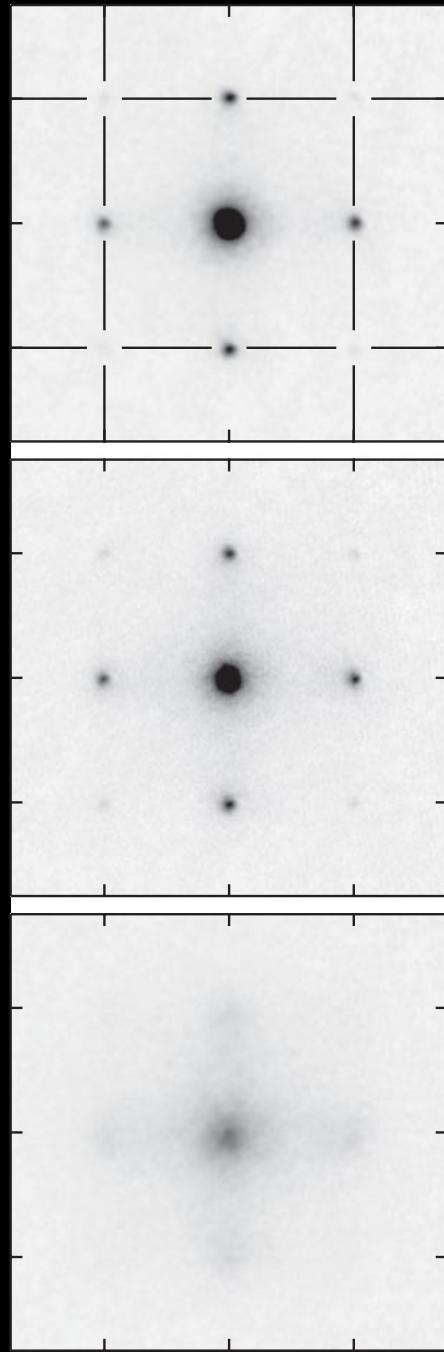
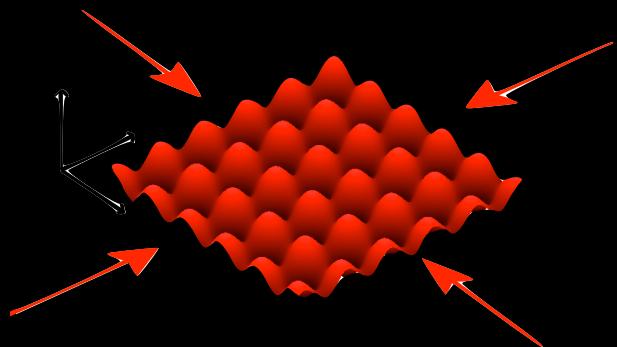
## Ultracold neutral atoms

$\sim 10^{14} \text{ cm}^{-3}$  or  $100 \text{ ng/cm}^3$

(air is  $\sim 1 \text{ mg/cm}^3$ )

Are these materials?



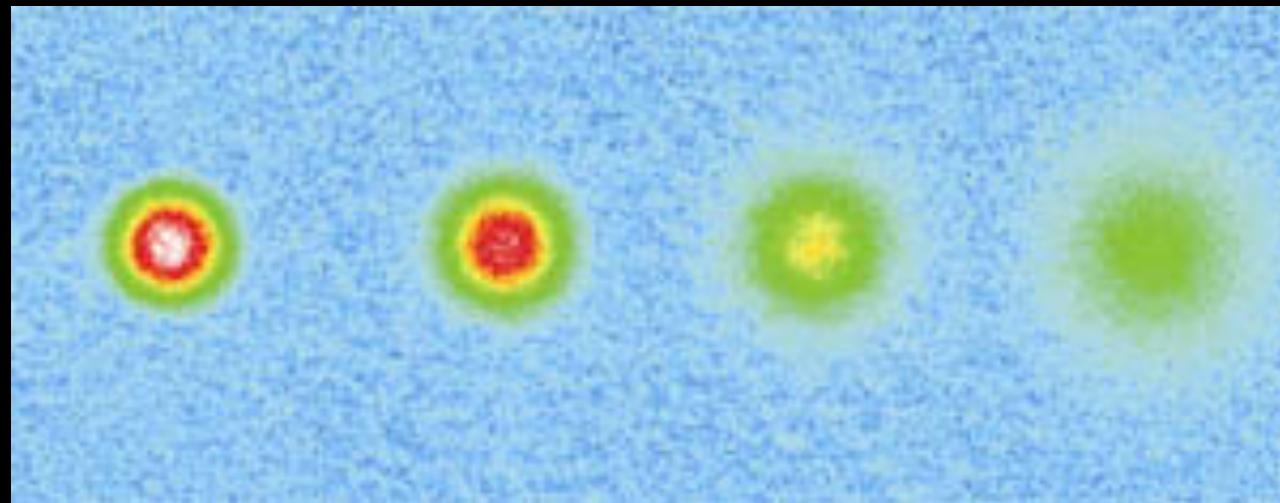


They can be fluids

They can be insulators

They can be bosons

They can be fermions

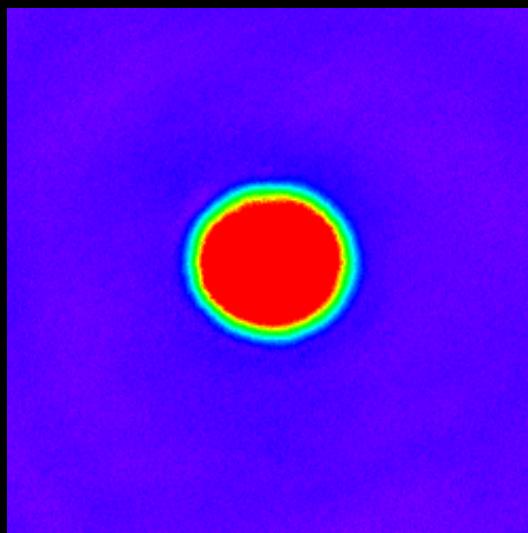


They can be molecules

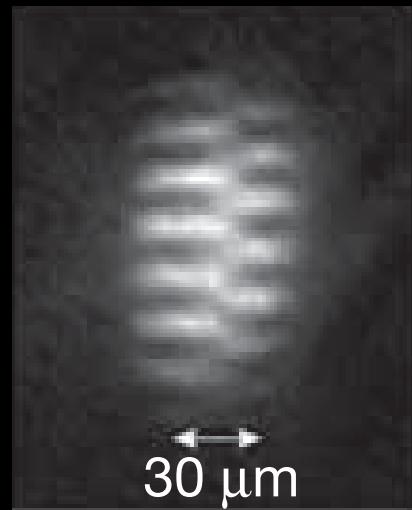
They can be atoms

e.g., Regal Nature (2003)

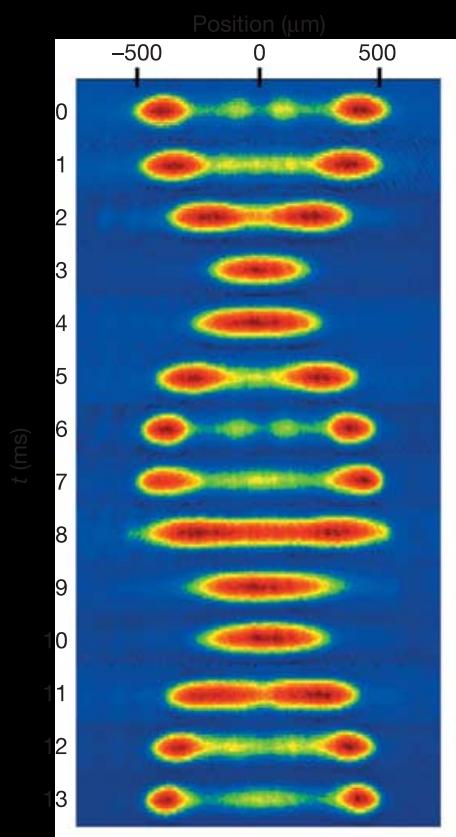
They can be 3D



They can be 2D



They can be 1D



e.g., Hadzibabic *Nature* (2006)

e.g., Kinoshita *Nature* (2006)

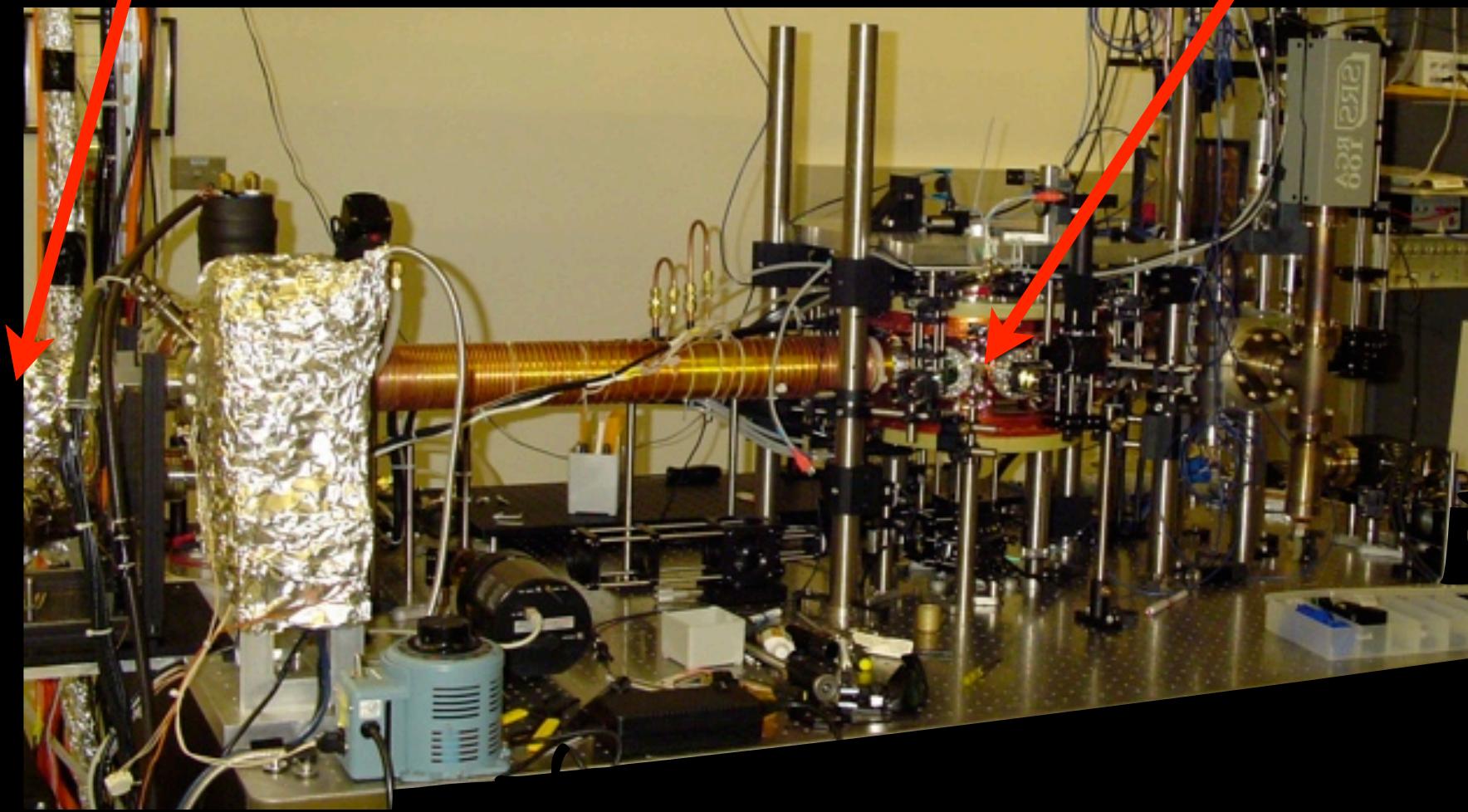
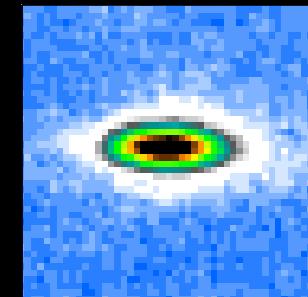
Starts like this

400 K

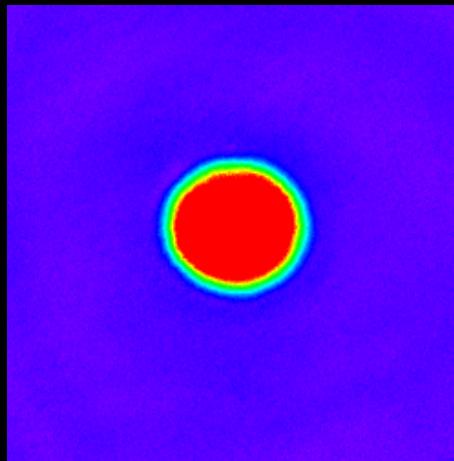


Ends here every 20 s

50 nK



# What are materials?

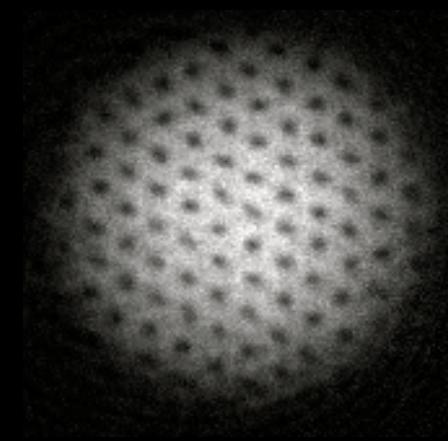


Ian's answer: "stuff"

In a finite volume of space

Have "mechanical properties"

Or "fluid properties"



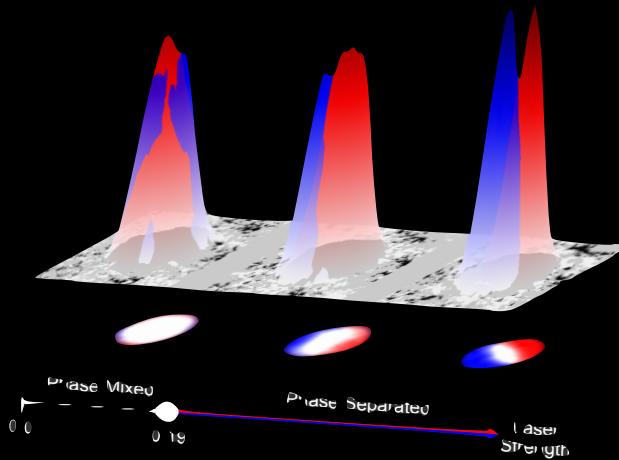
Cold atoms are: good materials

Numerous properties can  
be controlled on many timescales

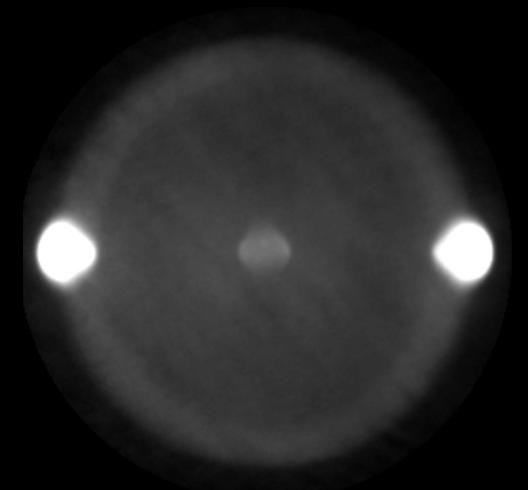
Very simple Hamiltonian

Cold atoms are: bad materials

Short lived

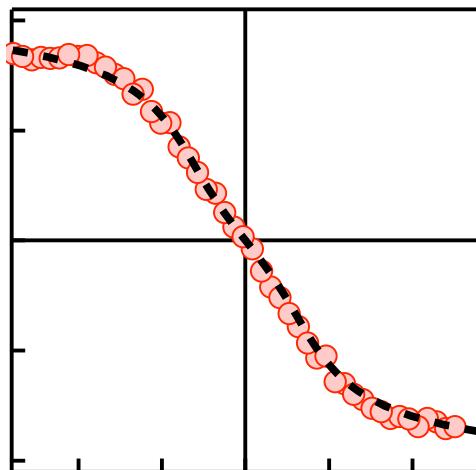


Interesting features **all** added  
by hand (complex experiments).

