

Spins with neutral atoms

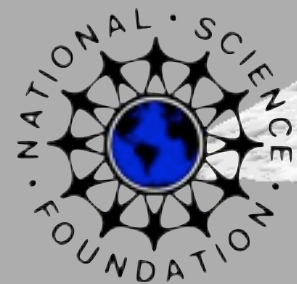
I. B. Spielman

Team

K. Jiménez-García,
R. A. Williams, L. J. LeBlanc, A. R. Perry, and M. Beeler
Now with real job: Y.-J. Lin

Senior coworkers

J. V. Porto, and W. D. Phillips



NIST

National Institute of Standards and Technology



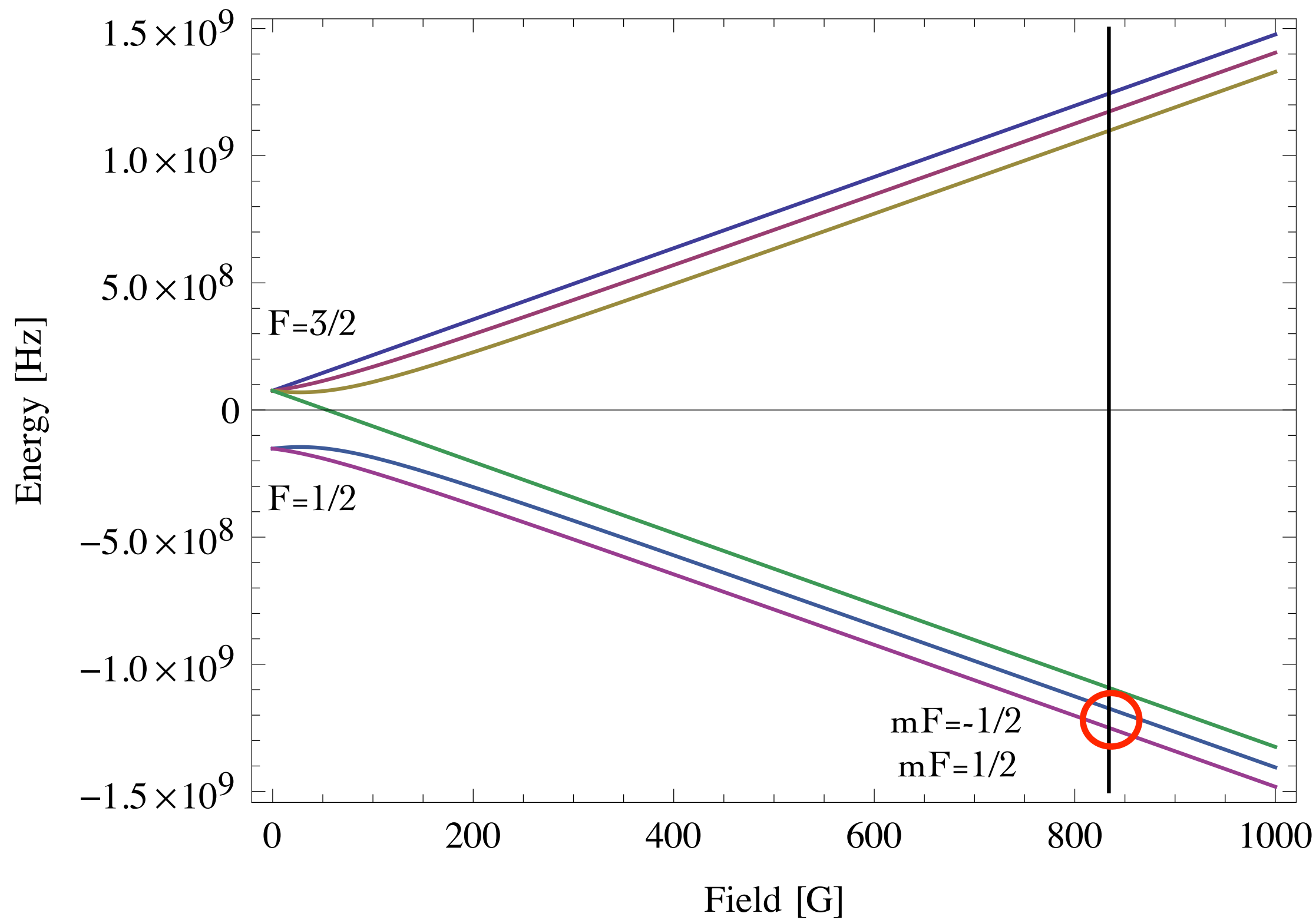
Dec. 2011

Rubidium 87

PERIODIC TABLE																	
Atomic Properties of the Elements																	
NIST National Institute of Standards and Technology Technology Administration, U.S. Department of Commerce																	
Physics Laboratory physics.nist.gov																	
Standard Reference Data Group www.nist.gov/srd																	
18 VIIIA																	
2 ¹ S ₀																	
He Helium 4.002602 1s ² 24.5874																	
13 14 15 16 17																	
IIIA IVA VA VIA VIIA																	
5 6 7 8 9 10 11 12																	
B C N O F Ne																	
Boron Carbon Nitrogen Oxygen Fluorine Neon																	
10.811 12.0107 14.0067 15.9994 18.9984032 20.1797																	
1s ² 2s ² 2p 1s ² 2s ² 2p ² 1s ² 2s ² 2p ³ 1s ² 2s ² 2p ⁴ 1s ² 2s ² 2p ⁵ 1s ² 2s ² 2p ⁶																	
8.2980 11.2603 14.5341 13.6181 17.4228 21.5645																	
13 14 15 16 17 18																	
Al Si P S Cl Ar																	
Aluminum Silicon Phosphorus Sulfur Chlorine Argon																	
26.981538 28.0855 30.973761 32.065 35.453 39.948																	
[Ne]3s ² 3p [Ne]3s ² 3p ² [Ne]3s ² 3p ³ [Ne]3s ² 3p ⁴ [Ne]3s ² 3p ⁵ [Ne]3s ² 3p ⁶																	
5.9858 8.1517 10.4867 10.3600 12.9676 15.7596																	
19 20 21 22 23 24 25 26 27 28 29 30																	
K Ca Sc Ti V Cr Mn Fe Co Ni Cu Zn																	
Potassium Calcium Scandium Titanium Vanadium Chromium Manganese Iron Cobalt Nickel Copper Zinc																	
39.0983 40.078 44.955910 47.867 50.9415 51.9961 54.938049 55.845 58.933200 58.6934 63.546 65.409																	
[Ar]4s [Ar]4s [Ar]3d4s ² [Ar]3d ² 4s ² [Ar]3d ³ 4s ² [Ar]3d ⁵ 4s [Ar]3d ⁵ 4s ² [Ar]3d ⁶ 4s ² [Ar]3d ⁷ 4s ² [Ar]3d ⁸ 4s ² [Ar]3d ⁹ 4s [Ar]3d ¹⁰ 4s ²																	
4.3407 6.1132 6.5615 6.8281 6.7462 6.7665 7.4340 7.9024 7.8810 7.6398 7.7264 9.3942																	
31 32 33 34 35 36																	
Ga Ge As Se Br Kr																	
Gallium Germanium Arsenic Selenium Bromine Krypton																	
69.723 72.64 74.92160 78.96 79.904 83.798																	
[Ar]3d ¹⁰ 4s ² 4p [Ar]3d ¹⁰ 4s ² 4p ² [Ar]3d ¹⁰ 4s ² 4p ³ [Ar]3d ¹⁰ 4s ² 4p ⁴ [Ar]3d ¹⁰ 4s ² 4p ⁵ [Ar]3d ¹⁰ 4s ² 4p ⁶																	
5.9993 7.8994 7.8867 9.7524 11.8138 13.9996																	
37 38 39 40 41 42 43 44 45 46 47 48																	
Rb Sr Y Zr Nb Mo Tc Ru Rh Pd Ag Cd																	
Rubidium Strontium Yttrium Zirconium Niobium Molybdenum Technetium Ruthenium Rhodium Palladium Silver Cadmium																	
85.4678 87.62 88.90585 91.224 92.90638 95.94 101.07 102.90550 106.42 107.8682 112.411 114.818																	
[Kr]5s [Kr]5s [Kr]4d5s ² [Kr]4d ² 5s ² [Kr]4d ⁴ 5s [Kr]4d ⁵ 5s [Kr]4d ⁵ 5s ² [Kr]4d ⁶ 5s [Kr]4d ⁷ 5s [Kr]4d ⁸ 5s [Kr]4d ⁹ 5s [Kr]4d ¹⁰ 5s ²																	
4.1771 5.6959 6.2173 6.6339 6.7589 7.0924 7.28 7.3605 7.4589 7.5762 8.9938 9.7864																	
49 50 51 52 53 54																	
In Sn Sb Te I Xe																	
Indium Tin Antimony Tellurium Iodine Xenon																	
114.818 118.710 121.760 127.60 126.90447 131.293																	
[Kr]4d ¹⁰ 5s ² 5p [Kr]4d ¹⁰ 5s ² 5p ² [Kr]4d ¹⁰ 5s ² 5p ³ [Kr]4d ¹⁰ 5s ² 5p ⁴ [Kr]4d ¹⁰ 5s ² 5p ⁵ [Kr]4d ¹⁰ 5s ² 5p ⁶																	
5.7864 7.3439 8.6084 9.0096 10.4513 12.1298																	
55 56 57 58 59 60 61 62 63 64 65 66																	
Cs Ba La Ce Pr Nd Pm Sm Eu Gd Tb Dy																	
Cesium Barium Lanthanum Cerium Praseodymium Neodymium Promethium Samarium Europium Gadolinium Terbium Dysprosium																	
132.90545 137.327 138.9055 140.116 140.90765 144.24 150.36 151.964 157.25 158.92534 162.500 164.93032																	
[Xe]6s [Xe]6s [Xe]4f145d ¹ 6s ² [Xe]4f145d ² 6s ² [Xe]4f145d ³ 6s ² [Xe]4f145d ⁴ 6s ² [Xe]4f145d ⁵ 6s ² [Xe]4f145d ⁶ 6s ² [Xe]4f145d ⁷ 6s ² [Xe]4f145d ⁸ 6s ² [Xe]4f145d ⁹ 6s ² [Xe]4f145d ¹⁰ 6s ²																	
3.8939 5.2117 6.8251 7.5496 7.8640 7.8335 8.4382 8.9670 8.9588 9.2255 10.4375 10.6782																	
67 68 69 70 71																	
Ho Er Tm Yb Lu																	
Holmium Erbium Thulium Ytterbium Lutetium																	
164.93032 167.259 168.93421 173.04 174.967																	
[Xe]4f146s ² [Xe]4f146s ² [Xe]4f146s ² [Xe]4f146s ² [Xe]4f146s ²																	
6.0215 6.1077 6.1843 6.2542 5.4259																	
72 73 74 75 76 77 78 79 80 81 82 83 84 85 86																	
Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn																	
Hafnium Tantalum Tungsten Rhenium Osmium Iridium Platinum Gold Mercury Thallium Lead Bismuth Polonium Astatine Radon																	
178.49 180.9479 183.84 186.207 192.22 192.217 195.078 196.96655 200.59 204.3833 207.2 208.98038 208.9804 210 210 222																	
[Xe]4f145d ² 6s ² [Xe]4f145d ³ 6s ² [Xe]4f145d ⁴ 6s ² [Xe]4f145d ⁵ 6s ² [Xe]4f145d ⁶ 6s ² [Xe]4f145d ⁷ 6s ² [Xe]4f145d ⁸ 6s ² [Xe]4f145d ⁹ 6s ² [Xe]4f145d ¹⁰ 6s																	

