TAMU Winterschool: Spin in cold atom systems

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$\boxed{\text{Ian's outline}}$

I. Meaning of "spin" in atomic systems

$(L,S) \to J, (J+I) \to F$ coupling

~~II. Important levels of Alkali atoms nS → nP transition~~

III. Structure in ground state (hyperfine)

IV. Couplings to spin states

- Zeeman → Breit-Rabi equation, RF coupling
- ~~Electric Dipole (Raman)~~

V. Example (A): creating spin-orbit coupling

- Select two levels (pseudo spins), then Raman Couple → SOC
- Introduce RWA, compare RF v.s. Raman (No SOC → SOC)

VI. Interactions: Spinor condensate physics

VII. ~~Example (B): Phase separation in a two component gas.~~

This lecture doesn't derive the underlying physics, and instead seeks to reveal the effective Hamiltonians which matter for cold atom physics.

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Spin in atomic systems, Basics

Today I will consider alkali atoms in their ground "s" orbital.

In this situation we will need to consider only the nuclear and electronic angular momenta

$S = \frac{1}{2}$ and $I = \ldots$ (we will see specifics later).

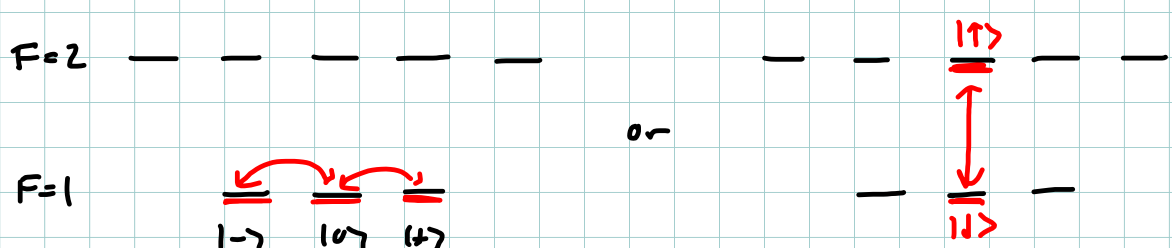
Lets think about I: it is a combenation of nuclear spin and orbital angular momentum.

So already we know what I call "spin" isn't.

The name of the game is to construct a total angular momentum

$F = I + S$ (or $\overbrace{L+S}^{J}+I$), which takes on values

$f = i \pm \frac{1}{2}$, e.g., F=1 and F=2 for ^{87}Rb. Given this we select some set of interesting pseudospins.



In this lecture I will explore some of the single particle and interaction physics that might go into such a selection.

Hyperfine structure + Zeeman Effect

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Here I will consider the properties of atoms in their ground electronic state in a uniform magnetic field.

I just introduced the components of $L=0$ atoms which make the spin

$$\vec{F} = \vec{S} + \vec{I}.$$

In atoms these are coupled by the Hyperfine interaction (Magnetic dipole)

$H_{HF} = A(\vec{I}\cdot\vec{S})$, using the standard trick $F^2 = (S+I)^2 = S^2 + 2S.I + I^2$

or $\frac{1}{2}(F^2 - S^2 - I^2) = \vec{S}\cdot\vec{I}$

So $$\boxed{\bar{H}_{HF} = \frac{A}{2}\left(\hat{F}^2 - \hat{S}^2 - \hat{I}^2\right) \rightarrow \frac{\hbar^2 A}{2}\left[f(f+1) - s(s+1) - i(i+1)\right]}$$

now in the Alkali atoms $S = \frac{1}{2}$, so $F = I \pm \frac{1}{2}$, for $I > 0$
$= \frac{1}{2}$ for $I = 0$

$$\boxed{E_{HF} = \frac{A}{2}\left[\pm\left(i+\frac{1}{2}\right) - \frac{1}{2}\right], \quad \Delta_{HFS} = A\left[i + \frac{1}{2}\right]}$$

Now some examples:

	^{6}Li	^{7}Li	^{23}Na	^{39}K	^{40}K	^{41}K	^{85}Rb	^{87}Rb	^{133}Cs
I	1	3/2	3/2	3/2	4	3/2	5/2	3/2	7/2
Δ_{HFS}	228 MHz	804 MHz	1.77 GHz	462 MHz	1.29 GHz	254 MHz	3.03 GHz	6.83 GHz	9.19 GHz
A	152 MHz	402 MHz	885 MHz	231 MHz	−286 MHz	127 MHz	1.61 GHz	3.42 GHz	2.30 GHz

So there are a wide range of possible hyperfine splittings which — as we shall see — has important effects.