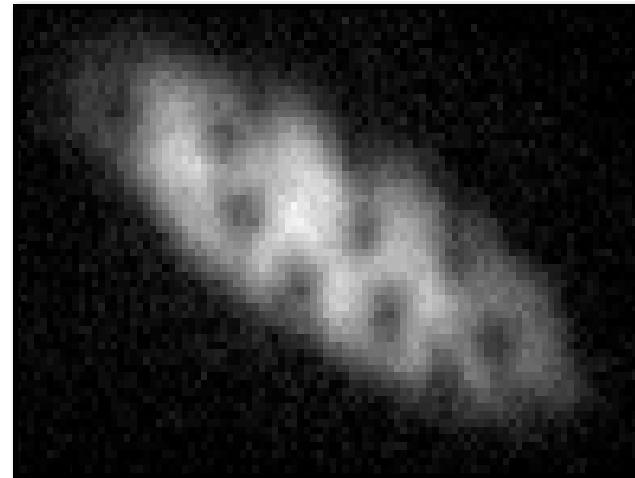


# Artificial gauge fields

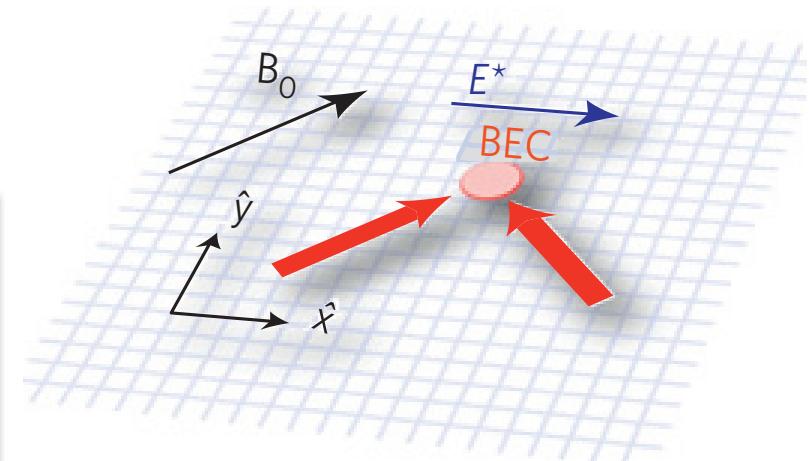
Vector potential



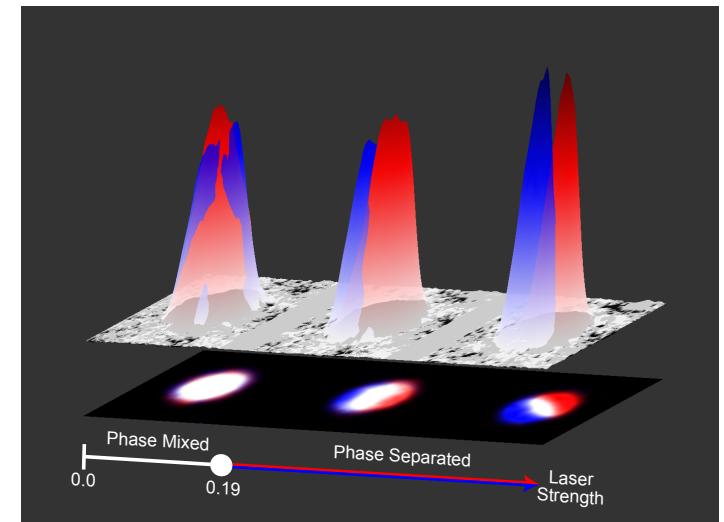
Magnetic field



Electric field



Spin Orbit coupling



$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

Can be a matrix

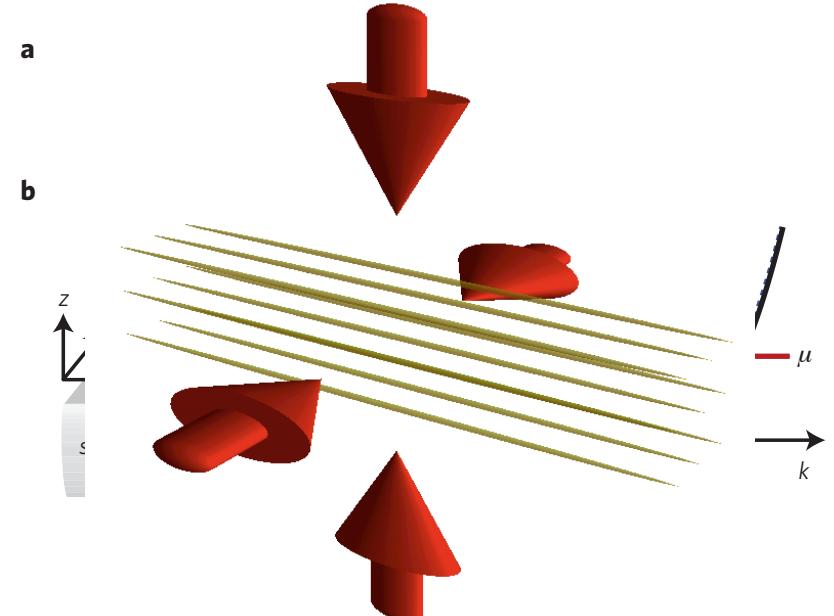
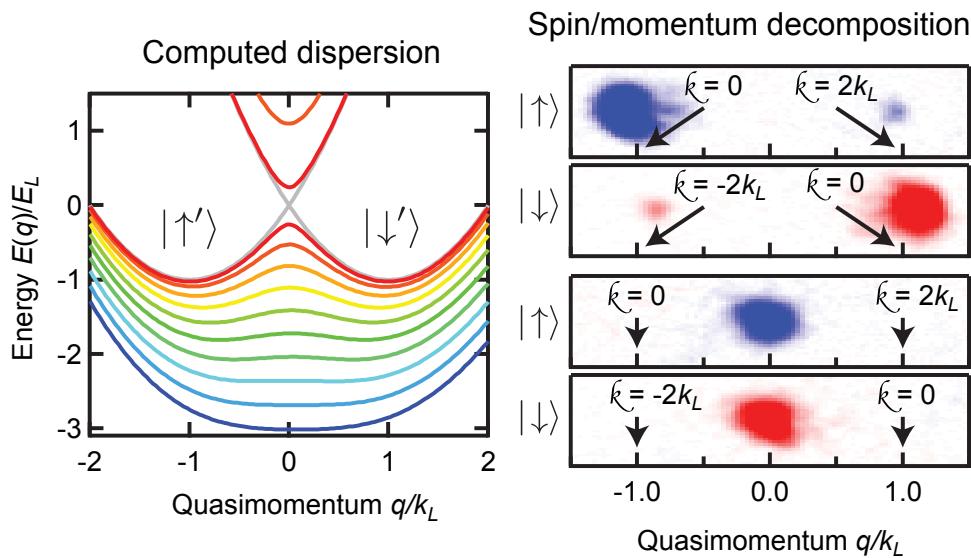
# Spin orbit coupling of pseudo spin-1/2 atoms

## Current experiments with Bosons

Spin orbit coupling for pseudo spin 1/2 **Bosons** (testbed platform)

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

for  $\Psi^\dagger(\mathbf{x}) = \{\psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x})\}$



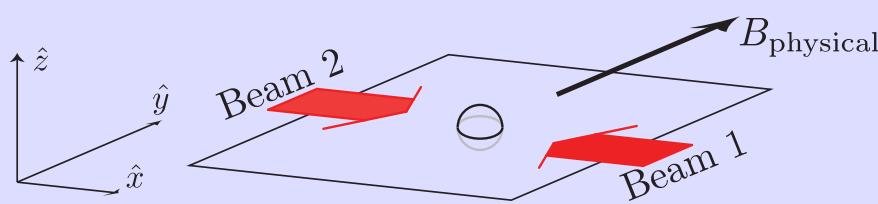
Refs.

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# Spin orbit coupling with ultracold atoms

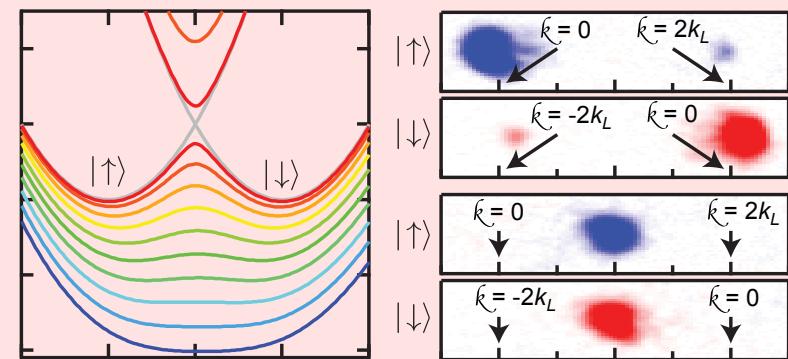
## Raman dressed states

### Concept



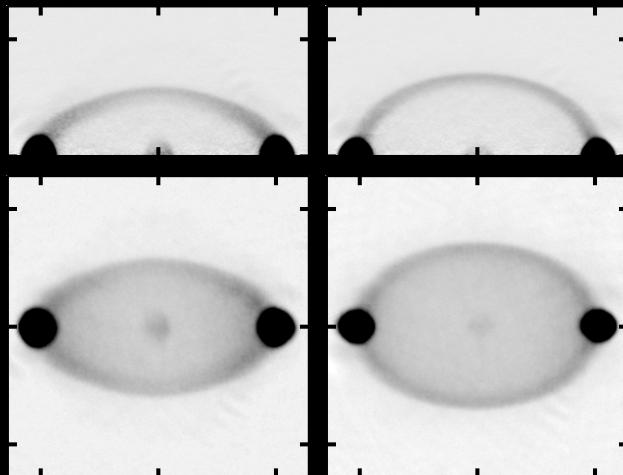
## Spin orbit coupling, engineered

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \mathbf{1} - [\mathbf{B} + \mathbf{B}_{SO}(\hat{\mathbf{k}})] \cdot \hat{\mathbf{p}} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \mathbf{1} + \frac{\Omega}{2} \hat{\sigma}_z + \frac{\delta}{2} \hat{\sigma}_y + 2\alpha \hat{k}_x \hat{\sigma}_y$$

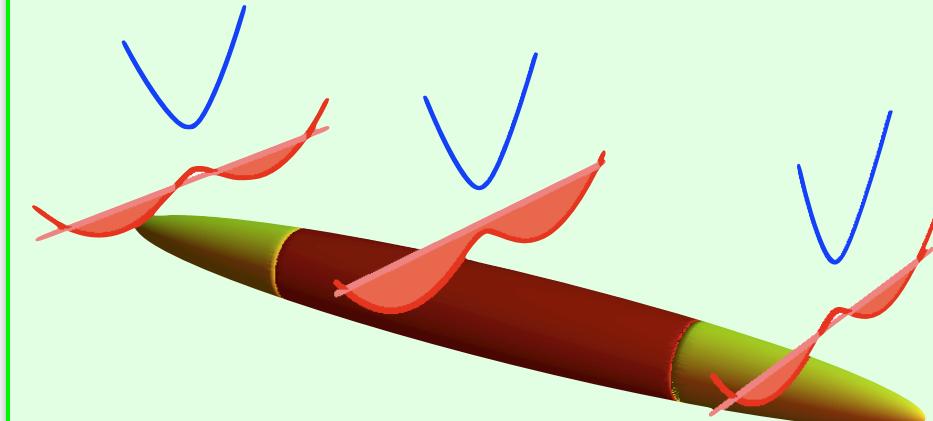


## Interactions, rearranged

effective d- and g- wave interactions in bosons



## Fermions, the future

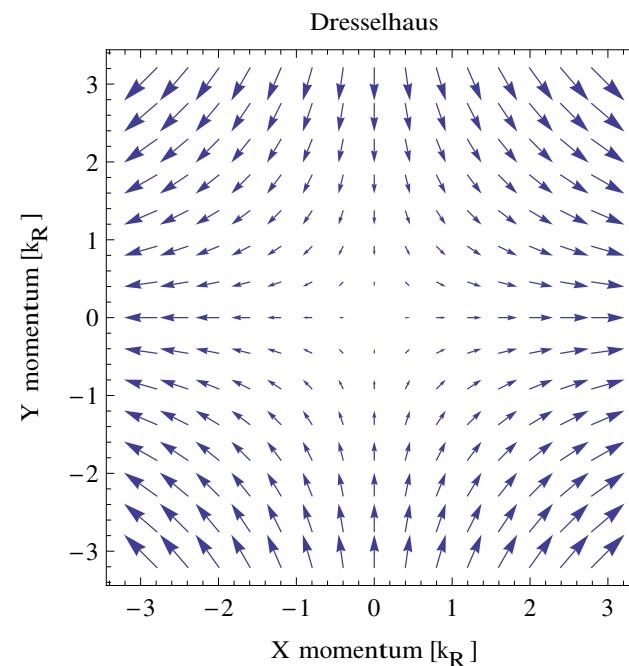
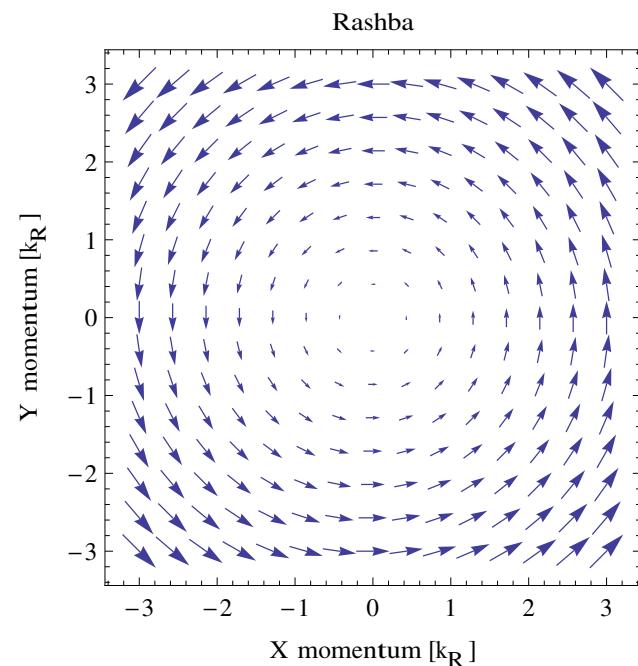


# Spin orbit coupling: what do we desire?

## Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbf{\hat{1}} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

$\alpha$  gives the strength of the Rashba coupling;  $\beta$  yields the linear Dresselhaus coupling; and  $\delta$  produces a Zeeman splitting

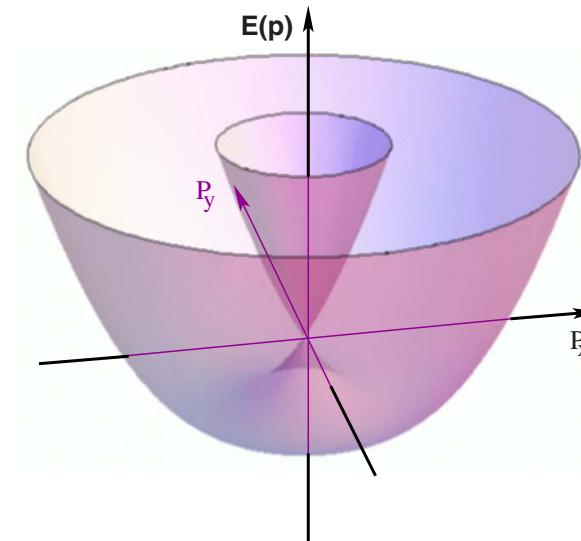
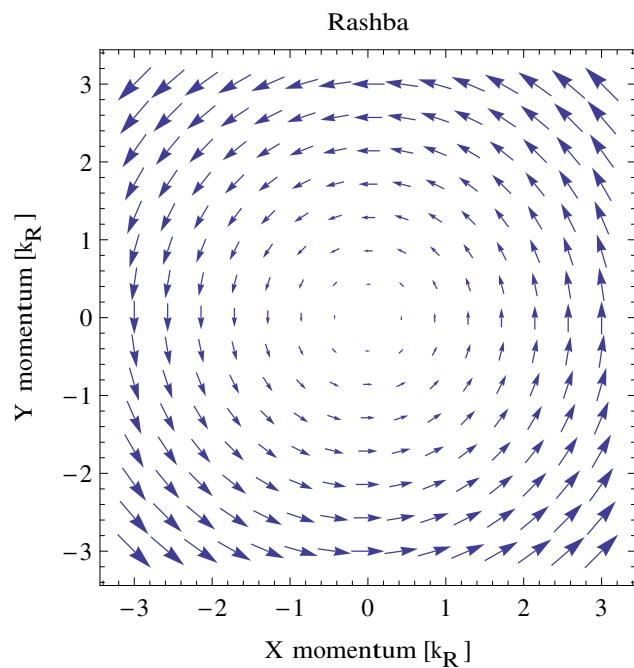


# Spin-orbit coupling: Rashba

## Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbf{\hat{1}} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

Pure Rashba:  $\beta = 0$



Cold atom Refs.