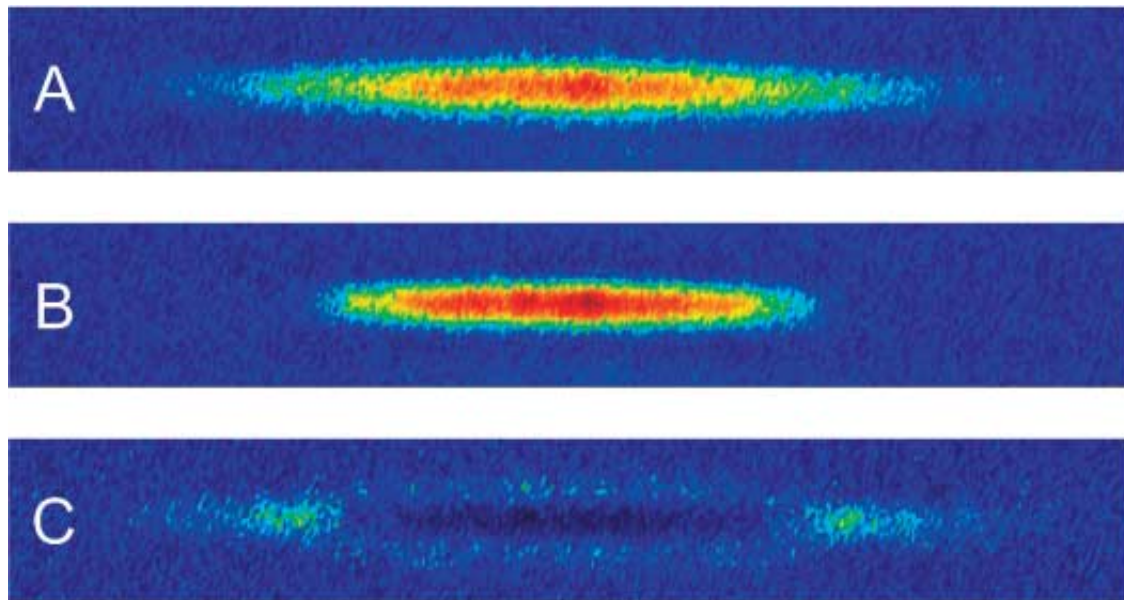


Li, 834 Feshbach Labeled

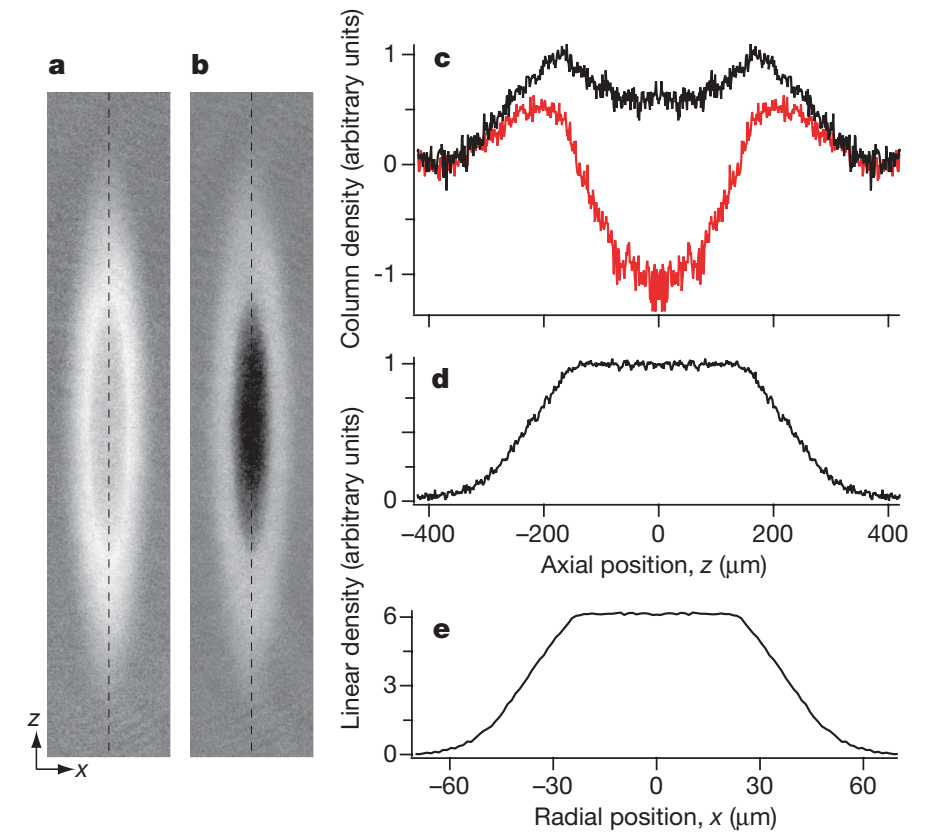


BEC-BCS Crossover physics

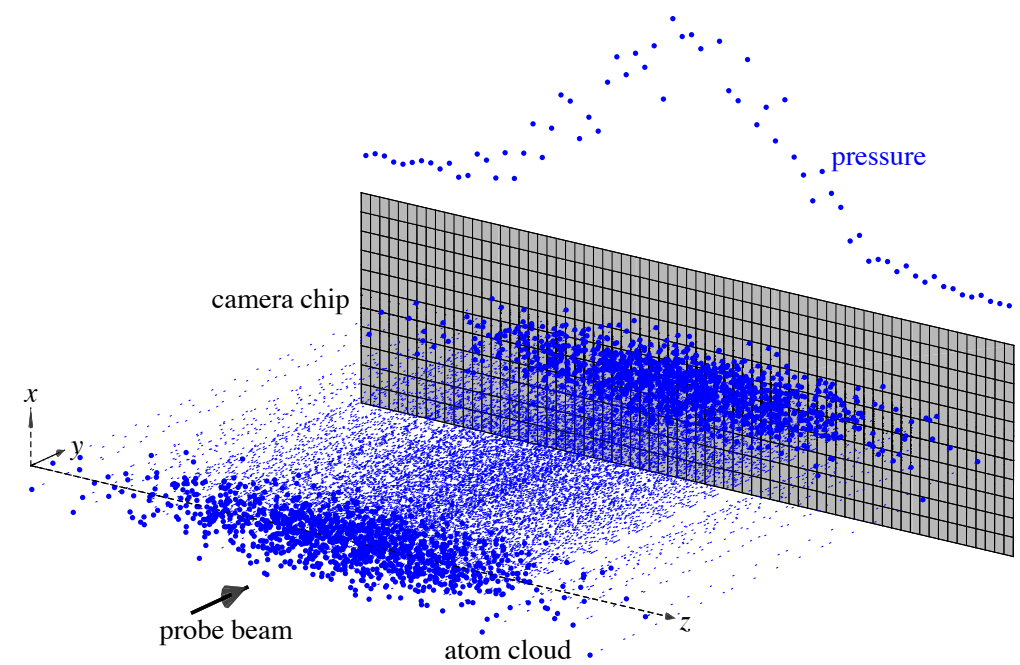
Hulet (Rice)



Ketterle (MIT)