Hyperfine structure + Zeeman Effect II

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Now with the hyperfine effect in the bag we move on to the Zeeman effect.

$$H_Z = \mu_B (g_s \vec{S} + g_I \vec{I}) \cdot \vec{B}$$

sometimes $\mu_B/\hbar$ depending on units (atomic physicists like Hz, or s^{-1})

So all together we consider $\boxed{H_g = A(I \cdot S) + \mu_B (g_s S + g_I I) \cdot B}$

the solution to this equation is the Breit-Rabi equation

For $s = 1/2$, and $B = B_z \hat{e}_z$

Recall that $J_\pm = J_x \pm i J_y$ and $J_\pm |j, m_j\rangle = \hbar [(j \mp m_j)(j+1 \pm m_j)]^{1/2} |j, m_j \pm 1\rangle$

So $J_x = \frac{1}{2}(J_+ + J_-)$, $J_y = \frac{1}{2i}(J_+ - J_-)$ giving $I \cdot S = I_z S_z + \frac{1}{2} I_+ S_- + \frac{1}{2} I_- S_+$

Thus in the uncoupled basis H_g couples $|i, m_i; m_s = 1/2\rangle$ to $|i, m_i + 1; -1/2\rangle$

which may be graphically depicted like:

$m_F = -i - 1/2$	$m_F = -i + 1/2$	....	$m_F = i - 1/2$	$m_F = i + 1/2$
(*)	$\|i, -i; 1/2\rangle$	...	$\|i, i-1; 1/2\rangle$	$\|i, i; 1/2\rangle$
$\downarrow$	$\updownarrow$		$\updownarrow$	
$\|i, -i; -1/2\rangle$	$\|i, -i+1; -1/2\rangle$		$\|i, i; -1/2\rangle$	

(*) "streached states"

So H_g decomposes into 2x2 blocks labeled by m_F as above

$$\frac{\check{H}(m_F)}{\hbar} = \left(\begin{array}{c:c} \mu_B B\left[\left(\frac{g_s - g_i}{2}\right) + g_i m_F\right] + \frac{\hbar A}{2}(m_F - 1/2) & \frac{\hbar A}{2}\left(i + \frac{1}{2} + m_F\right)^{1/2}\left(i + \frac{1}{2} - m_F\right)^{1/2} \\ \hdashline c.c. & -\mu_B B\left[\left(\frac{g_s - g_i}{2}\right) - g_i m_F\right] - \frac{\hbar A}{2}(m_F + 1/2) \end{array} \right)$$

I will compute the eigenvalues of this matrix on the next page.

M_J doublets #'d by M_F $\longrightarrow \begin{pmatrix} |i, m_F - 1/2; 1/2\rangle \\ |i, m_F + 1/2, -1/2\rangle \end{pmatrix}$

Hyperfine structure + Zeeman Effect III

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To recap, I started with

$$\boxed{H_J = A(I\cdot S) + \mu_B(g_S S + g_I I)\cdot B}$$

and transformed to

$$\frac{\check{H}(m_F)}{\hbar} = \begin{pmatrix} \mu_B B\left[\frac{(g_S - g_i)}{2} + g_i m_F\right] + \frac{\hbar A}{2}(m_F - \tfrac{1}{2}) & \frac{\hbar A}{2}\left(i+\tfrac{1}{2}+m_F\right)^{1/2}\left(i+\tfrac{1}{2}-m_F\right)^{1/2} \\ \text{c.c.} & -\mu_B B\left[\frac{(g_S-g_i)}{2} - g_i m_F\right] - \frac{\hbar A}{2}(m_F+\tfrac{1}{2}) \end{pmatrix}$$