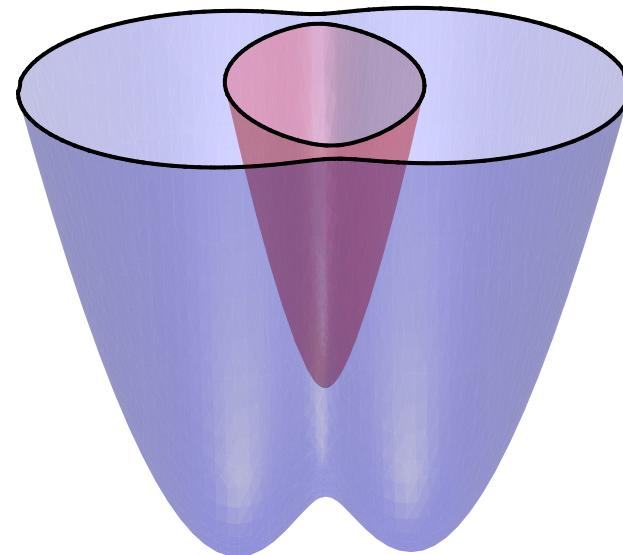
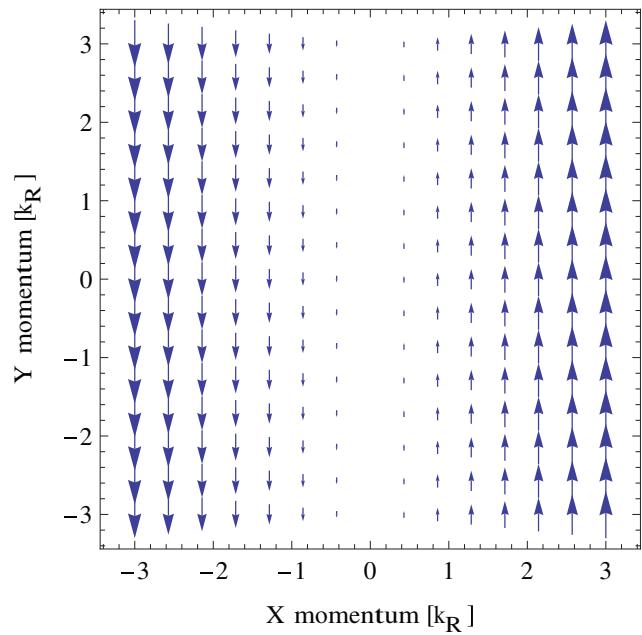
T. D. Stanescu and B. Anderson and V. Galitski PRA (2008)

# Spin-orbit coupling: Rashba = Dresselhaus

## Spin-orbit coupling

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \mathbf{1} + 2\alpha k_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z$$

Equal Rashba and Dresselhaus:  $\alpha = \beta$



## GaAs Refs.

J. D. Koralek et al, Nature (2009); C. H. L. Quay et al, Nat. Phys. (2010)

# Non-abelian gauge fields and Spin-Orbit coupling

## Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbf{\check{1}} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

## Uniform Non-abelian gauge field

$$\check{H} = \frac{\hbar^2}{2m} \left[ \left( k_x \check{1} - \frac{q}{\hbar} \check{A}_x \right)^2 + \left( k_y \check{1} - \frac{q}{\hbar} \check{A}_y \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_z + E_0 \check{1}$$

Spin-orbit coupling *is* a (sometimes) non-abelian gauge field!

$$\check{H} = \frac{\hbar^2}{2m} \left\{ \left[ (k_x \check{1} + \frac{1}{2} (\alpha \check{\sigma}_y + \beta \check{\sigma}_x) \right]^2 + \left[ (k_y \check{1} - \frac{1}{2} (\alpha \check{\sigma}_x + \beta \check{\sigma}_y) \right]^2 \right\} + \frac{\delta}{2} \check{\sigma}_z + E_0 \check{1}$$

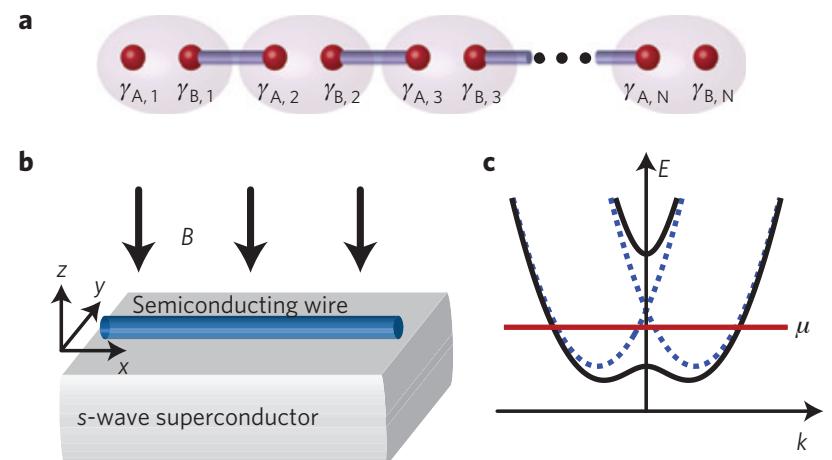
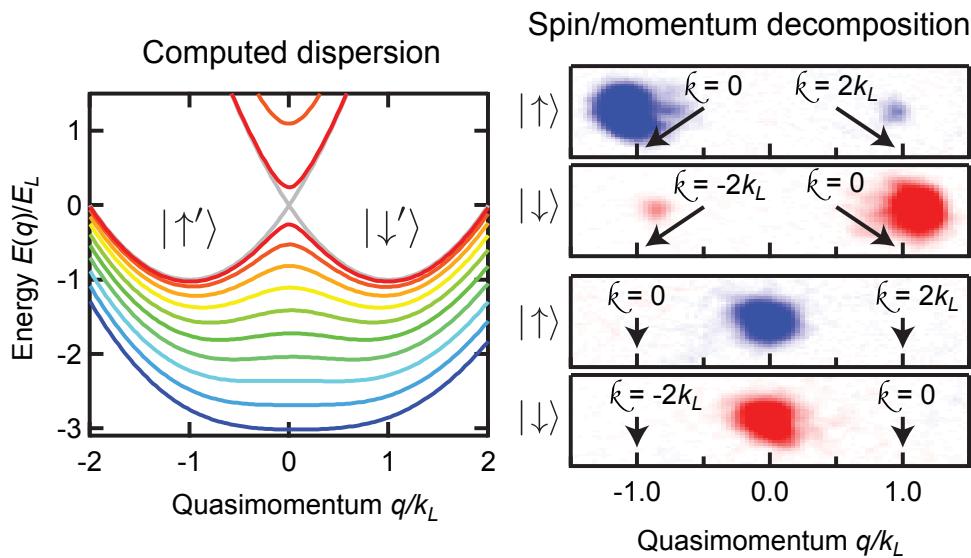
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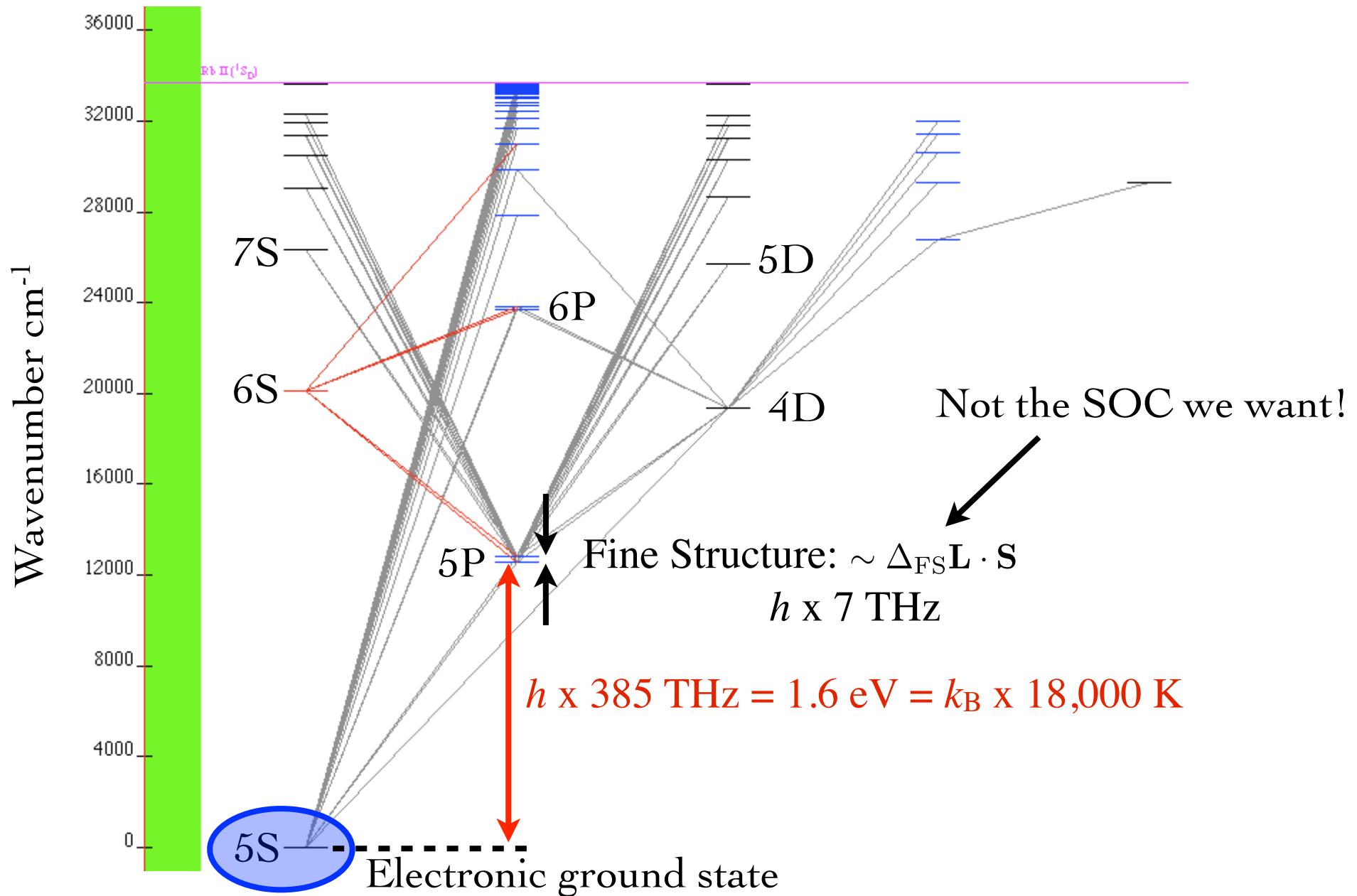
for  $\Psi^\dagger(\mathbf{x}) = \{\psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x})\}$



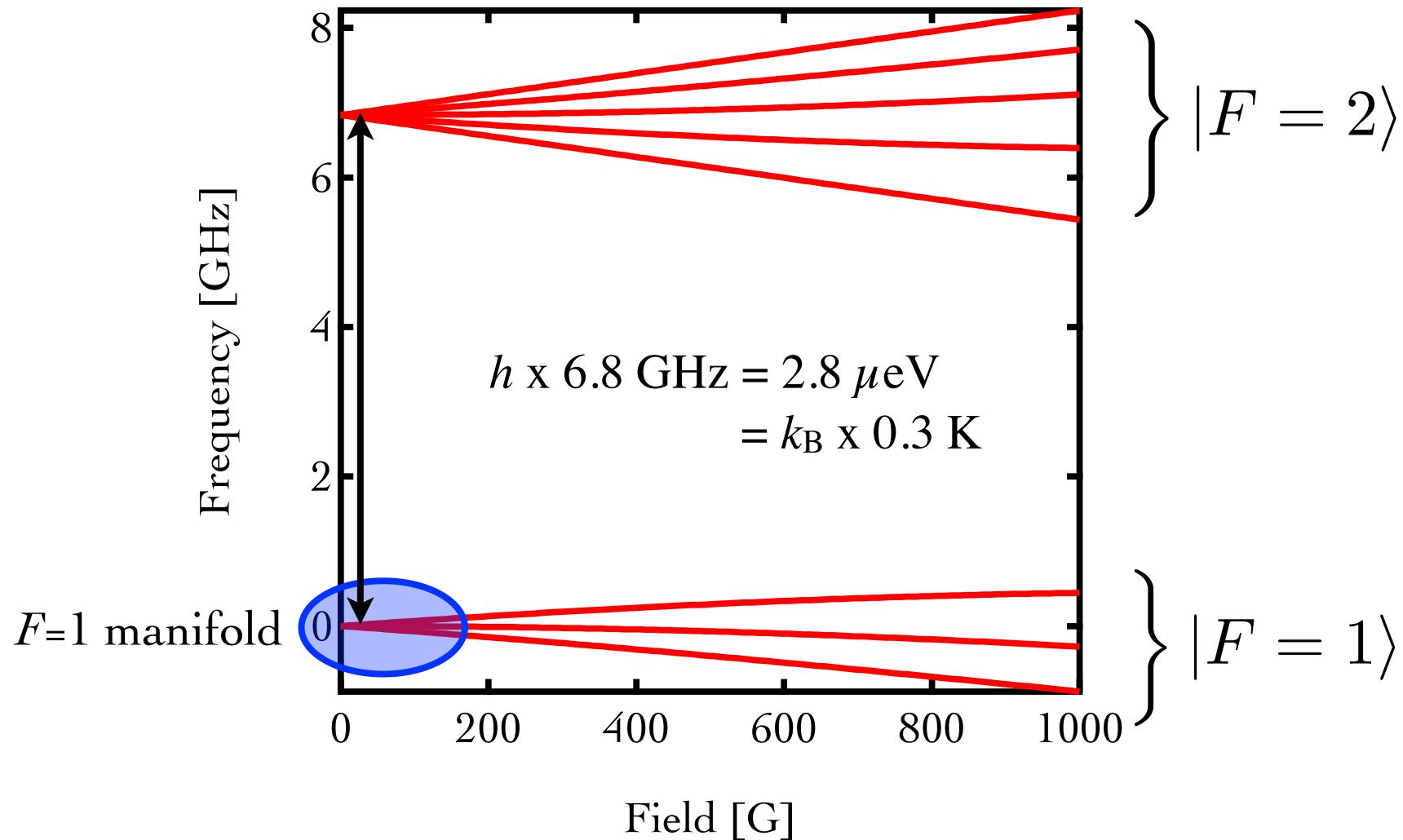
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# Rubidium 87: “The GaAs of atoms”



# Rubidium 87: $5S_{1/2}$ ground state

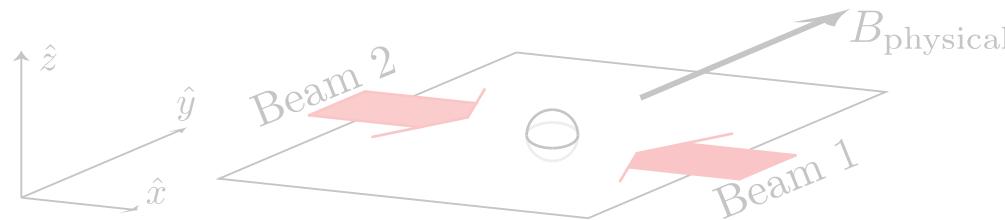


# Engineered spin-orbit coupling

## Momentum representation

$$H = \sum_k \left\{ \left( \langle k-1, \uparrow | \quad \langle k+1, \downarrow | \right) \left( \begin{array}{cc} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{array} \right) \left( \begin{array}{c} |k-1, \uparrow \rangle \\ |k+1, \downarrow \rangle \end{array} \right) \right\}$$

## Geometry



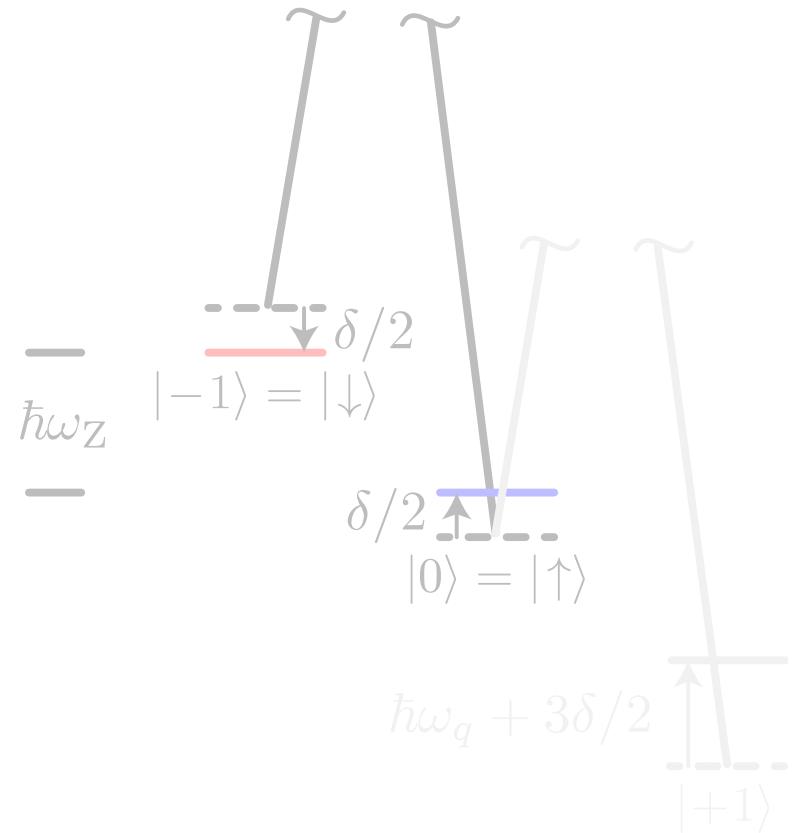
## Natural dimensions

Length:  $\lambda \approx 790$  nm

Momentum:  $\hbar k_r = 2\pi/\lambda$

Energy:  $E_r = \frac{\hbar^2 k_r^2}{2m} \approx h \times 3.4$  kHz = 14 peV

## Levels

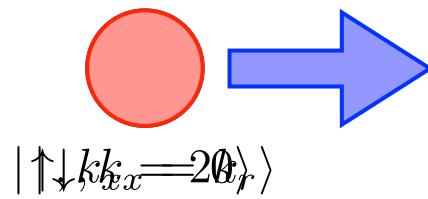
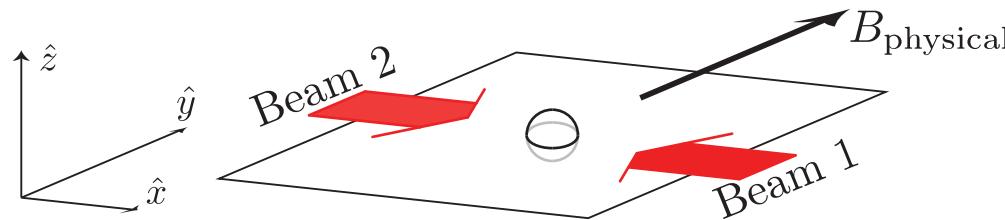