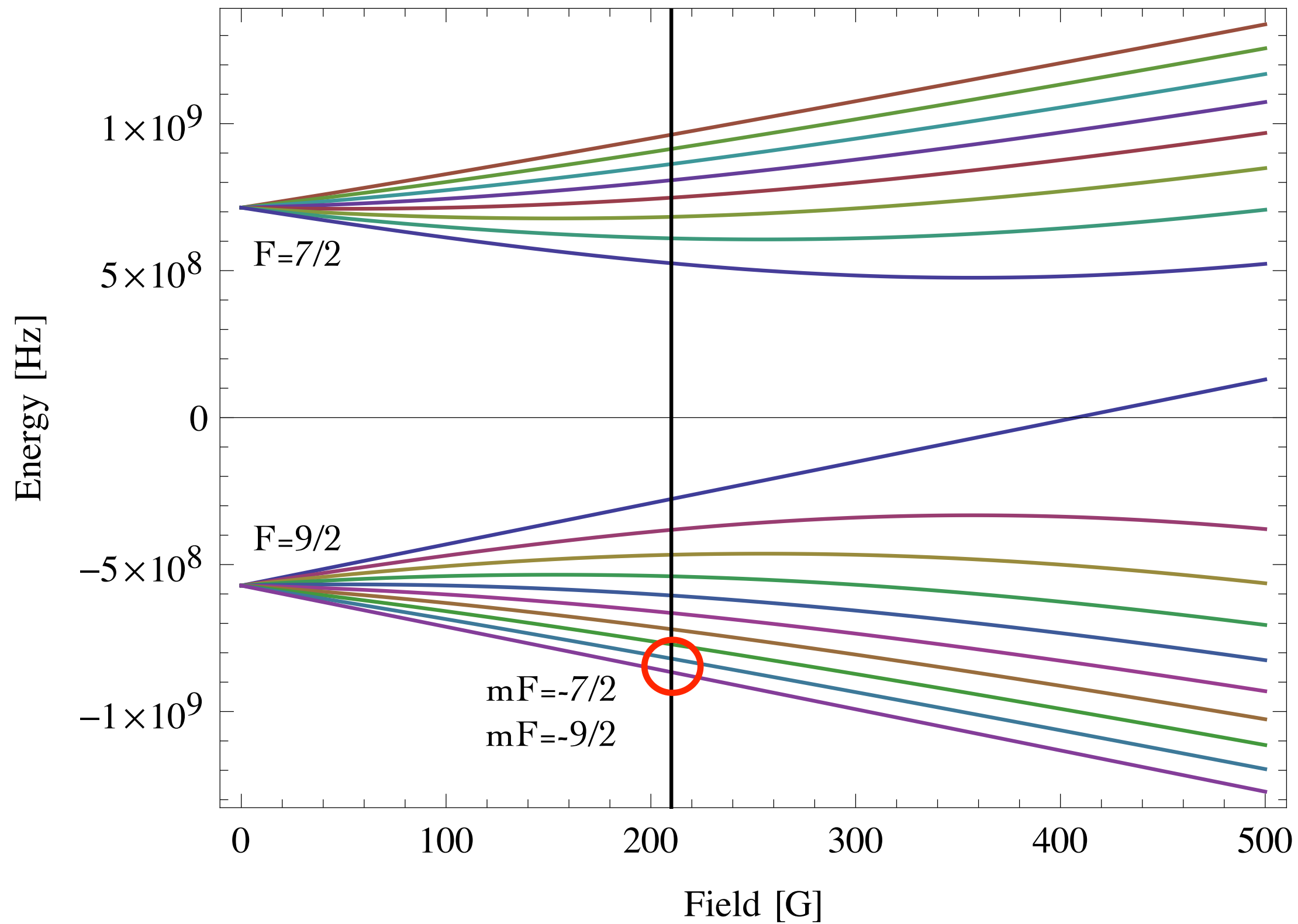


Solomon (ENS)