For today eigenvalues are enough:

$$\frac{E_{B-R}}{\hbar} = g_i \mu_B B m_F + \frac{\hbar A}{4}\left\{-1 \pm \left[(2i+1)^2 + 4m_F\left(\frac{2(g_s-g_i)\mu_B B}{A}\right) + \left(\frac{2(g_s-g_i)\mu_B B}{A}\right)^2\right]^{1/2}\right\}$$

This is the Breit-Rabi equation, the eigenstates of which are the "spin states" usually used in cold atom systems

⊗ Now lets look at some examples $\boxed{\text{See Slides}}$

Keep in mind that ~2kG = 0.2 T is near the upper field limit for cold atom experiments

Clock States

depending on one's aims it can be useful to select a pair of states who's energy difference is 1st order insensitive to external magnetic fields.

In ^{87}Rb near B=0 there are two such pairs

$$\{|F=1, m_F=0\rangle, |F=2, m_F=0\rangle\} \text{ and } \{|F=1, m_F=-1\rangle, |F=2, m_F=1\rangle\}$$

$\boxed{\text{See Slides}}$

RF coupling

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Lets focus on just one manifold at small field, then

Landé g factor

$$H = g_F \mu_B \vec{F}\cdot\vec{B}, \qquad g_F = g_i \pm \frac{(g_s - g_i)}{2i+1}$$

make $B = B_0 e_z + \vec{\delta B}\cos(\omega t)$

$$H = g_F \mu_B B_0 \hat{F}_z + g_F \mu_B \cos(\omega t)\ \vec{F}\cdot\delta B$$

now I make a transformation into a frame rotating with angular frequency ω and make the RWA

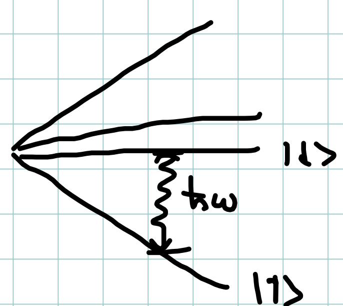
to get: $H = \Omega \cdot F$, where

$$\Omega = (\underbrace{+g_F \mu_B \delta B_y, -g_F \mu_B \delta B_x}_{\text{coupling terms.}}, \overbrace{g_F \mu_B B_0 - \omega}^{=\delta})$$

* only terms $\perp$ to bias matter

* phase of drive affects Ω_x, Ω_y

* frequency selects states



Dressed states: RF/Raman

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Now we have selected a pair of spin states and we want to couple them.

I. $$\hat{H} = \begin{pmatrix} \hbar^2 k^2/2m & 0 \\ 0 & \hbar^2 k^2/2m \end{pmatrix} + \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}$$

$$\check{H} = \frac{\hbar^2 k^2}{2m}\check{\mathbb{1}} + \frac{\delta}{2}\check{\sigma}_z + \frac{\Omega}{2}\check{\sigma}_x,$$ now some U solves this hamiltonian

Since U is position independent, it commutes with $k^2\check{\mathbb{1}}$

so $$UHU^\dagger = \frac{\hbar^2 k^2}{2m}\check{\mathbb{1}} + \frac{(\delta^2+\Omega^2)^{1/2}}{2}\check{\sigma}_z$$ standard RF dressed state

* RF evaporation
* RF dressed traps
* Quantum Control

⋮

II. Raman dressed ← "vector light shift"

momentum displacement operator $|k-2k_r\rangle\langle k|$

$$\check{H} = \frac{\hbar^2 k^2}{2m}\check{\mathbb{1}} + \frac{\delta}{2}\check{\sigma}_z + \frac{\Omega}{2}\begin{pmatrix} 0 & e^{-2ik_r x} \\ e^{2ik_r x} & 0 \end{pmatrix}$$