# Engineered spin-orbit coupling

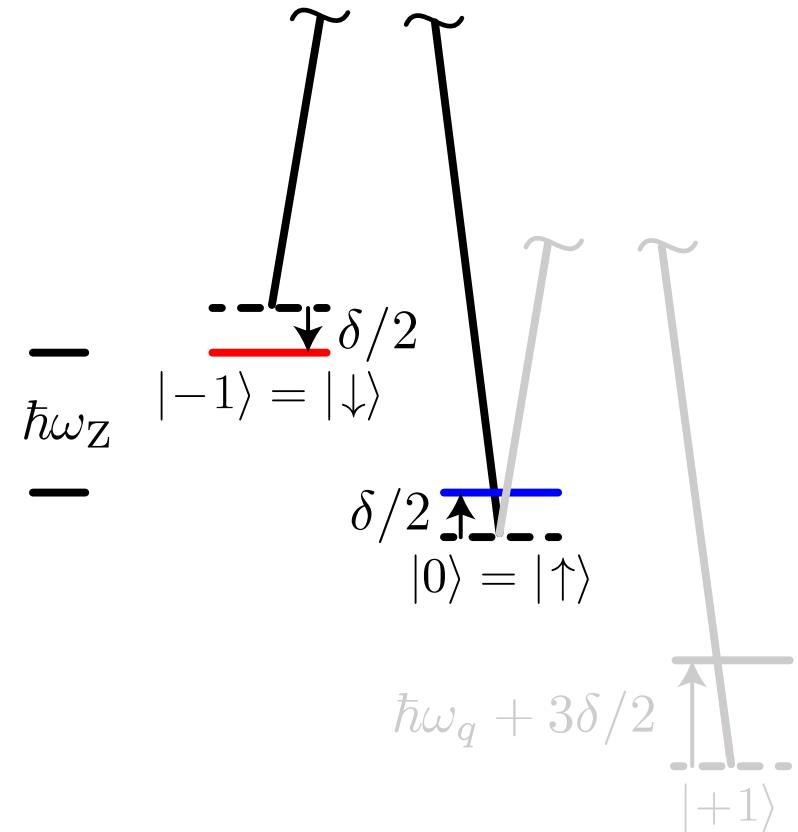
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## Geometry



## Levels

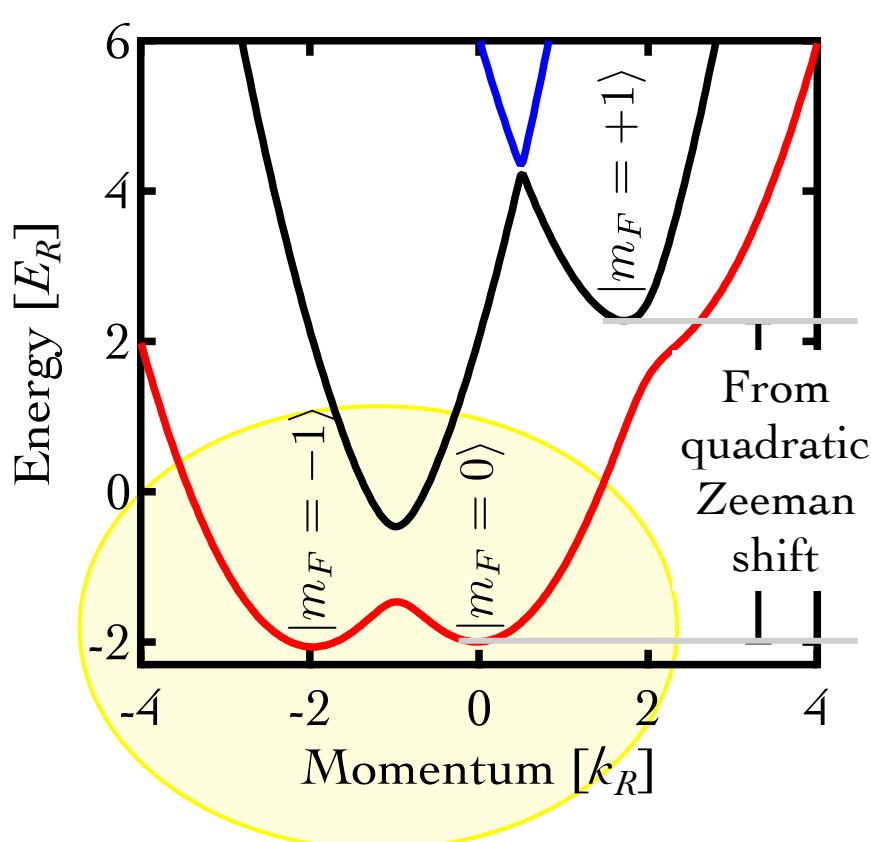


# Spin orbit coupling: origin

## Momentum representation

$$H = \sum_k \left\{ \left( \langle k-1, \uparrow | \quad \langle k+1, \downarrow | \right) \left( \begin{array}{cc} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{array} \right) \left( \begin{array}{c} |k-1, \uparrow \rangle \\ |k+1, \downarrow \rangle \end{array} \right) \right\}$$

Spin 1/2 bosons????



Transform to

$$\begin{aligned} \hat{H} &= \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \left( \frac{\delta}{2} + \frac{\hbar^2 k_R}{m} \hat{k}_x \right) \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z + \Delta E \hat{1} \\ &= \frac{\hbar^2}{2m} \left[ \left( \hat{k}_x \hat{1} + k_R \check{\sigma}_y \right)^2 + \left( \hat{k}_y \hat{1} - 0 \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \end{aligned}$$

NOTICE

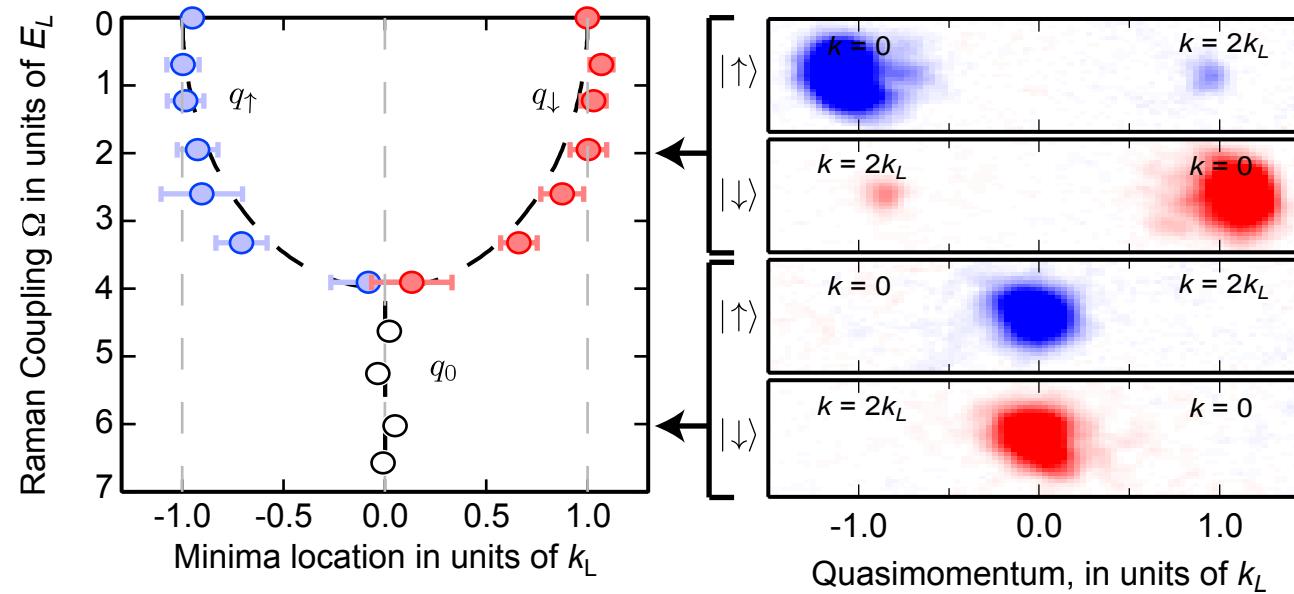
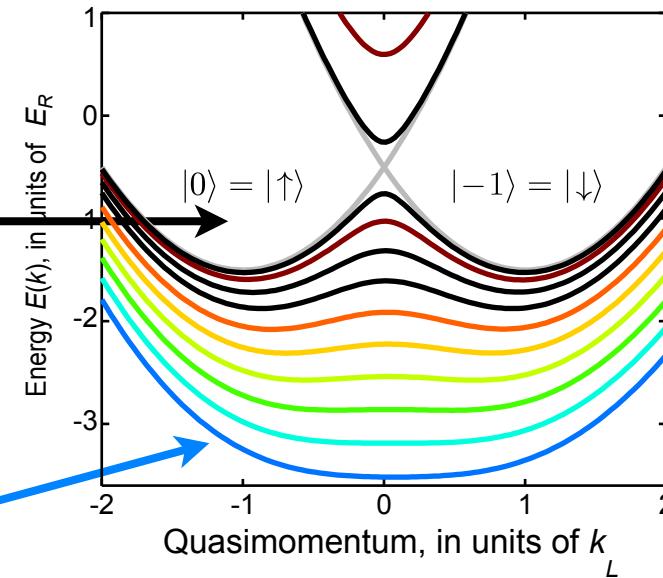
Written as a “2x2” vector potential, this S-O coupling is Abelian,  $A_y = 0$ : so  $[A_x, A_y] = 0$ .

However, the Hamiltonian is non-trivial owing to the Zeeman field along  $z$ .

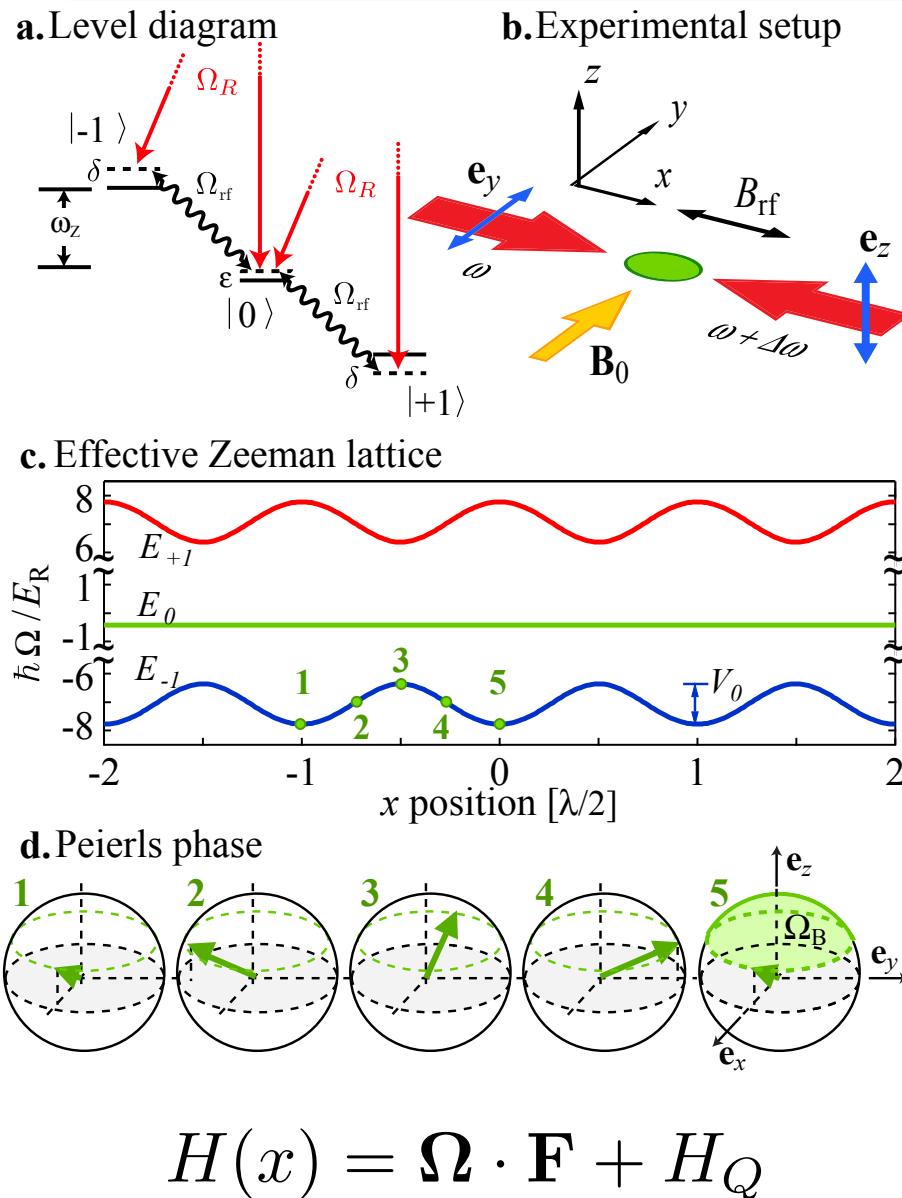
# Typical data

Double Well  
“Spin-orbit limit”

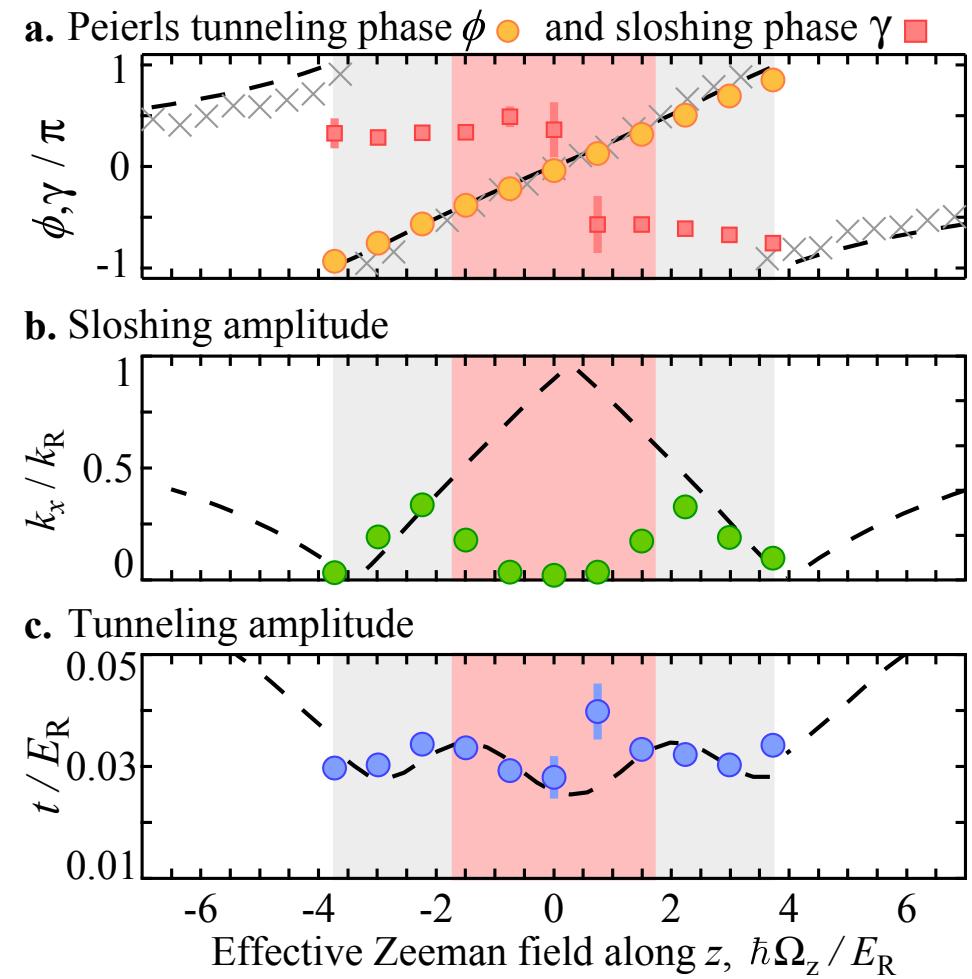
Single minimum  
“vector potential limit”



# Peierls lattice



$$\boldsymbol{\Omega} = \{\Omega_{\text{rf}} + \Omega_R \cos(2k_R x), -\Omega_R \sin(2k_R x), \sqrt{2}\delta\}$$



$$H = \sum_j [t \exp(i\phi) \hat{a}_{j+1}^\dagger \hat{a}_j + \text{h.c.}]$$

Refs.