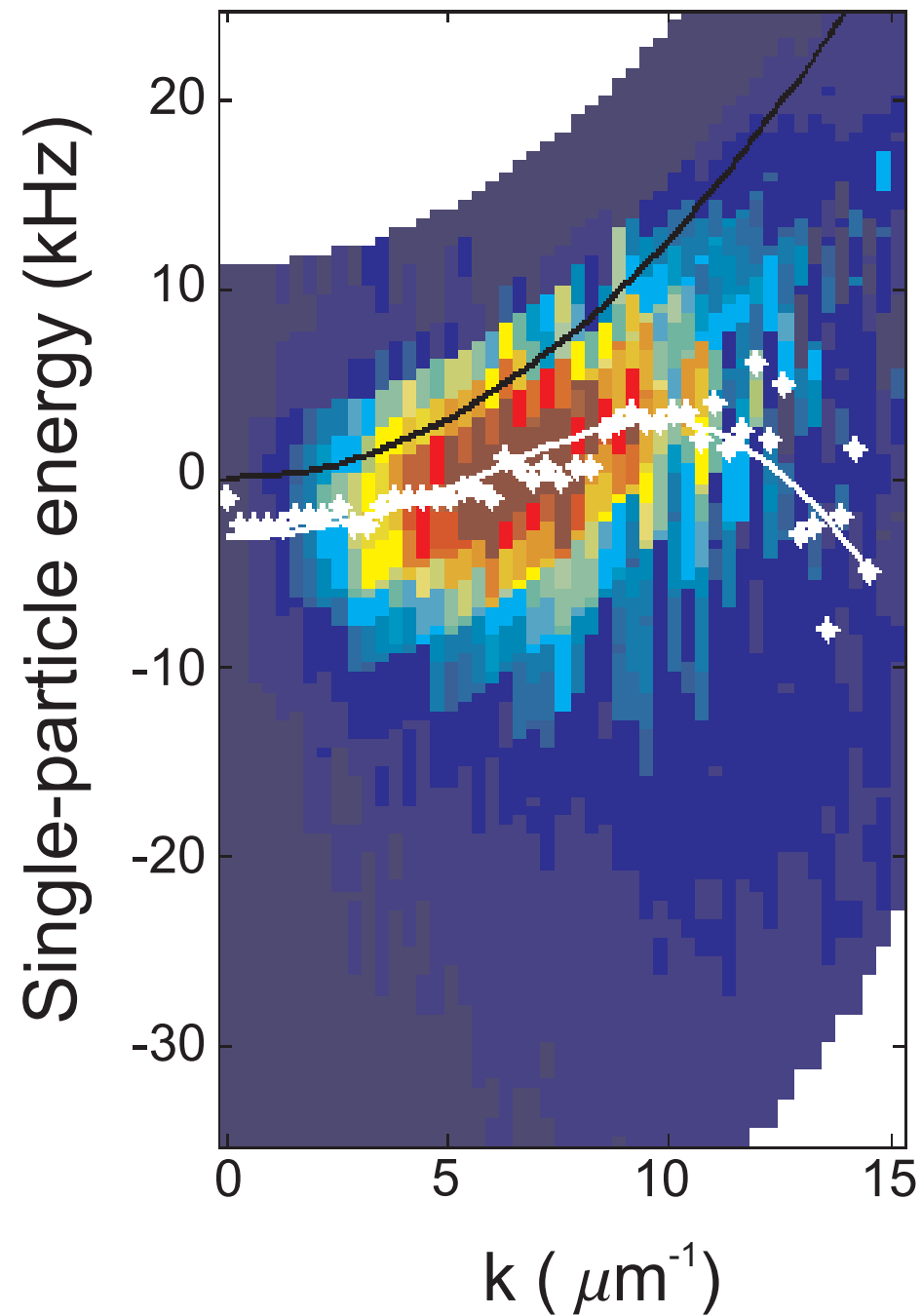


^{40}K has inverted hyperfine

^{40}K , 210 G Feshbach Labeled

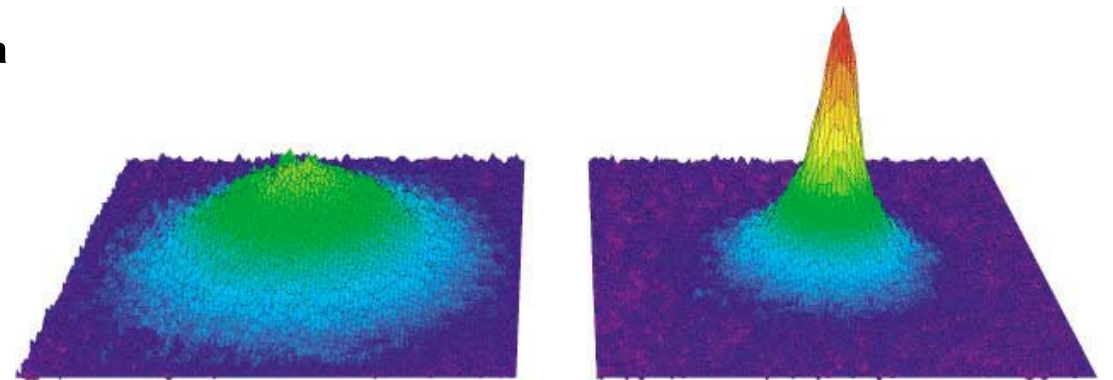


ARPES analogy

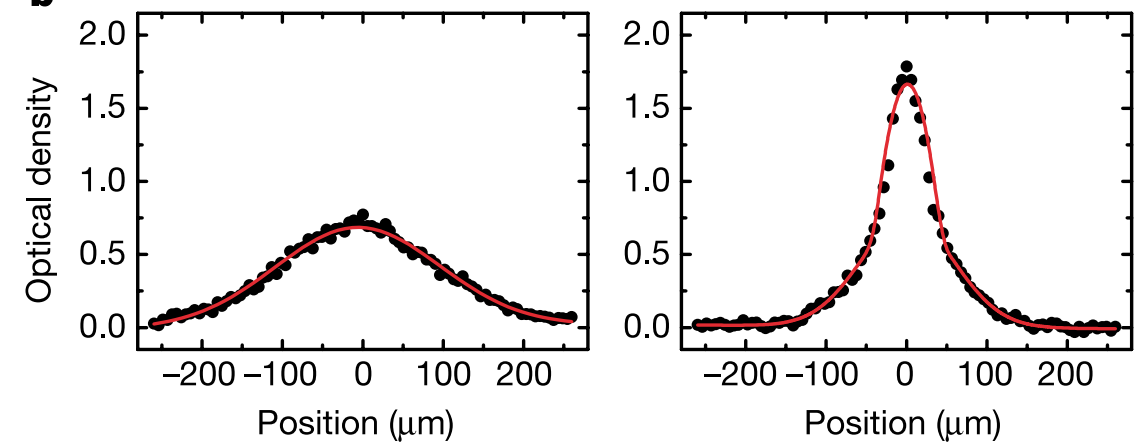