It's most simple to write this out:

$$= \int \frac{dk}{2\pi}\frac{\hbar^2}{2m}\left(k^2|k,\uparrow\rangle\langle k,\uparrow| + k^2|k,\downarrow\rangle\langle k,\downarrow|\right) + \frac{\Omega}{2}\left(|k-2k_r,\uparrow\rangle\langle k,\downarrow| + H.C.\right) + \frac{\delta}{2}\check{\sigma}_z$$

now I change my labeling so $|k,\uparrow\rangle \rightarrow |k+k_R,\uparrow'\rangle$
$|k,\downarrow\rangle \rightarrow |k-k_R,\downarrow'\rangle$

$$= \int \frac{dk}{2\pi}\frac{\hbar^2}{2m}\left(k^2|k+k_R,\uparrow'\rangle\langle k+k_R,\uparrow'| + k^2|k-k_R,\downarrow'\rangle\langle k-k_R,\downarrow'|\right) + \frac{\Omega}{2}\left(|k-k_R,\uparrow'\rangle\langle k-k_R,\downarrow'| + H.C.\right) + \frac{\delta}{2}\check{\sigma}_z$$, change integration

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Dressed states: RF/Raman

Now we have selected a pair of spin states and we want to couple them.

$$= \int \frac{dk}{2\pi} \frac{\hbar^2}{2m} \left(k^2 |k+k_R, \uparrow'\rangle\langle k+k_R, \uparrow'| + k^2 |k-k_R, \downarrow'\rangle\langle k-k_R, \downarrow'| \right) + \frac{\Omega}{2} \left(|k-k_R, \uparrow'\rangle\langle k-k_R, \downarrow'| + H.c. \right) + \frac{\delta}{2} \check{\sigma}_z$$

, change integration

$$= \int \frac{dq}{2\pi} \left(|q,\uparrow'\rangle, |q,\downarrow'\rangle \right) \underbrace{\begin{pmatrix} \frac{\hbar^2}{2m}(q-k_R)^2 + \frac{\delta}{2} & \frac{\Omega}{2} \\ \frac{\Omega}{2} & \frac{\hbar^2}{2m}(q+k_R)^2 - \frac{\delta}{2} \end{pmatrix}}_{= \check{H}(q)} \begin{pmatrix} |q,\uparrow'\rangle \\ |q,\downarrow'\rangle \end{pmatrix}$$

$$\check{H}(q) = \frac{\hbar^2 q^2}{2m} \check{\mathbb{1}} + \left(-\frac{\hbar^2 q k_R}{m} \right) \check{\sigma}_z + \frac{\delta}{2} \check{\sigma}_z + \frac{\Omega}{2} \check{\sigma}_x$$

Rotation by $\pi/2$ about e_y (spins only)

$$\boxed{H(q) = \frac{\hbar^2 q^2}{2m} \check{\mathbb{1}} + \frac{\hbar^2 q k_R}{2m} \check{\sigma}_x - \frac{\delta}{2} \check{\sigma}_x + \frac{\Omega}{2} \check{\sigma}_z}$$

$\boxed{\text{To Experiment}}$

Interactions

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So far my discussion has focused on single particle physics, but the "good stuff" has to do with interacting systems.

For the interatomic $V(r) \sim -C_6/r^6$ potential, it is possible to make a pseudo-potential approximation when

$k \ll \frac{2\pi}{a_{vdw}}$, $a_{vdw} \sim 5nm$ (generally ok)

Then $V(r) = g\,\delta(r_1 - r_2)$ for a single spin.

Lets consider two atoms with angular momentum F colliding with B=0. This implies that angular momentum is conserved

so $$\hat{V} = \delta(r_1 - r_2) \sum_{G=0}^{2F} g_G \hat{P}_G, \quad \text{where } g_G = \frac{4\pi\hbar^2 a_G}{m} \quad \text{and} \quad \hat{P}_G = \sum_{m_G=-G}^{G} |G, m_G\rangle\langle G, m_G|$$

for Bosons only even G are allowed and odd for Fermions.
This is the first time spin statistics has come up!