K. Jimenez-Garcia *et al* (in preparation)

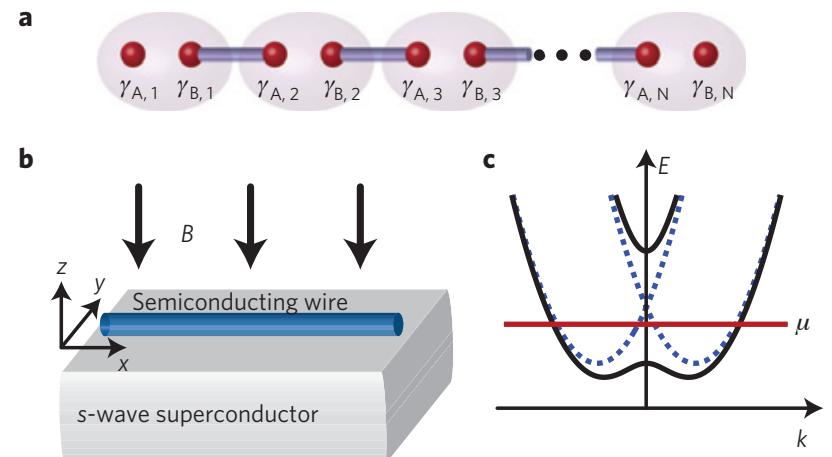
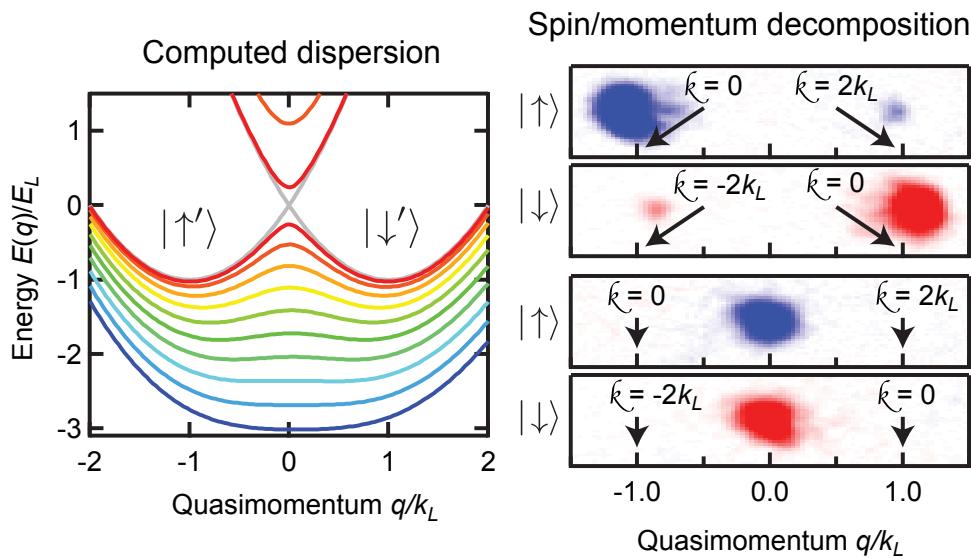
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for  $\Psi^\dagger(\mathbf{x}) = \{\psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x})\}$



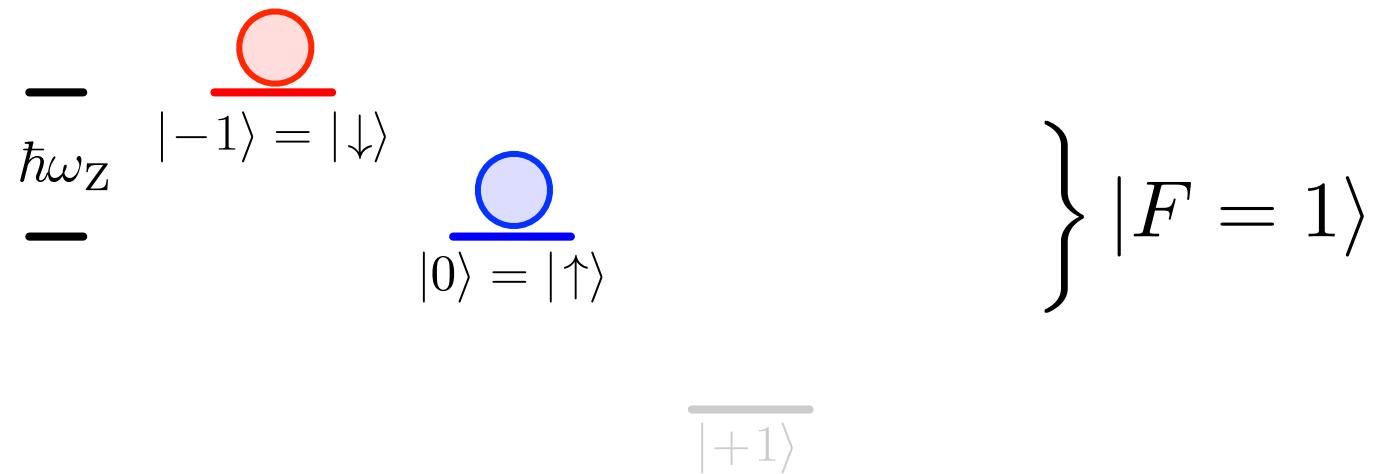
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# Effective Hamiltonian for dressed spins

## Two pseudo-spin contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[ \left( c_0 + \frac{c_2}{2} \right) (\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2} (\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2) + c_2 \hat{\rho}_\downarrow \hat{\rho}_\uparrow \right] :$$



# Effective Hamiltonian for dressed spins

---

## Two pseudo-spin contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[ \left( c_0 + \frac{c_2}{2} \right) (\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2} (\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2) + c_2 \hat{\rho}_\downarrow \hat{\rho}_\uparrow \right] :$$

$m_F = -1, m_F = 0$  mixture: miscible for  $^{87}\text{Rb}$

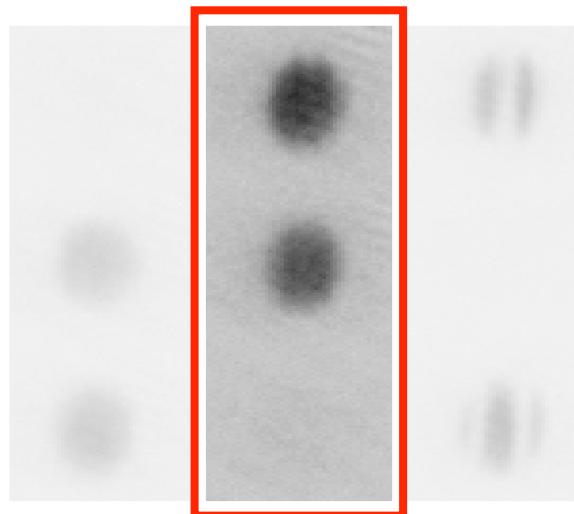
$$c_0 = 7.8 \times 10^{-12} \text{ Hz} \cdot \text{cm}^3$$

$$c_2 = -3.6 \times 10^{-14} \text{ Hz} \cdot \text{cm}^3$$

$$m_F = -1$$

$$m_F = 0$$

$$m_F = +1$$

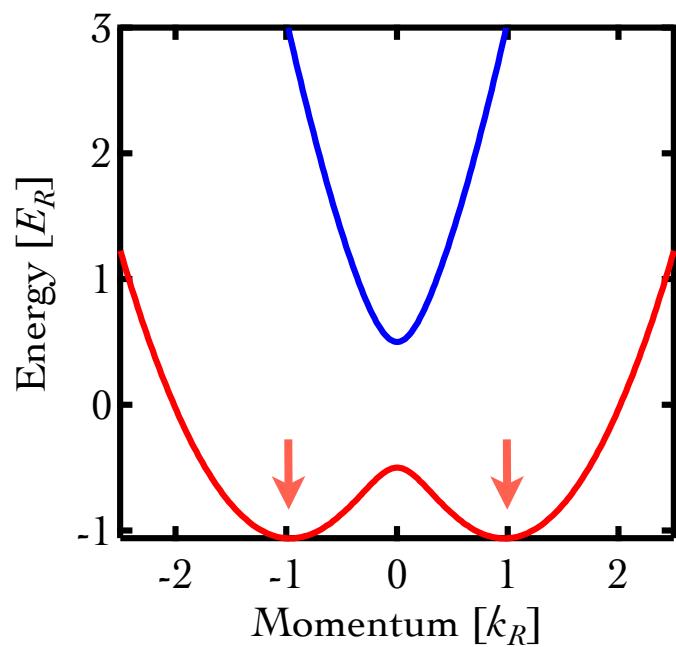


Ph.D. Thesis of Ming-Shien Chang  
(Chapman group)

# Effective Hamiltonian for dressed spins

## Two pseudo-spin contact interactions

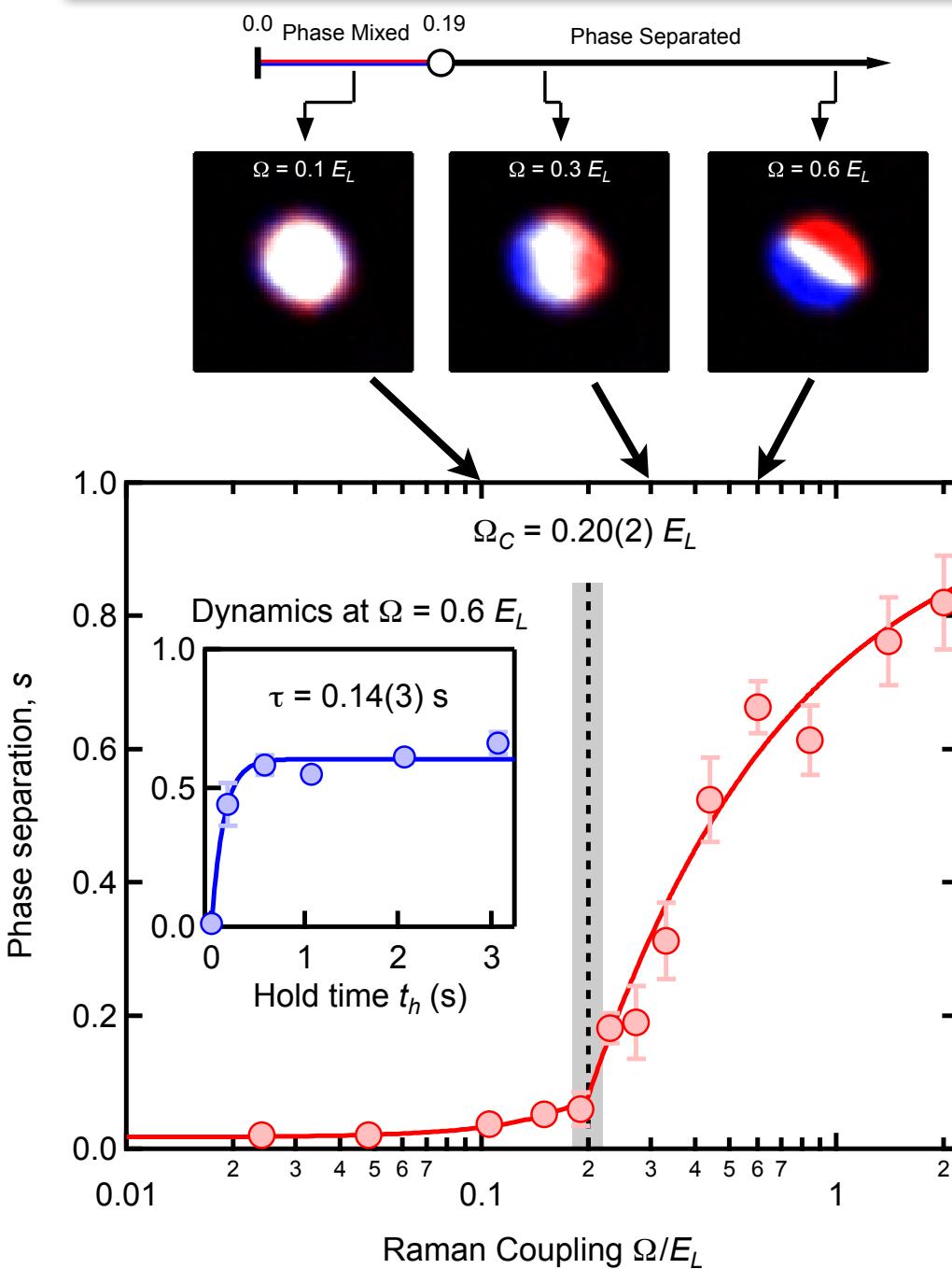
$$\begin{aligned}\hat{H}_{\text{int}} &= \frac{1}{2} \int d^3r : \left[ \left( c_0 + \frac{c_2}{2} \right) (\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2} (\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2) + c_2 \hat{\rho}_\downarrow \hat{\rho}_\uparrow \right] : \\ &\rightarrow \frac{1}{2} \int d^3r : \left[ \left( c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow'} + \hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow'}^2 - \hat{\rho}_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \hat{\rho}_{\downarrow'} \hat{\rho}_{\uparrow'} \right] :\end{aligned}$$



## Spin-orbit term

$$c'_{\uparrow,\downarrow} \approx c_0 \frac{\Omega_R^2}{8}$$

# Transition from miscible to immiscible



A quantum phase transition  
Previously unexpected

Our MFT prediction  
Phase separation at  $\Omega = 0.19 E_L$

Ref.  
Y.-J. Lin et al Nature (2011),

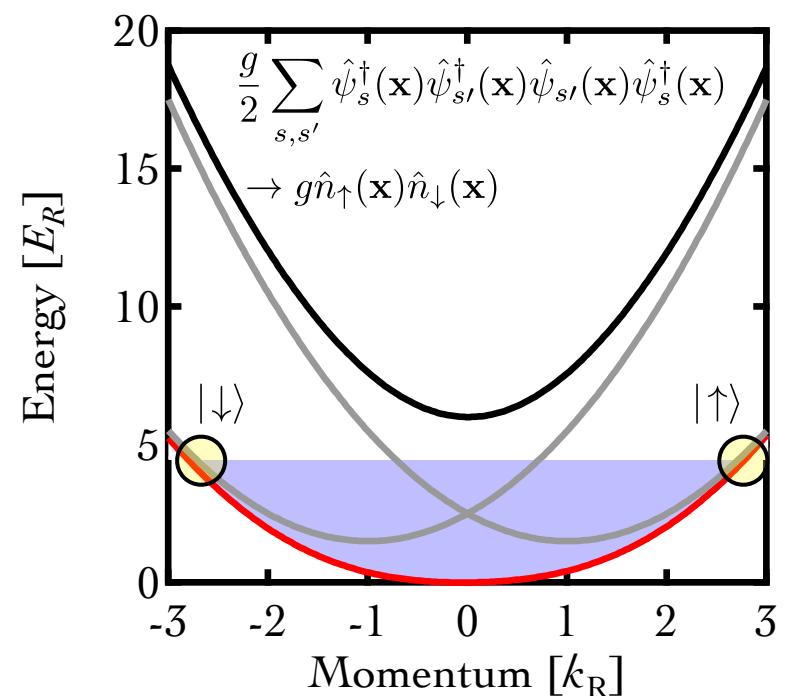
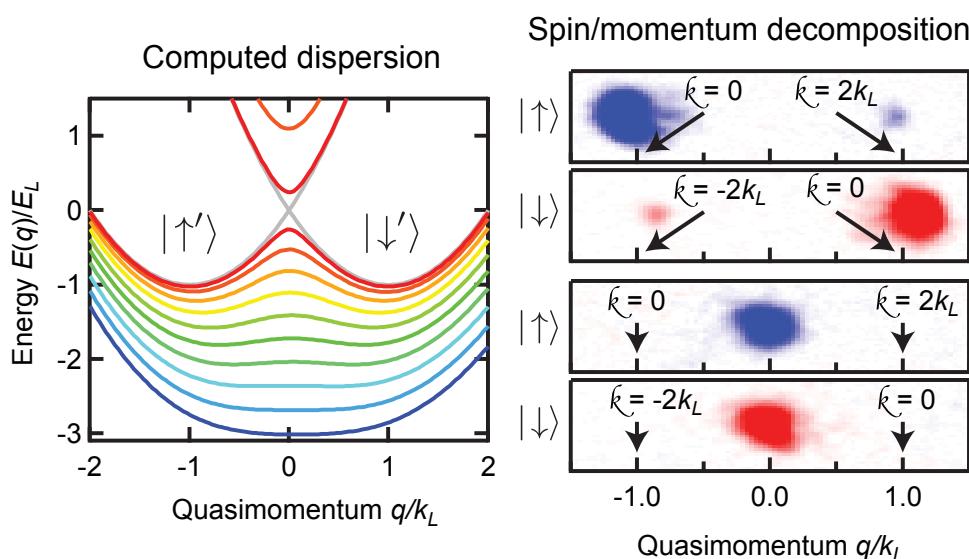
# Spin orbit coupling of pseudo spin-1/2 atoms

## Current experiments with Bosons

Spin orbit coupling for pseudo spin 1/2 **Bosons** (testbed platform)

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \hat{\sigma}_y + \frac{\Omega}{2} \hat{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

for  $\Psi^\dagger(\mathbf{x}) = \{\psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x})\}$



Refs.

Cold atom experiments: Y.-J. Lin et al Nature (2011), R. A. Williams Science (accepted, 2011)  
Theory: C. Zhang et al, PRL (2008), J. D. Sau et al, PRL (2010), J. Alicea et al, N. Physics (2011)

# Modified interactions: optical screening

Interacting fermions in a **single component** gas

Effective  $p$ -wave interactions!!