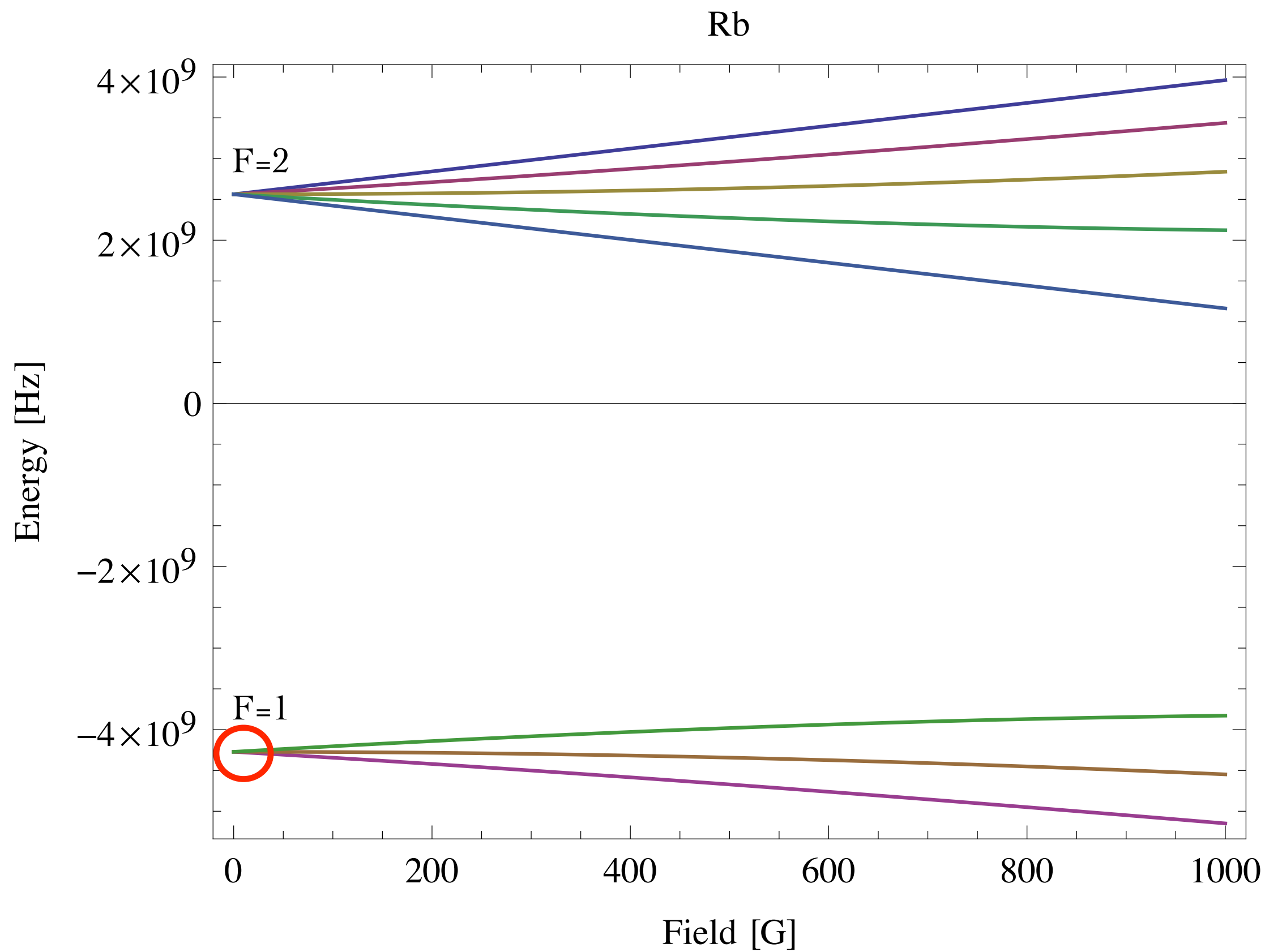
BEC of molecules

a

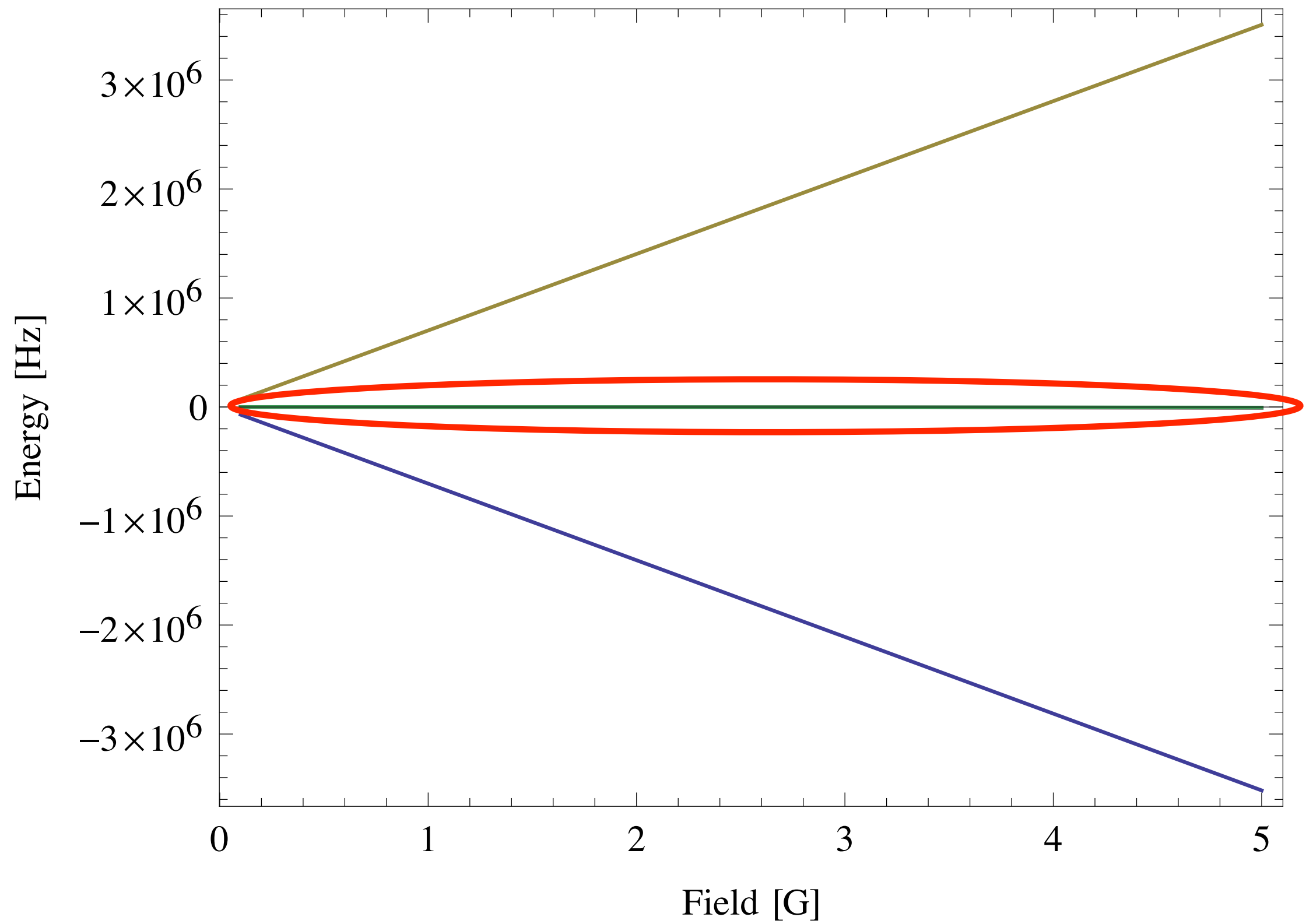


b

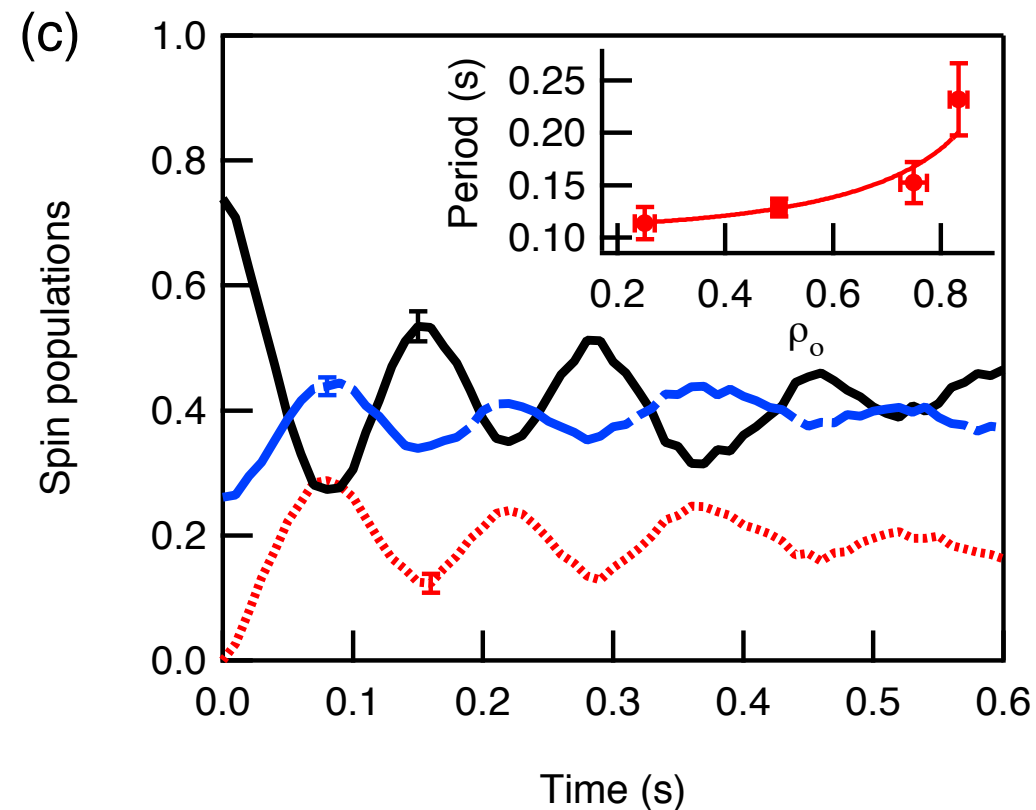
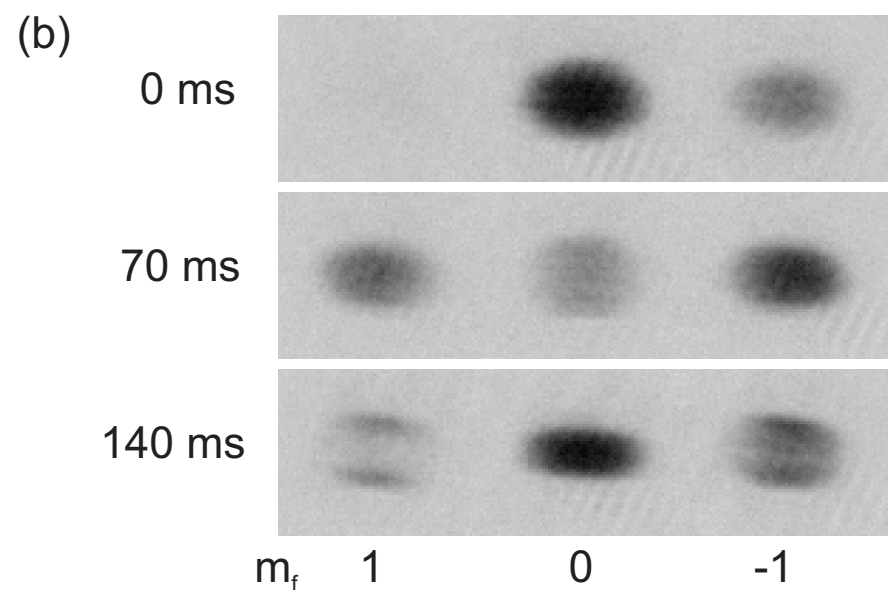
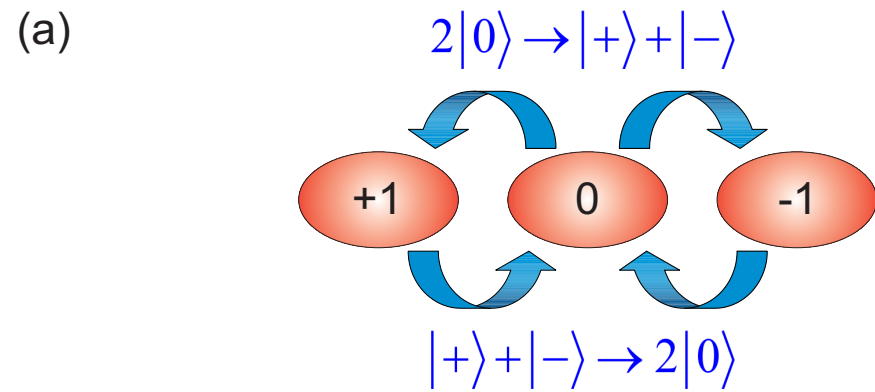