So we have the closure relations for Bosons and Fermions

$$\underbrace{\mathbb{1} = \sum_{\substack{G=0 \\ \text{even}}}^{2F} \hat{P}_G}_{\text{Bosons}}, \quad \underbrace{\mathbb{1} = \sum_{\substack{G=0 \\ \text{odd}}}^{2F} \hat{P}_G}_{\text{Fermions}}$$

Lastly recall $$\hat{F}_1 \cdot \hat{F}_2 = \frac{\hat{G}^2 - \hat{F}_1^2 - \hat{F}_2^2}{2} = \frac{\hbar^2}{2}\left[g(g+1) - 2f(f+1)\right].$$

multiplying this by our projector: $$\hat{F}_1 \cdot \hat{F}_2 = \sum_{G=0}^{2F} \hat{F}_1 \cdot \hat{F}_2 \hat{P}_G = \sum_{G=0}^{2F} \lambda_G \hat{P}_G$$

At this point it's difficult to go farther w/o picking F

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Interactions : Examples

$$\lambda_G = \frac{\hbar^2}{2}\left[g(g+1) - 2f(f+1)\right]$$

$$\boxed{\hat{V} = \delta(r_1 - r_2)\sum_{G=0}^{2F} g_G \hat{P}_G}$$

$$\boxed{\hat{F}_1\cdot\hat{F}_2 = \sum_{G=0}^{2F} \hat{F}_1\cdot\hat{F}_2\,\hat{P}_G = \sum_{G=0}^{2F} \lambda_G \hat{P}_G}$$

$F=0$ (Bosons, trivial): $V = \delta(r_1-r_2)\, g_0 \hat{P}_0$, but $\mathbb{1} = P_0$

$$\hat{V} = g_0\,\delta(r_1 - r_2)$$

$F=1$ (Bosons): $V = \delta(r_1 - r_2)(g_0 P_0 + g_2 P_2)$

This can be rewritten using $\hat{F}_1\cdot\hat{F}_2 = -2P_0 + P_2$

$$\boxed{\begin{gathered} V = \delta(r_1 - r_2)\left[c_0(\hat{P}_0 + \hat{P}_2) + c_2 F_1\cdot F_2\right] \\ \text{with } c_0 = \frac{g_0 + 2g_2}{3},\; c_2 = \frac{g_2 - g_0}{3} \end{gathered}}$$

This is pretty stunning:

The most general potential is $V = \frac{1}{2}\int d^3x\, g_{ijkl}\, \psi_i^\dagger(x)\psi_j^\dagger(x)\psi_k(x)\psi_l(x)$

which has 81 coefficients

exchange symmetry $g_{ijkl} = g_{jikl}$ and time reversal $g_{ijkl} = g_{klij}$

reduces this to a still large 21!

wheres the above gives just 2!

If I look at just the c_0 term: $V_0 = \frac{c_0}{2}\int d^3x\, \psi_i^\dagger(x)\psi_j^\dagger(x)\psi_j(x)\psi_i(x)$

$$= \frac{c_0}{2}\int d^3x :\left(\sum_i \hat{n}_i\right)^2: \quad \underline{\text{density}}\text{-}\underline{\text{density}}$$

Fermions

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$F = 1/2$ fermions

This is most simple in second quantized notation

$$V = \frac{1}{2}\int d^3x\, g_{ijkl}\, \psi_i^\dagger \psi_j^\dagger \psi_k \psi_l, \quad \text{with } \{\psi_i^\dagger, \psi_j\} = \delta_{ij}$$

$$\{\psi_i, \psi_j\} = 0$$

So $k \neq l$ and $i \neq j$

$$V = \frac{1}{2}\int d^3x \left(\psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\uparrow \psi_\downarrow\, g_{\uparrow\downarrow\uparrow\downarrow} + \psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow\, g_{\downarrow\uparrow\uparrow\downarrow} + \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow\, g_{\uparrow\downarrow\downarrow\uparrow} + \psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\downarrow \psi_\uparrow\, g_{\downarrow\uparrow\downarrow\uparrow}\right)$$

$$= \frac{(g_{\uparrow\downarrow\downarrow\uparrow} + g_{\downarrow\uparrow\uparrow\downarrow} - g_{\uparrow\downarrow\uparrow\downarrow} - g_{\downarrow\uparrow\downarrow\uparrow})}{2}\int d^3x\, \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

$$\boxed{V = \frac{\bar{g}}{2}\int d^3x\, \psi_i^\dagger \psi_j^\dagger \psi_j \psi_i = \frac{\bar{g}}{2}\int d^3x :(n_\uparrow + n_\downarrow)^2:}$$

so this is just a density-density interaction.

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