**Test with Bosons, look for d- and g- wave interactions**

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \hat{\sigma}_y + \frac{\Omega}{2} \hat{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

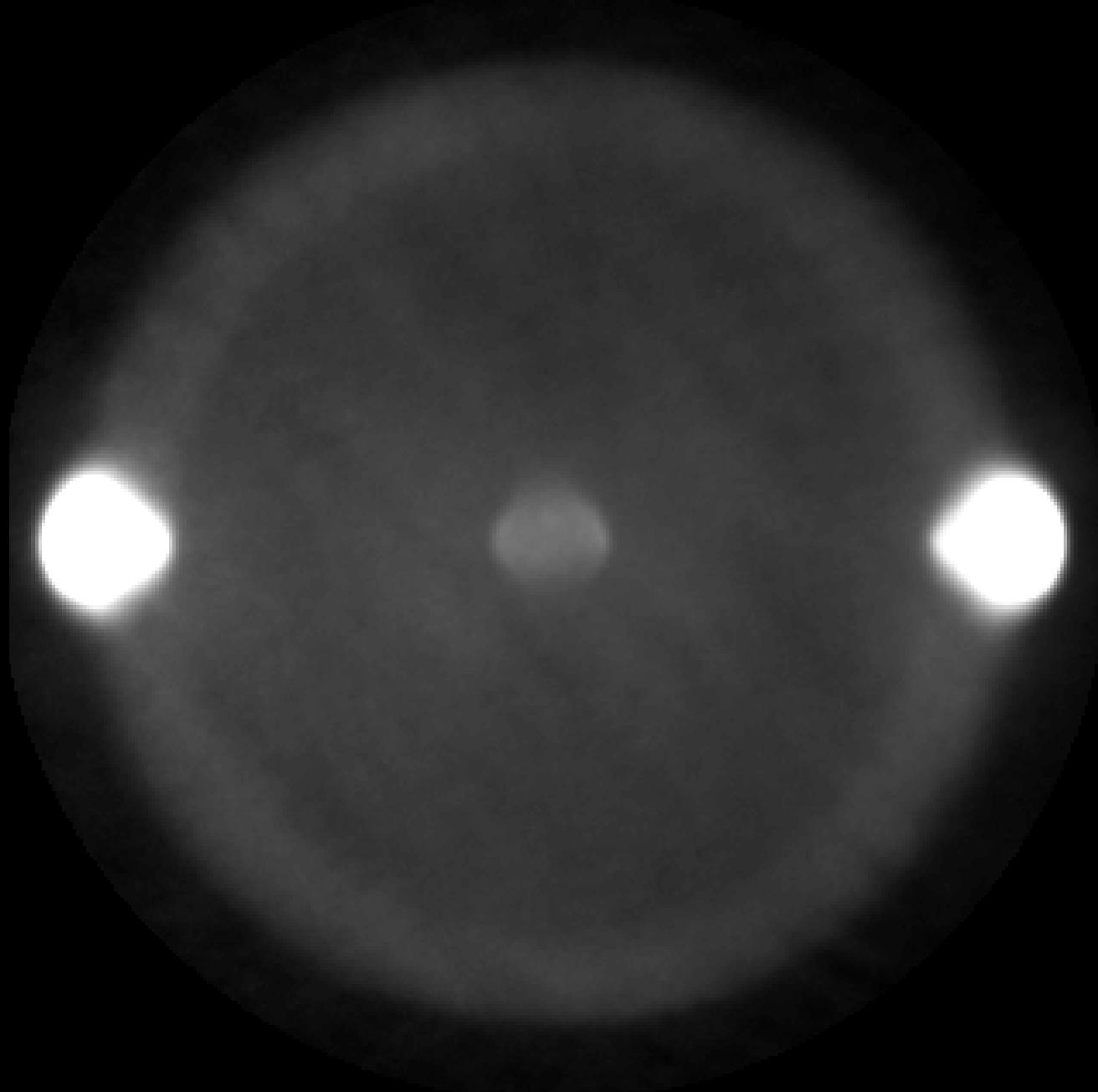
A dielectric function

$$V_{\text{eff}} = g \sum_{\sigma_1, \sigma_2} U_{-, \sigma_1}(\mathbf{k}_1) U_{\sigma_1, -}^\dagger(\mathbf{k}_3) U_{-, \sigma_2}(\mathbf{k}_2) U_{\sigma_2, -}^\dagger(\mathbf{k}_4)$$

$$= V(\mathbf{k}_1 - \mathbf{k}_3) \chi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

$$V_{\text{eff}}(\mathbf{x}) \approx g \frac{\Omega |x| + 2}{16} \left( e^{-\Omega |x|/2} \right) \delta(y) \delta(z)$$

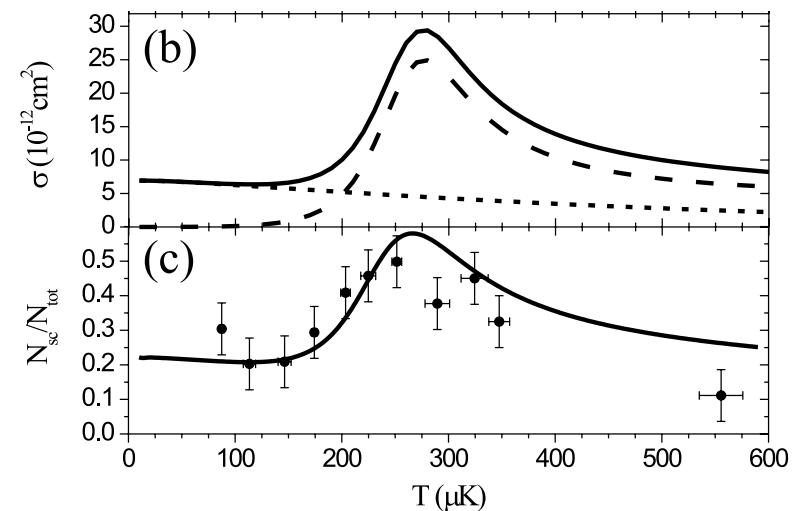
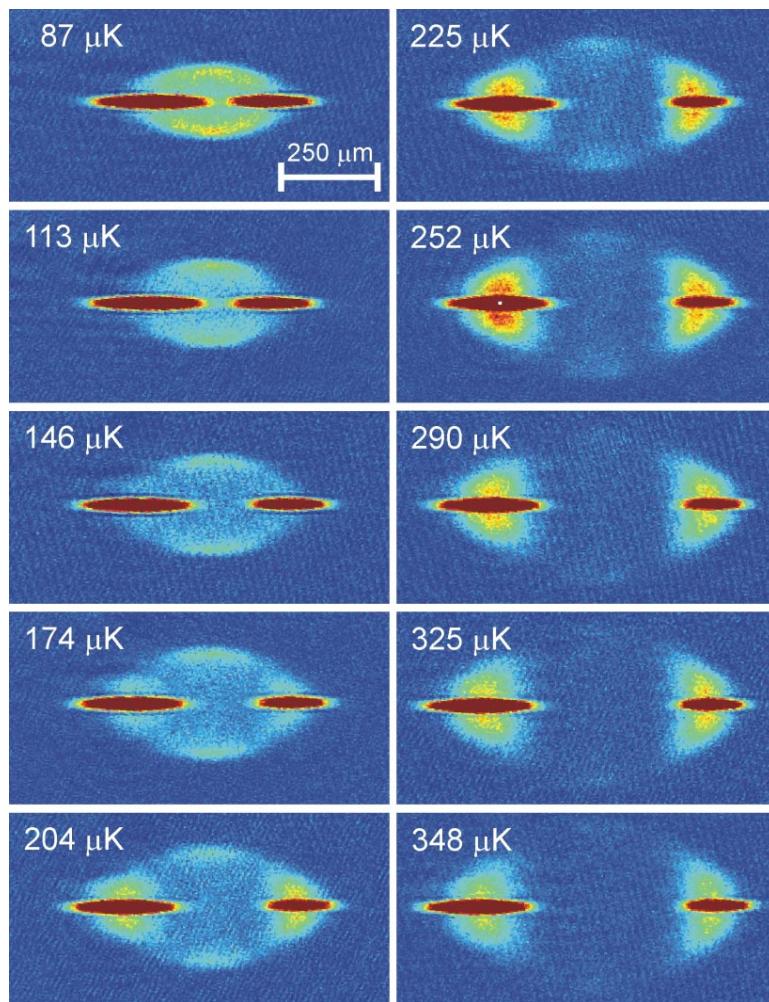
# Colliding BEC's



# Collisions as a probe of interatomic potentials

## Colliding BEC's

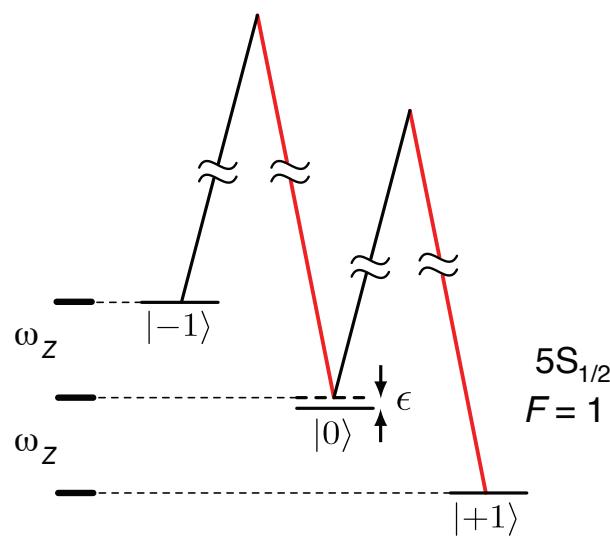
All s-wave



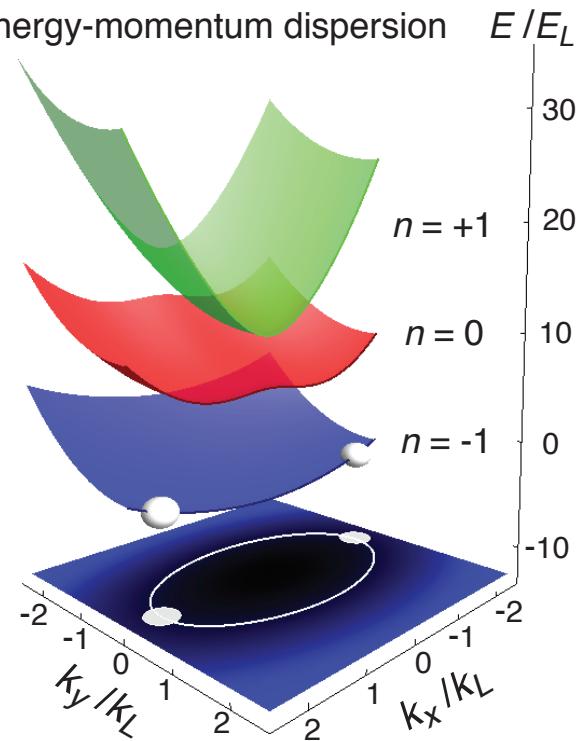
lots of d-wave

# Schematic

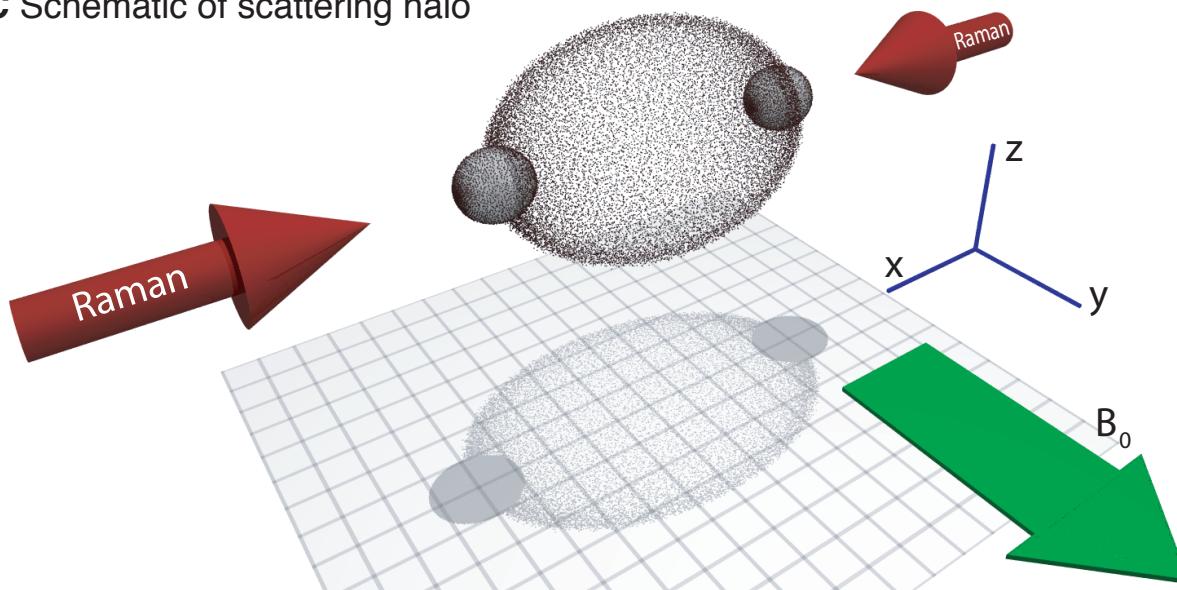
**A** Three-level coupling scheme



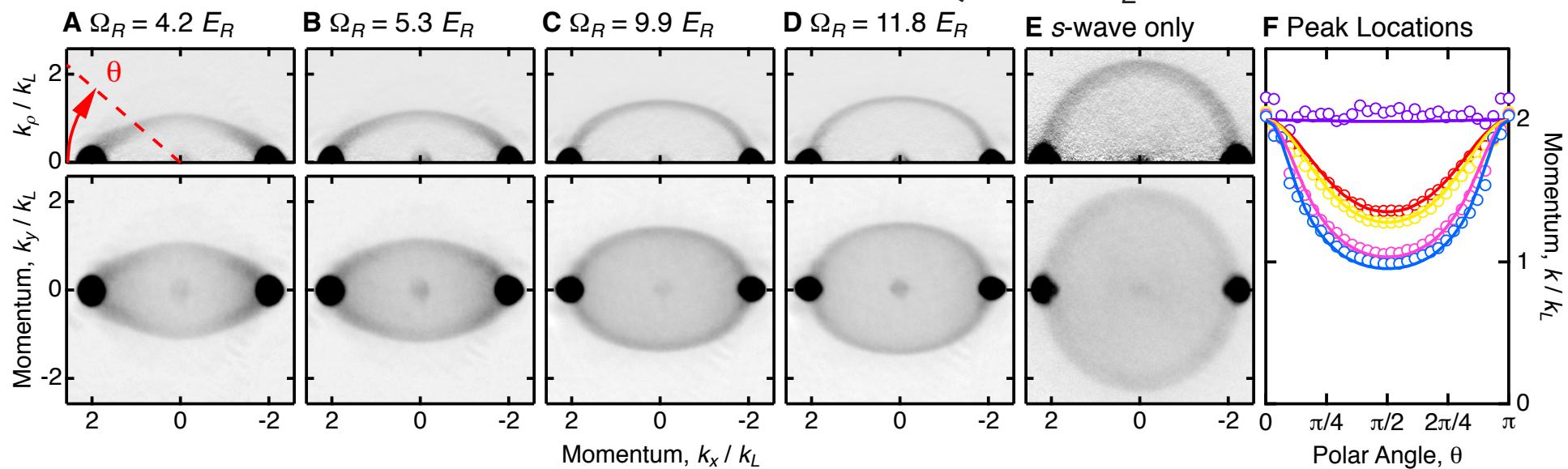
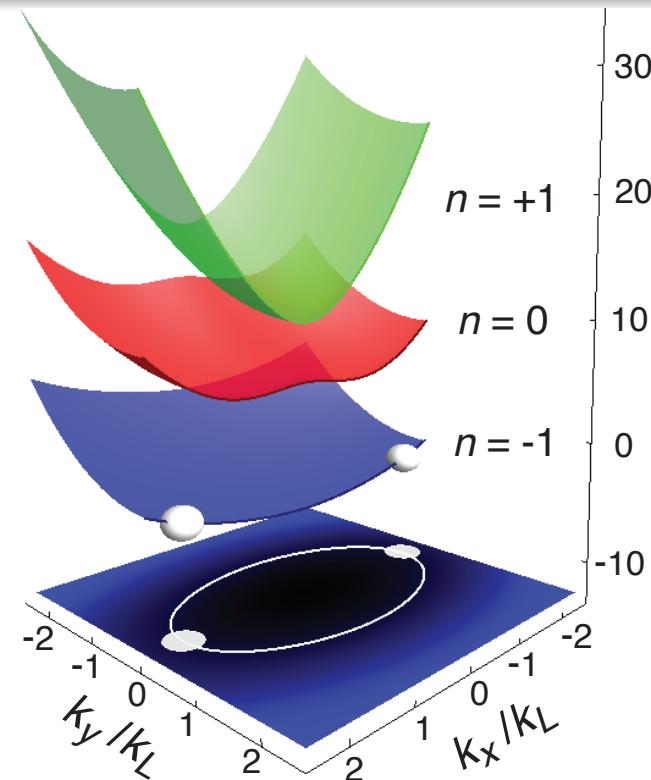
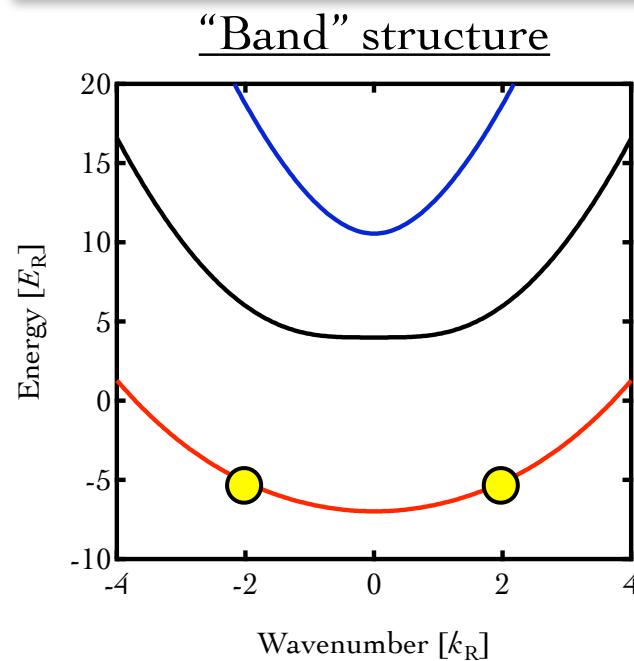
**B** Energy-momentum dispersion