^{87}Rb F=1 at low field

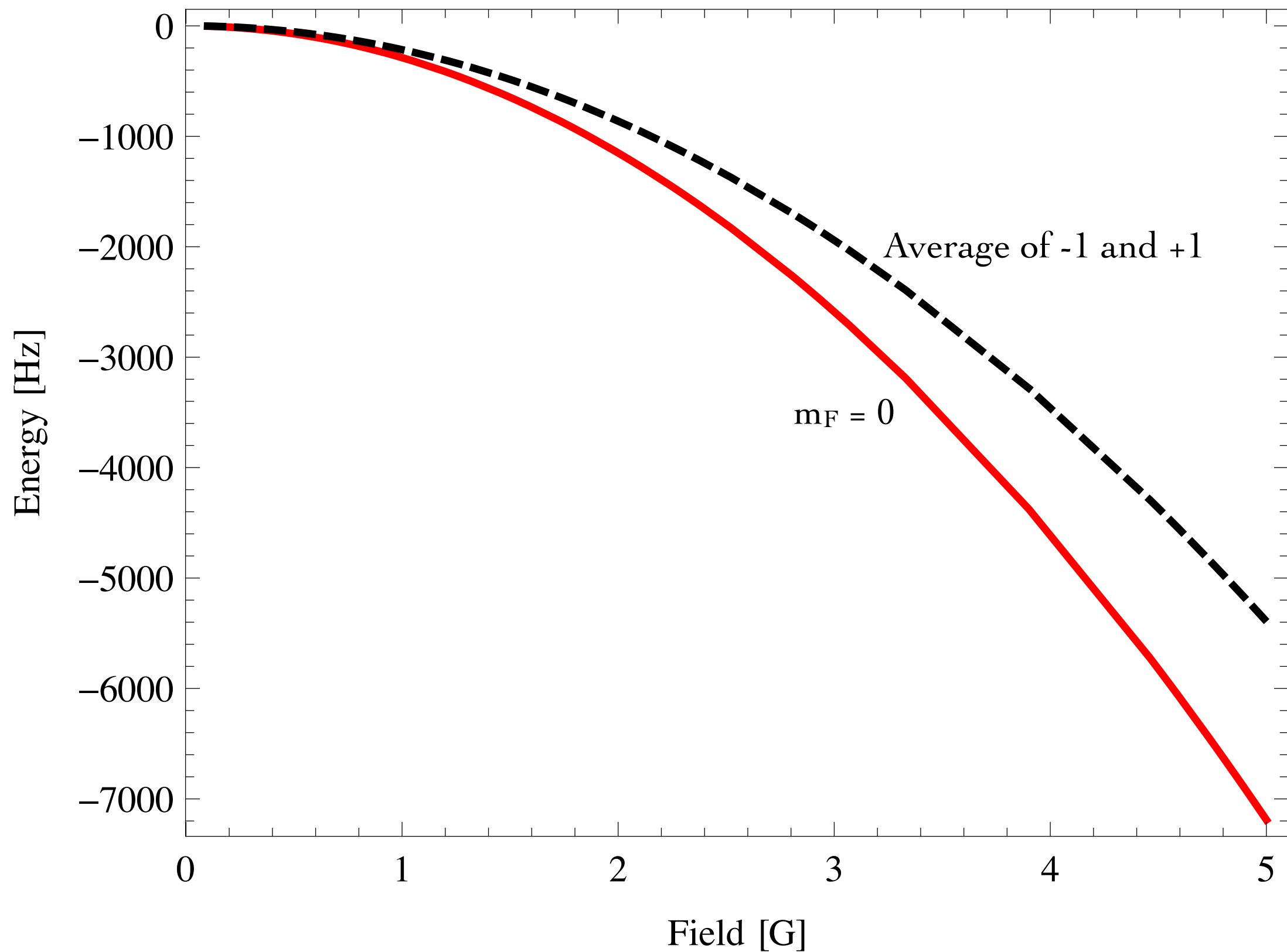


Spinor BEC's (Chapman group) for example

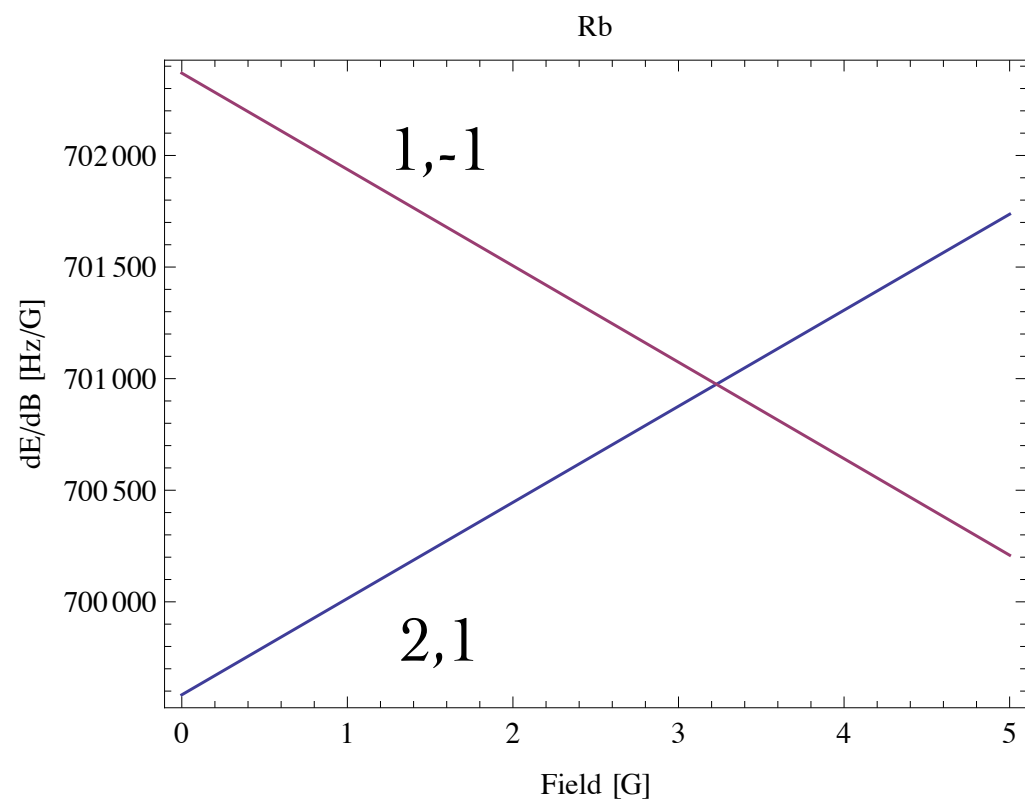
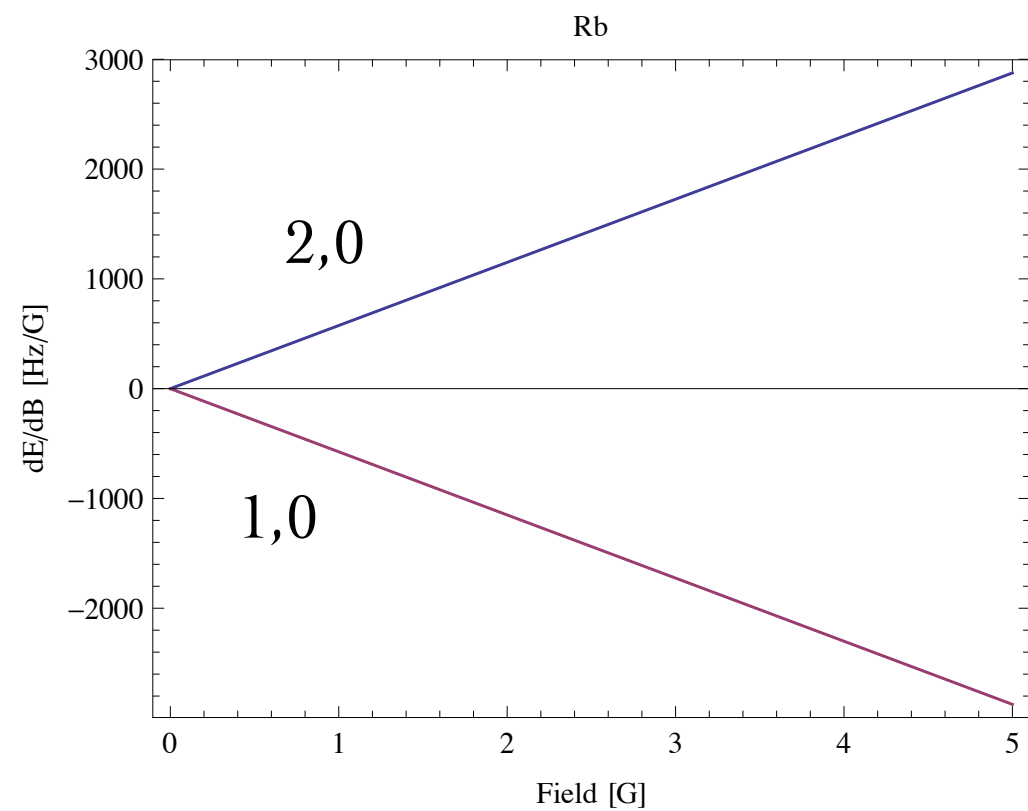
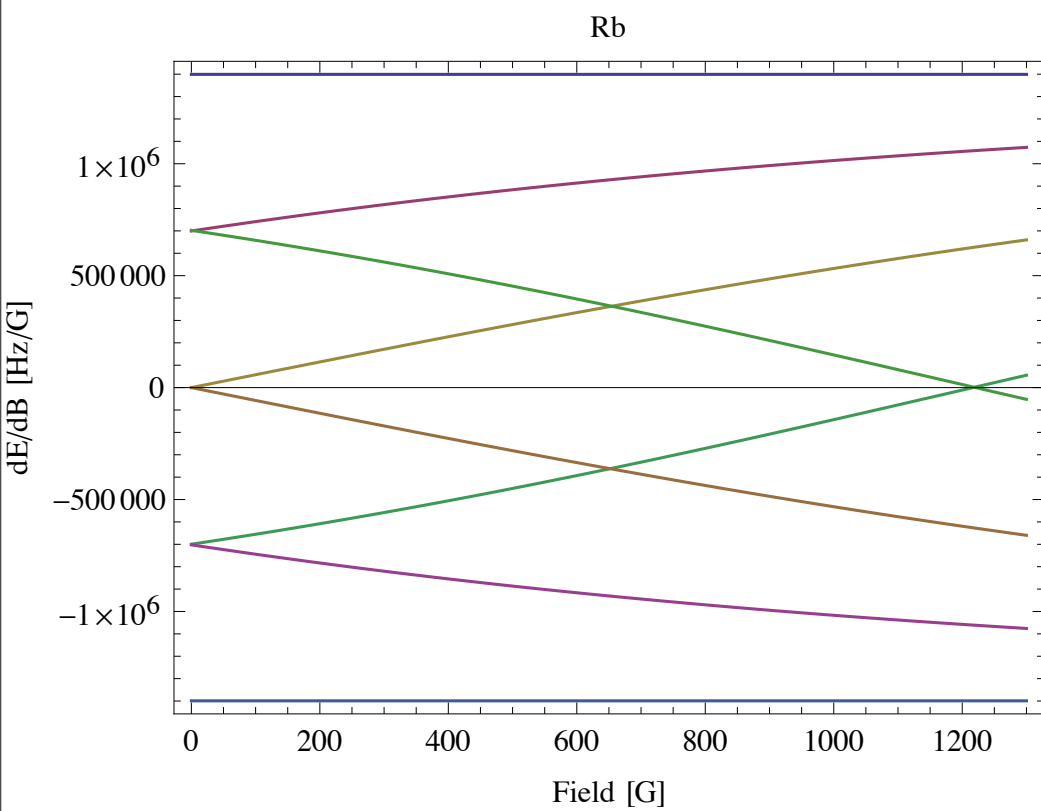


Three spins! How often do you see THAT with electrons...

^{87}Rb F=1 quadratic effects



^{87}Rb Clock states

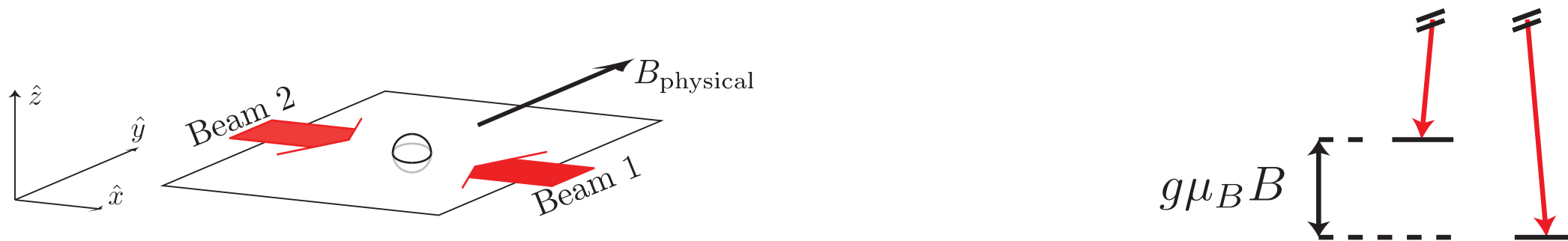


Standard picture

Space dependent coupling

Now we have no time-dependence, but instead space-dependence.

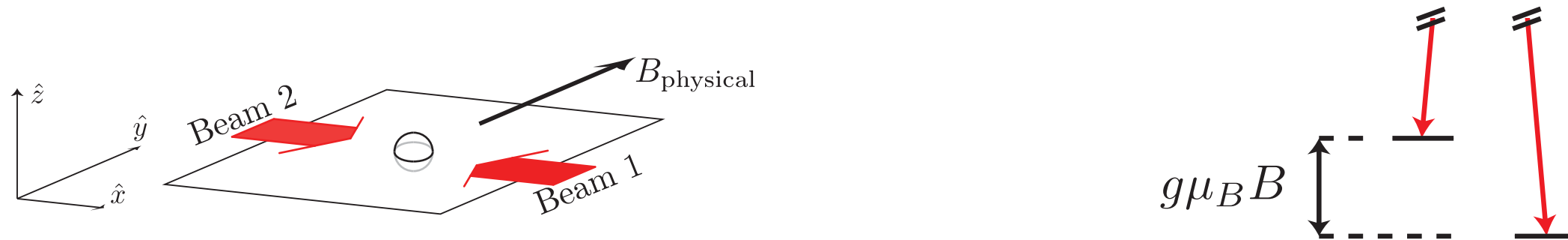
Geometry and initial Hamiltonian



$$\begin{aligned}\hat{H} &= \frac{\hbar^2 \hat{k}^2}{2m} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\Omega_R}{2} \begin{pmatrix} 0 & \exp(i2k_R x) \\ \exp(-i2k_R x) & 0 \end{pmatrix} + \begin{pmatrix} \delta/2 & 0 \\ 0 & -\delta/2 \end{pmatrix} \\ &= \frac{\hbar^2 \hat{k}^2}{2m} \hat{1} + \frac{\Omega}{2} [\cos(2k_R x) \check{\sigma}_x - \sin(2k_R x) \check{\sigma}_y] + \frac{\delta}{2} \check{\sigma}_z\end{aligned}$$

Exact picture

Geometry and initial Hamiltonian



Work in the momentum basis!

$$\begin{aligned}\hat{U}_1 \hat{H} \hat{U}_1^\dagger &= (\hat{k} \hat{1} - \check{\sigma}_z)^2 + \frac{\Omega}{2} \check{\sigma}_x + \frac{\delta}{2} \check{\sigma}_z \\ &= \begin{pmatrix} (k-1)^2 + \delta/2 & \Omega/2 \\ \Omega/2 & (k+1)^2 - \delta/2 \end{pmatrix}\end{aligned}$$

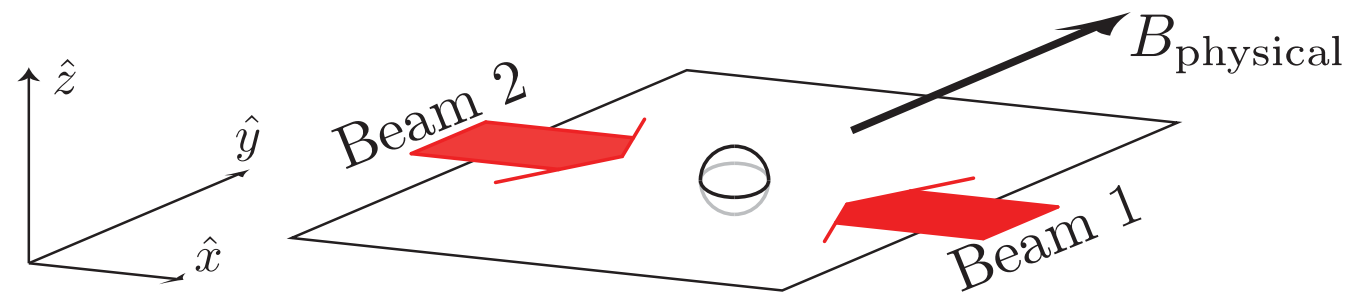
Numerically solve without approximation

Spin orbit coupling: origin

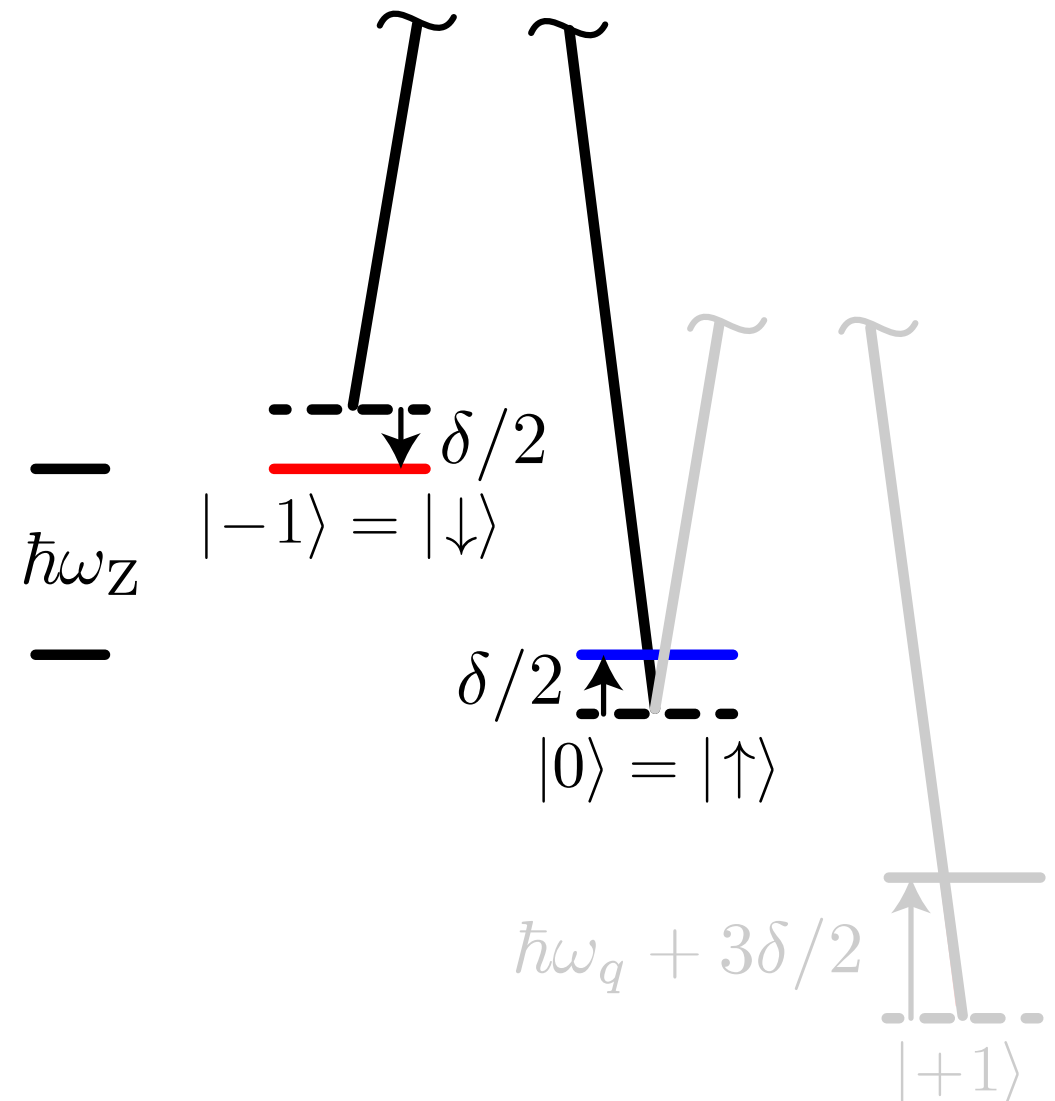
Momentum representation

$$H = \sum_k \left\{ \begin{pmatrix} \langle k-1, \uparrow | & \langle k+1, \downarrow | \end{pmatrix} \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1, \uparrow\rangle \\ |k+1, \downarrow\rangle \end{pmatrix} \right\}$$

Geometry



Levels



References

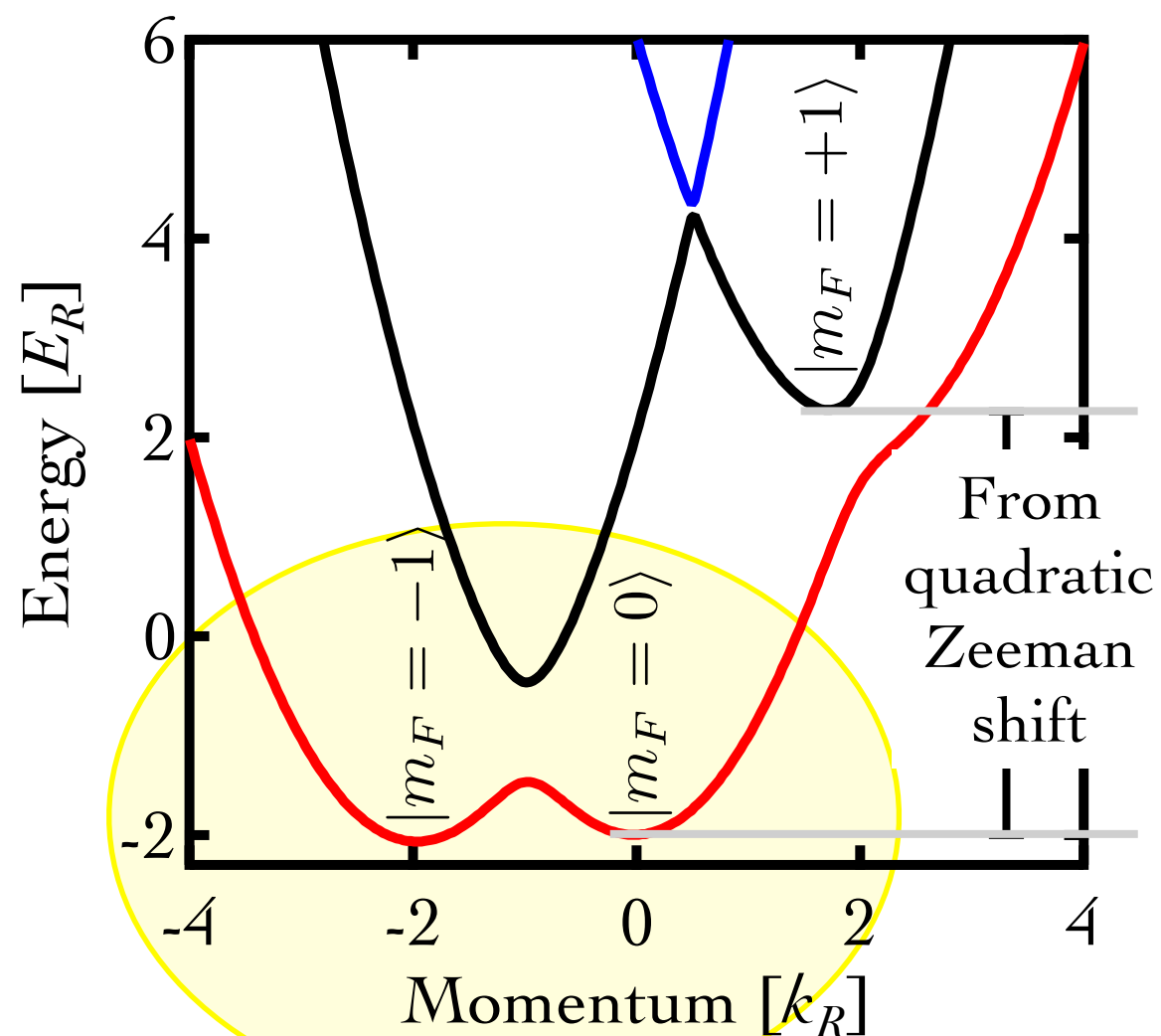
Y.-J. Lin *et al* Nature (2011)

Spin orbit coupling: origin

Momentum representation

$$H = \sum_k \left\{ \begin{pmatrix} \langle k-1, \uparrow | & \langle k+1, \downarrow | \end{pmatrix} \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1, \uparrow\rangle \\ |k+1, \downarrow\rangle \end{pmatrix} \right\}$$

Spin 1/2 bosons????