**C** Schematic of scattering halo



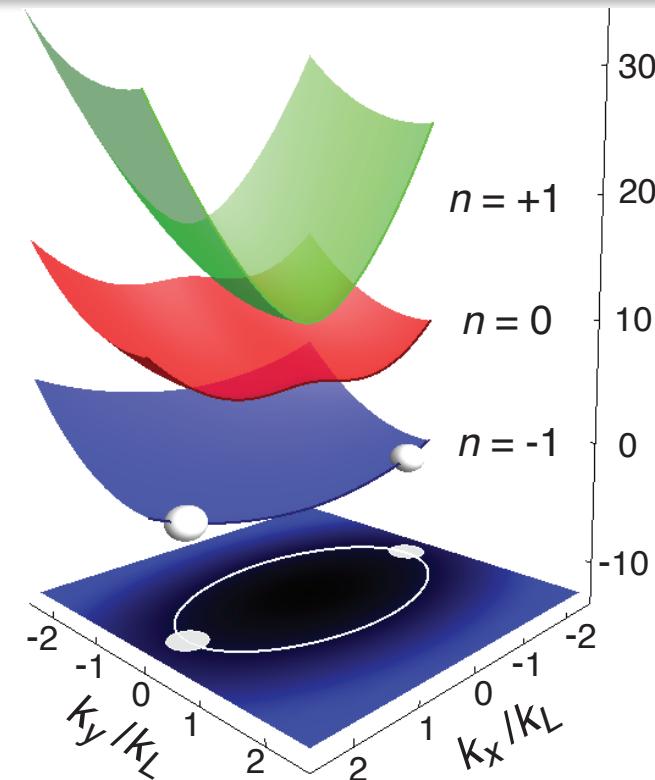
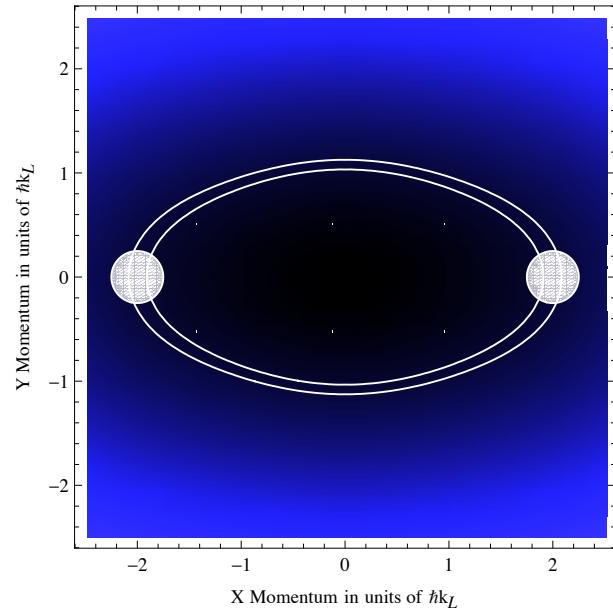
# Effective mass effect



## Density of states effect

$$g(E)\delta E = N(E + \delta E) - N(E)$$

$$\Omega_R = 6 E_L$$



$$\mathbf{A} \Omega_R = 4.2 E_R$$

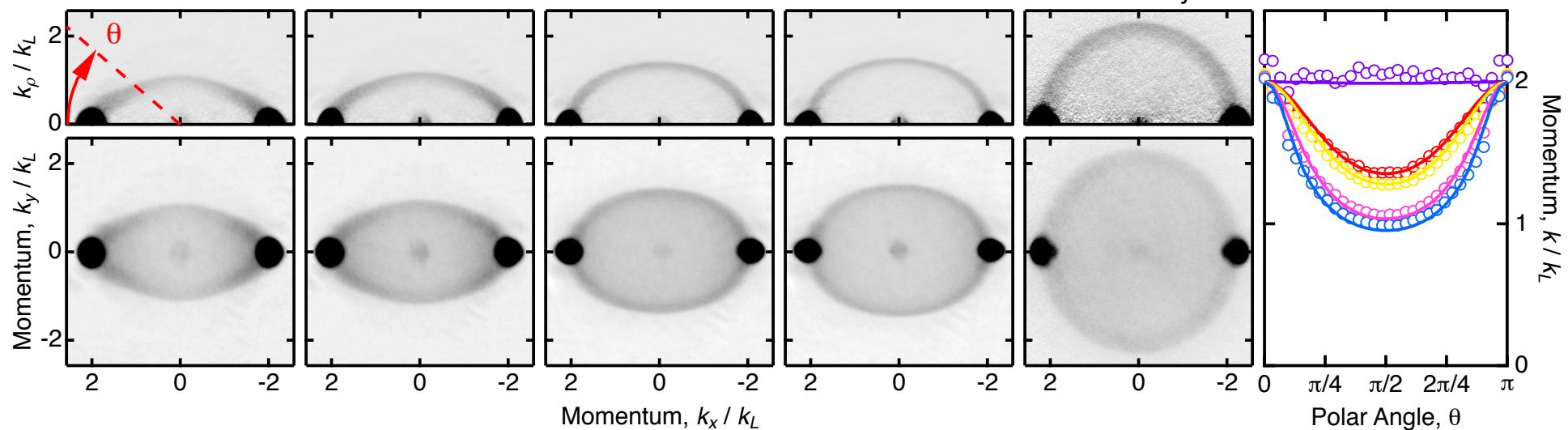
$$\mathbf{B} \Omega_R = 5.3 E_R$$

$$\mathbf{C} \Omega_R = 9.9 E_R$$

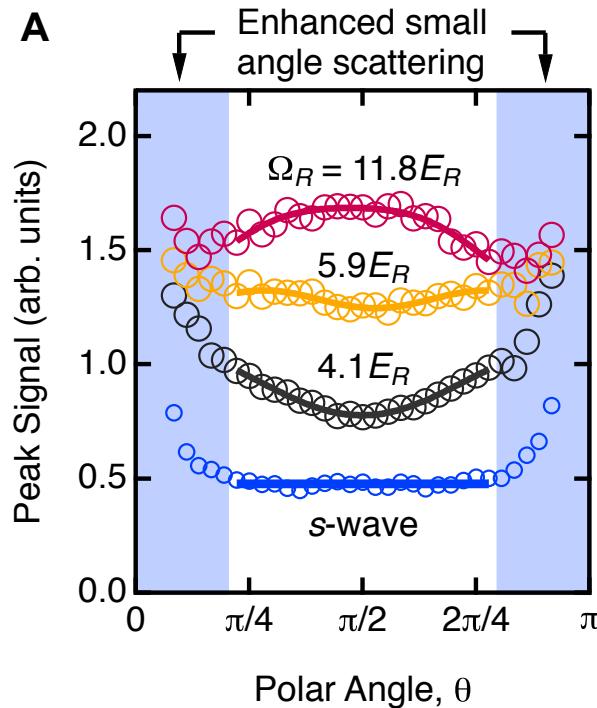
$$\mathbf{D} \Omega_R = 11.8 E_R$$

## **E s-wave only**

## F Peak Locations



# Modified collisions

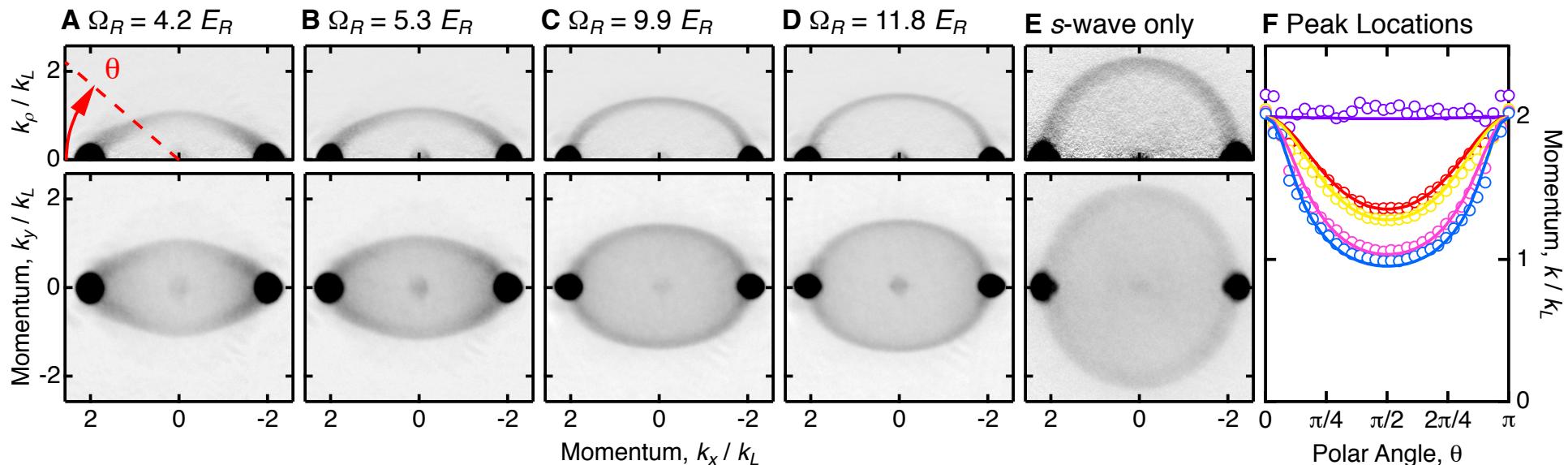


Remove density of states effect

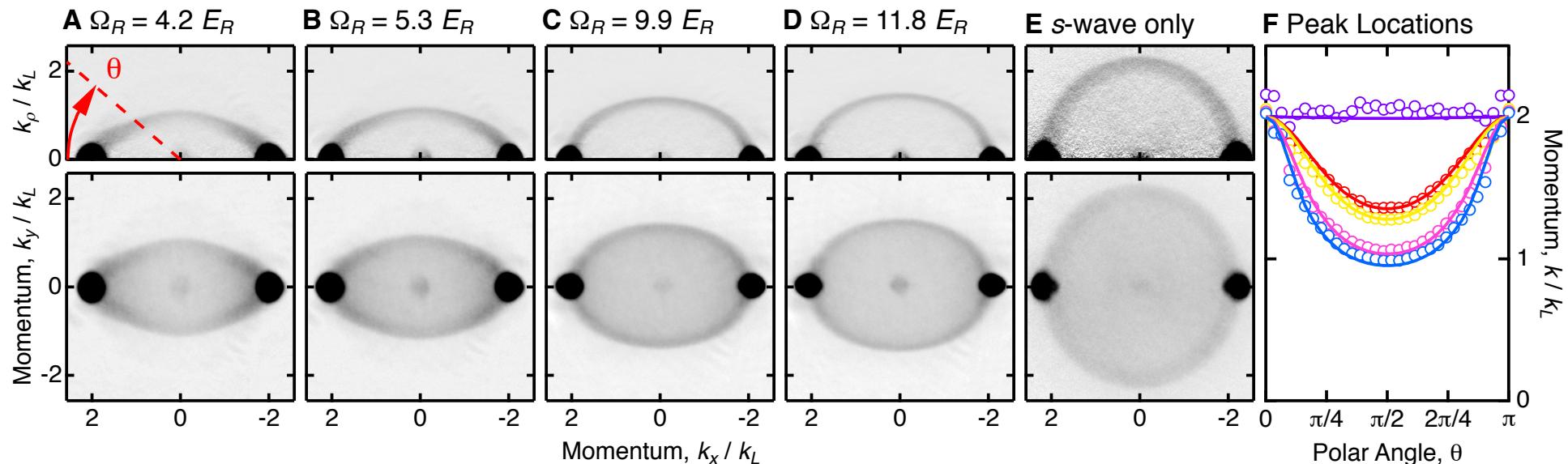
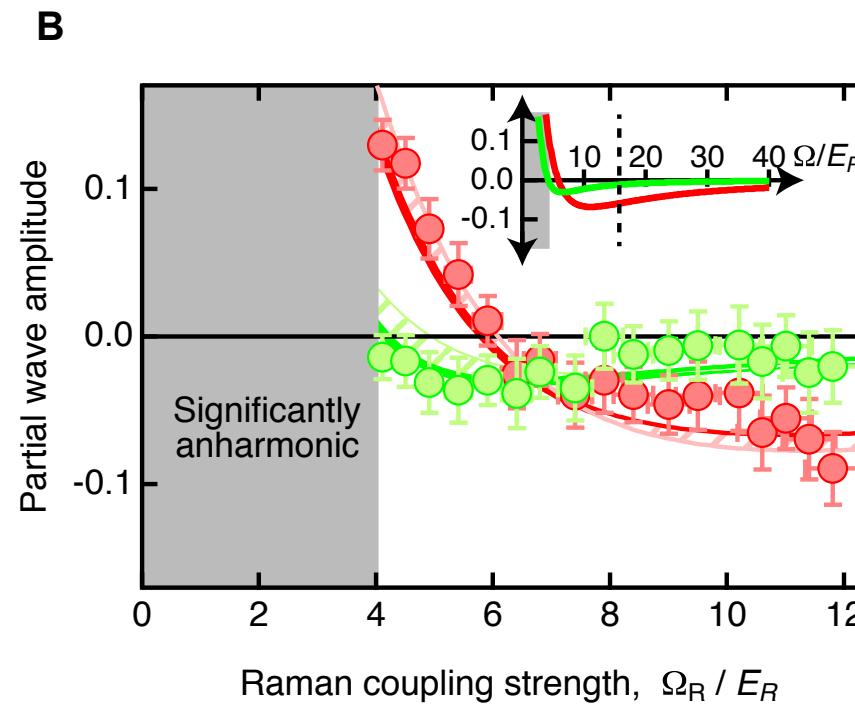
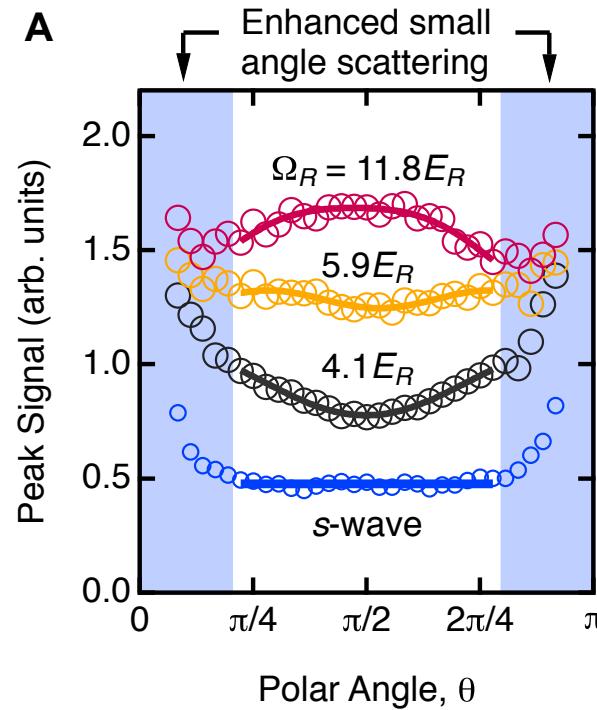
$$\Gamma_i = \frac{2\pi}{\hbar} \int_{S_i} \frac{dS_k}{(2\pi)^2} \frac{|\langle i | \hat{V} | S_k \rangle|^2}{|\nabla_k E(S_k)|}$$

Then fit to partial wave expansion

$$\left| \sum_l (\exp 2i\eta_l - 1) (2l + 1) P_l(\cos(\theta)) \right|^2$$



# Modified matrix element effect

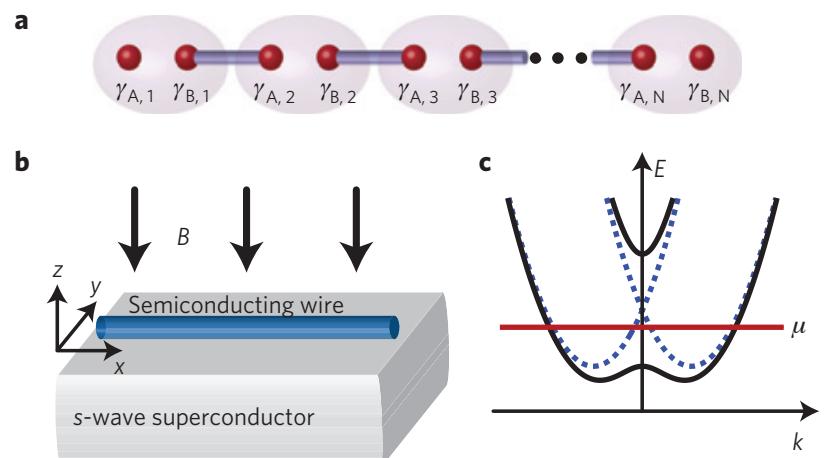
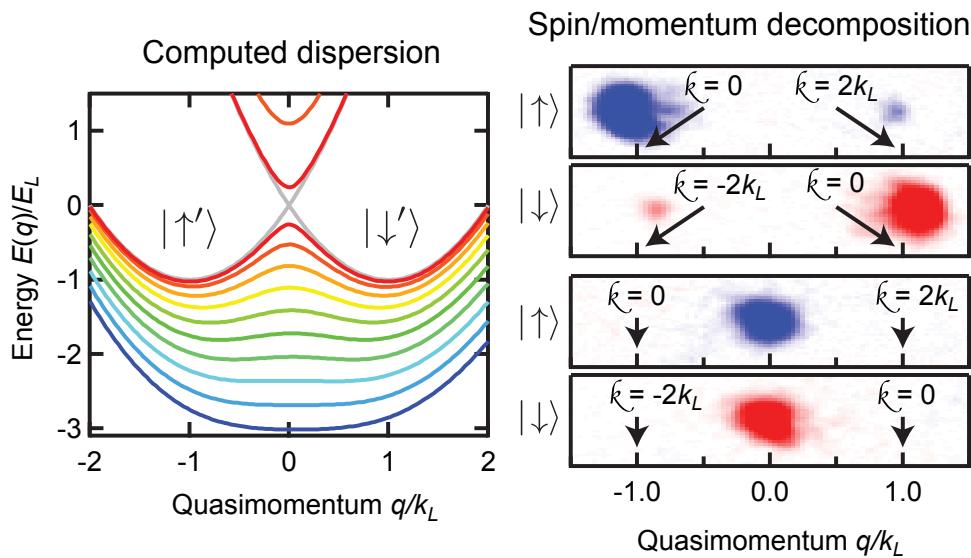


# Spin orbit coupling of pseudo spin-1/2 atoms

## Moving forward to Fermions

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

for  $\Psi^\dagger(\mathbf{x}) = \{\psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x})\}$



## Refs.

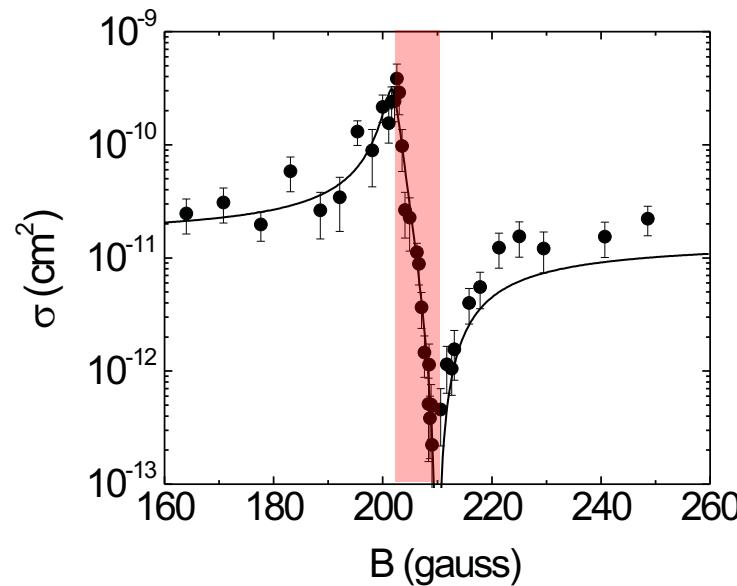
Cold atom experiments: Y.-J. Lin et al Nature (2011), R. A. Williams Science (accepted, 2011)  
 Theory: C. Zhang et al, PRL (2008), J. D. Sau et al, PRL (2010), J. Alicea et al, N. Physics (2011)

# Pairing: $^{40}\text{K}$

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + [\Delta \hat{\Psi}_\uparrow(\mathbf{x}) \hat{\Psi}_\downarrow(\mathbf{x}) + \text{h.c.}]$$

$E_{B,1D} \propto g^2$   
 $E_{B,3D} \propto e^{-\pi/k_F|a|}$ , with  $g = 2\pi\hbar^2 a/m$

In 1D and 2D robust pairing (at single particle level)  
at all attractive coupling strengths



Refs.

C. Regal (Ph.D. thesis); Bloch, I., Dalibard, J. & Zwerger, Rev. Mod. Phys. 80, 885–964 (2008).

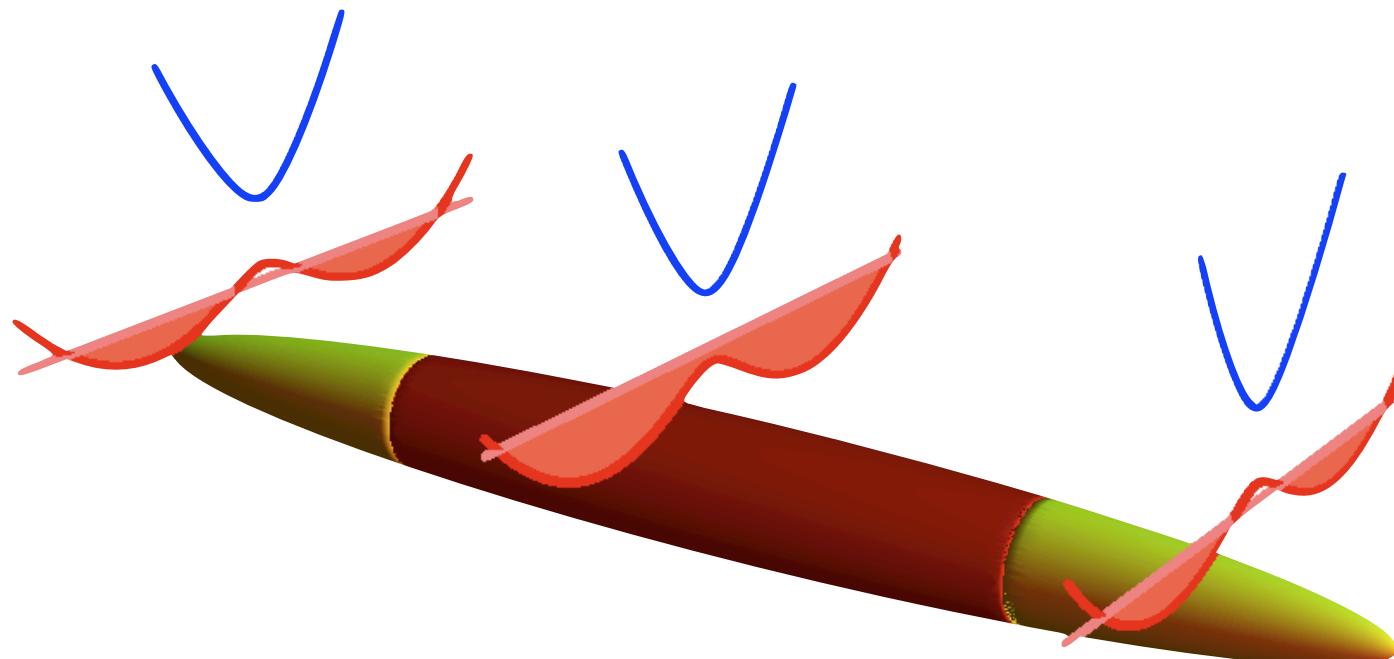
# Theory questions

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Pairing gap is *intrinsic* not from proximity effect

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \mathbf{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \sum_{s,s'} \hat{\psi}_s^\dagger(\mathbf{x}) \hat{\psi}_{s'}^\dagger(\mathbf{x}) \hat{\psi}_{s'}(\mathbf{x}) \hat{\psi}_s(\mathbf{x})$$

for  $\Psi^\dagger(\mathbf{x}) = \left\{ \psi_\uparrow^\dagger(\mathbf{x}), \psi_\downarrow^\dagger(\mathbf{x}) \right\}$



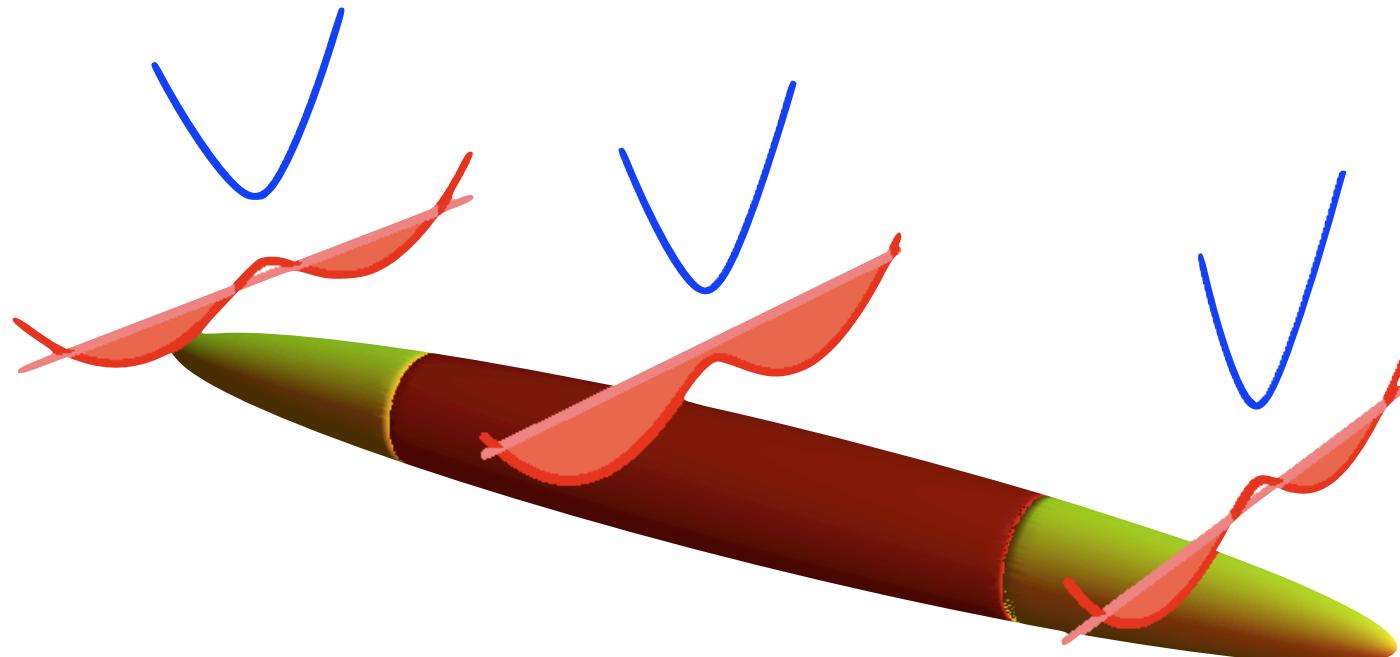
# Theory questions

---

Pairing gap is *intrinsic* not from proximity effect

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + [\Delta e^{i\varphi} \hat{\psi}_\uparrow(\mathbf{x}) \hat{\psi}_\downarrow(\mathbf{x}) + \text{h.c.}]$$

for  $\hat{\Psi}^\dagger(\mathbf{x}) = \left\{ \hat{\psi}_\uparrow^\dagger(\mathbf{x}), \hat{\psi}_\downarrow^\dagger(\mathbf{x}) \right\}$



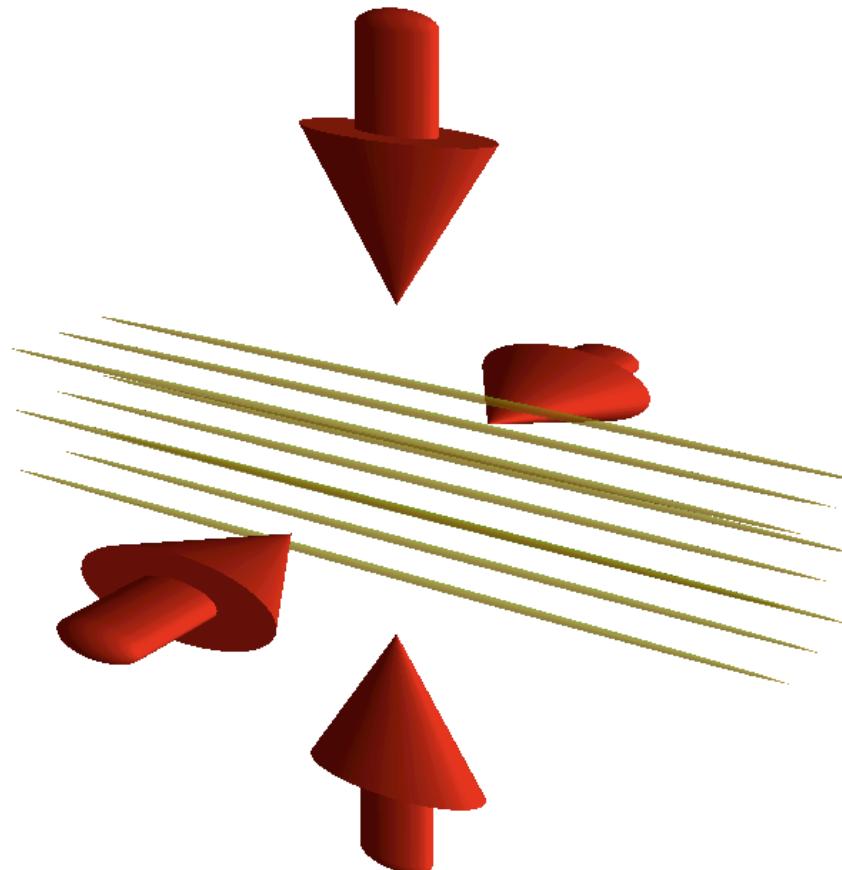
# Theory questions

---

Pairing gap is *intrinsic* not from proximity effect  
Coupling between needles important? Finite size important?

$$\hat{\mathcal{H}} = \hat{\Psi}^\dagger(\mathbf{x}) \left( \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \frac{\hbar^2 k_R}{m} \hat{k}_x \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \right) \hat{\Psi}(\mathbf{x}) + \left[ \Delta e^{i\varphi} \hat{\psi}_\uparrow(\mathbf{x}) \hat{\psi}_\downarrow(\mathbf{x}) + \text{h.c.} \right]$$

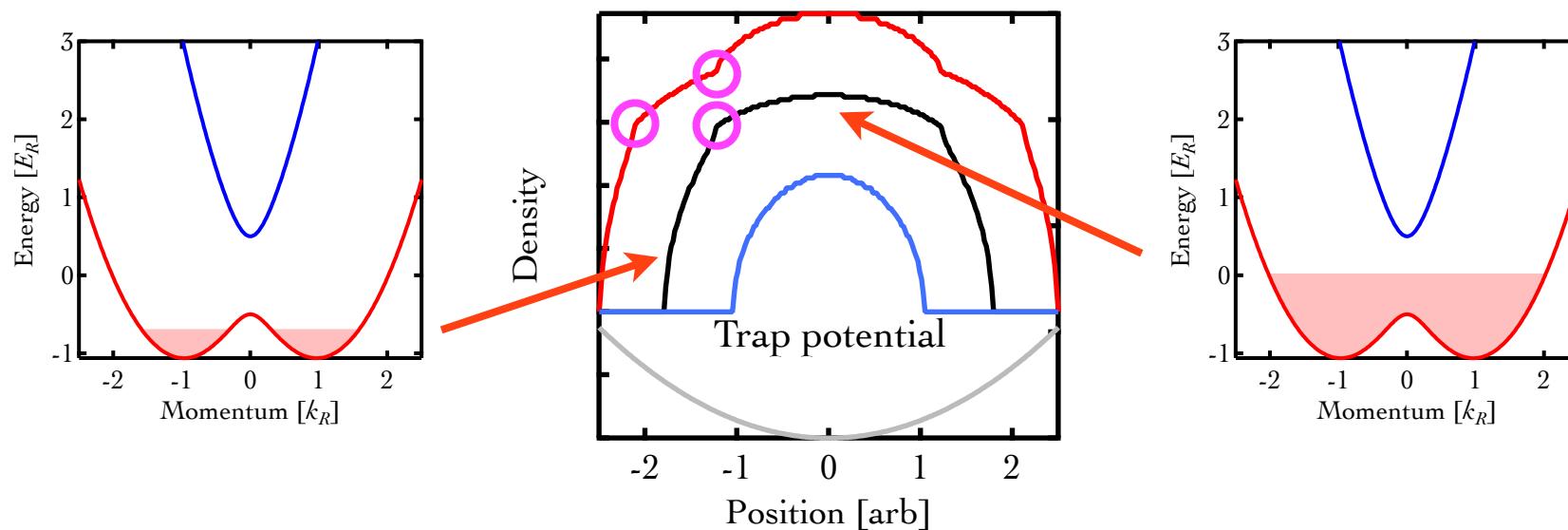
for  $\hat{\Psi}^\dagger(\mathbf{x}) = \left\{ \hat{\psi}_\uparrow^\dagger(\mathbf{x}), \hat{\psi}_\downarrow^\dagger(\mathbf{x}) \right\}$



# Theory questions

Pairing gap is *intrinsic* not from proximity effect  
Coupling between needles important? Finite size important?

What happens at  $\mu$  smoothly changes and crosses from TSC to SC phases?



If Majorana fermions's do form at these interfaces, how to detect?

# The ugly: $^{40}\text{K}$

Lifetime, lifetime, stability

Spontaneous emission  
(lights scattering)  
 $t \sim 0.5 - 1 \text{ s}$  (double well)  
 $t \sim 0.1 \text{ s}$  (single well)

Feshbach losses  
(molecule formation)  
 $t \sim 1 \text{ s}$  ( $1/k_F a \sim -1$ )  
 $t \sim 0.1 \text{ s}$  ( $1/k_F a \sim 0$ )

$\sim 200 \mu\text{G}$  absolute  
stability at 200 G  
(1 ppm)

# Gauge fields



# Mixtures