Transform to

$$\begin{aligned} \hat{H} &= \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \left(\frac{\delta}{2} + \frac{\hbar^2 k_R}{m} \hat{k}_x \right) \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z + \Delta E \hat{1} \\ &= \frac{\hbar^2}{2m} \left[\left(\hat{k}_x \hat{1} + k_R \check{\sigma}_y \right)^2 + \left(\hat{k}_y \hat{1} - 0 \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \end{aligned}$$

NOTICE

Written as a “2x2” vector potential, this S-O coupling is **NOT** non-Abelian (it is A_x)

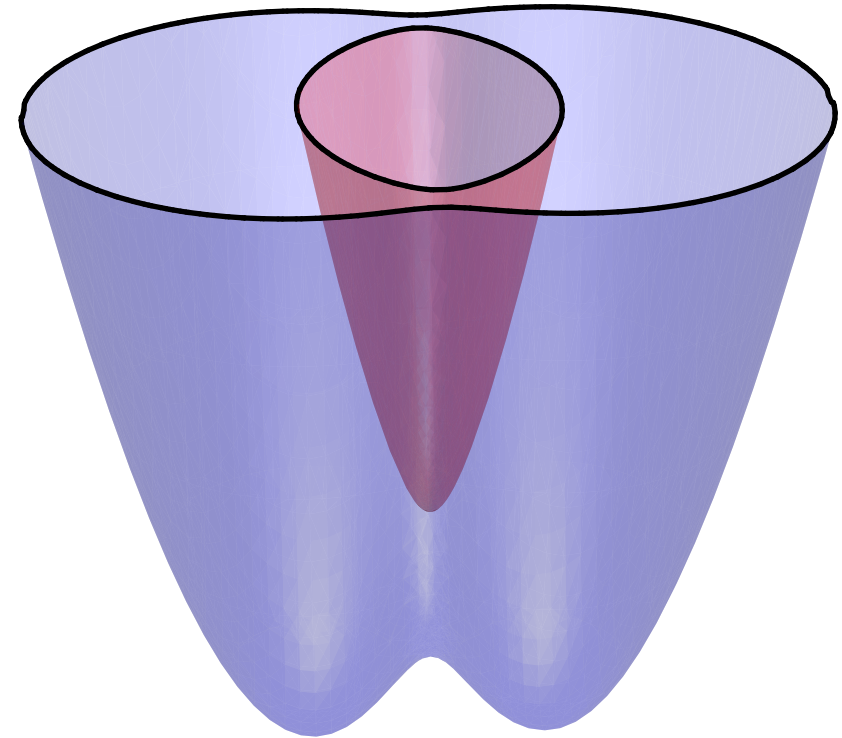
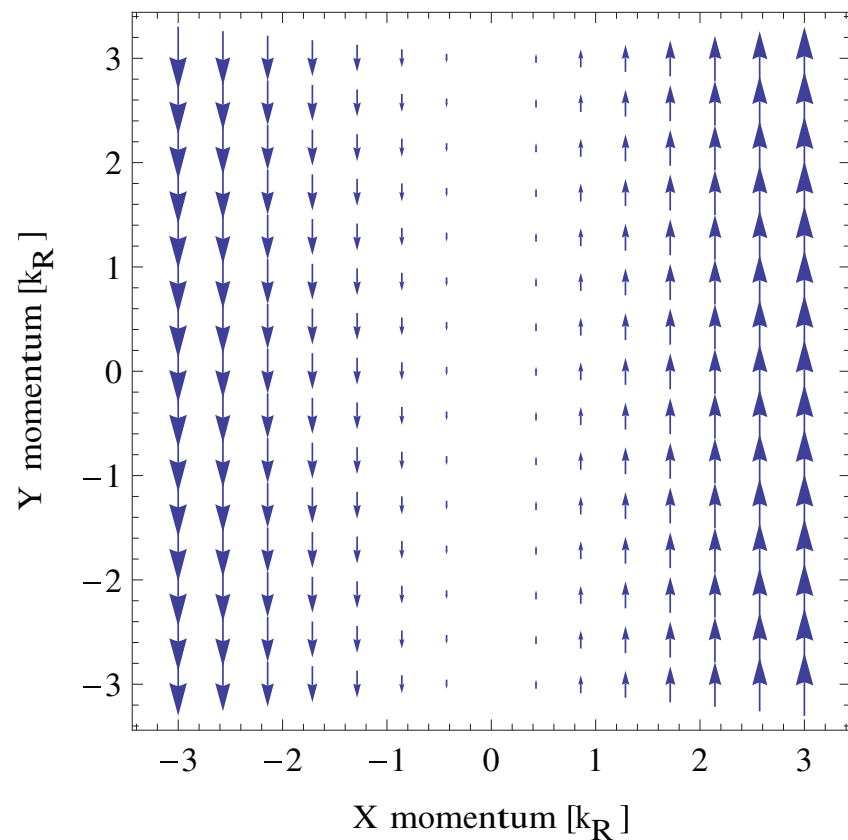
However, the Hamiltonian *is* non-trivial owing to the Zeeman field along z .

Spin-orbit coupling

Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \check{1} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y) .$$

Equal Rashba and Dresselhaus: $\alpha = \beta$



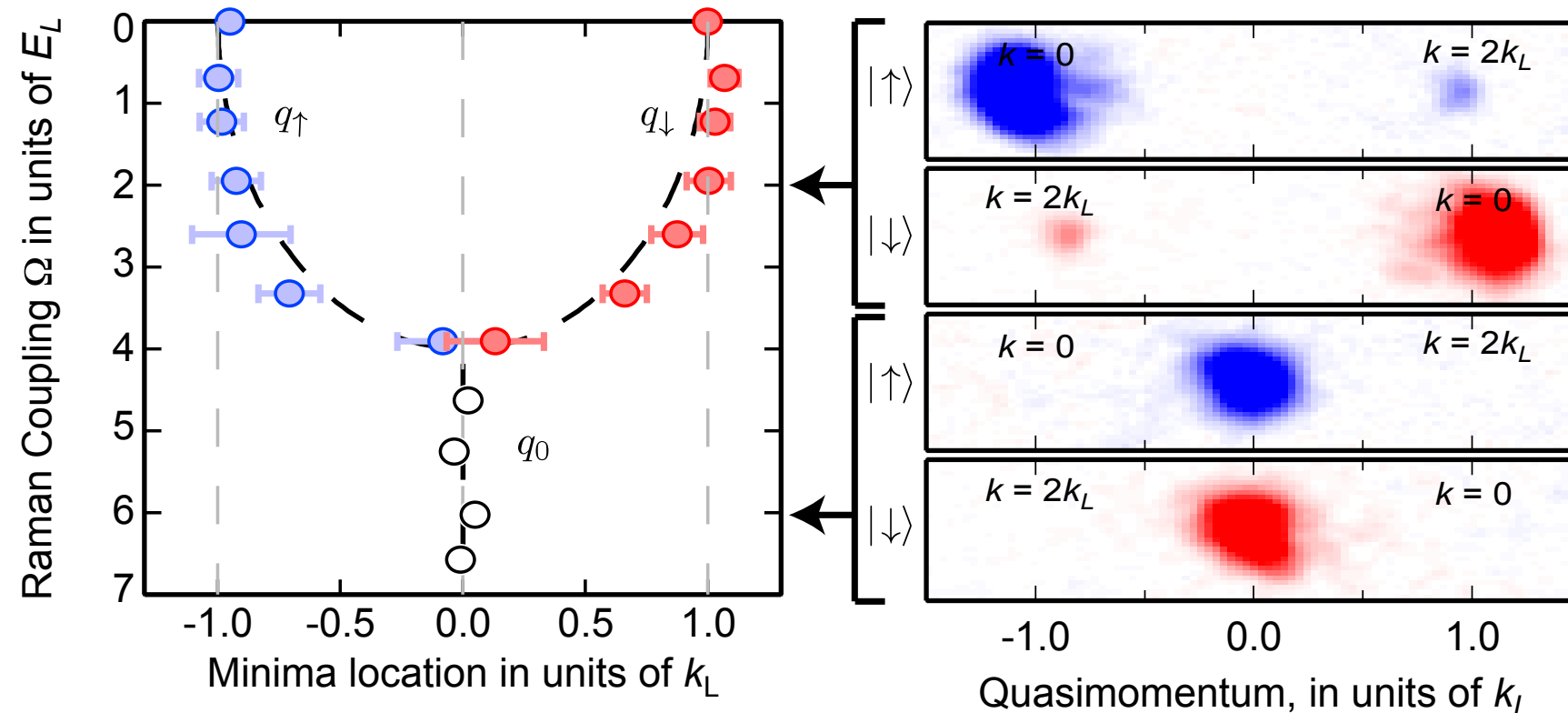
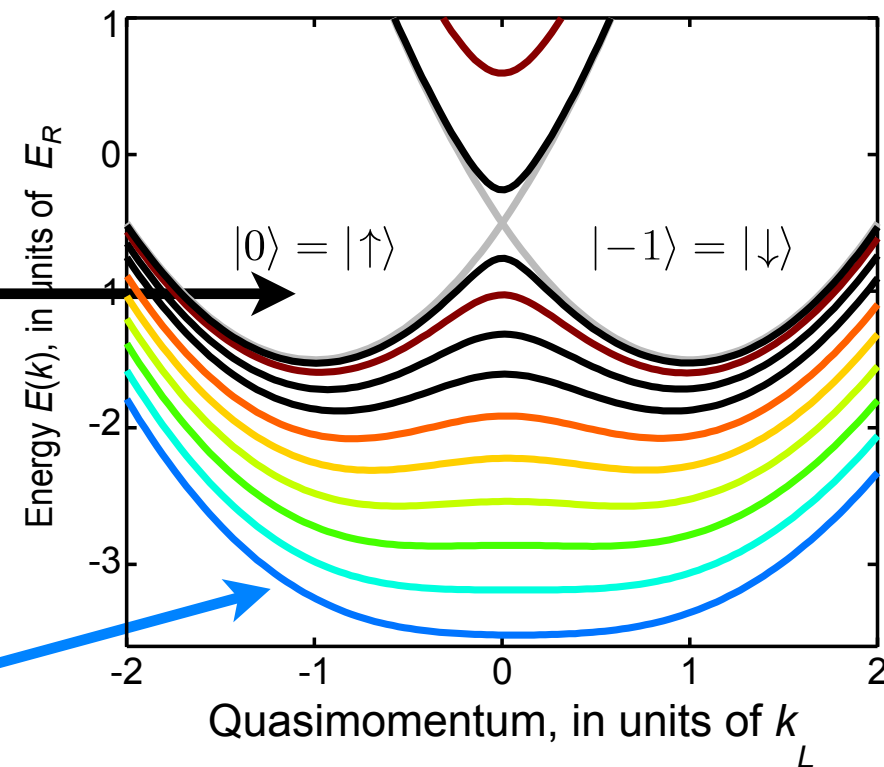
Refs.

T. D. Stanescu and B. Anderson and V. Galitski PRA (2008)

Typical data

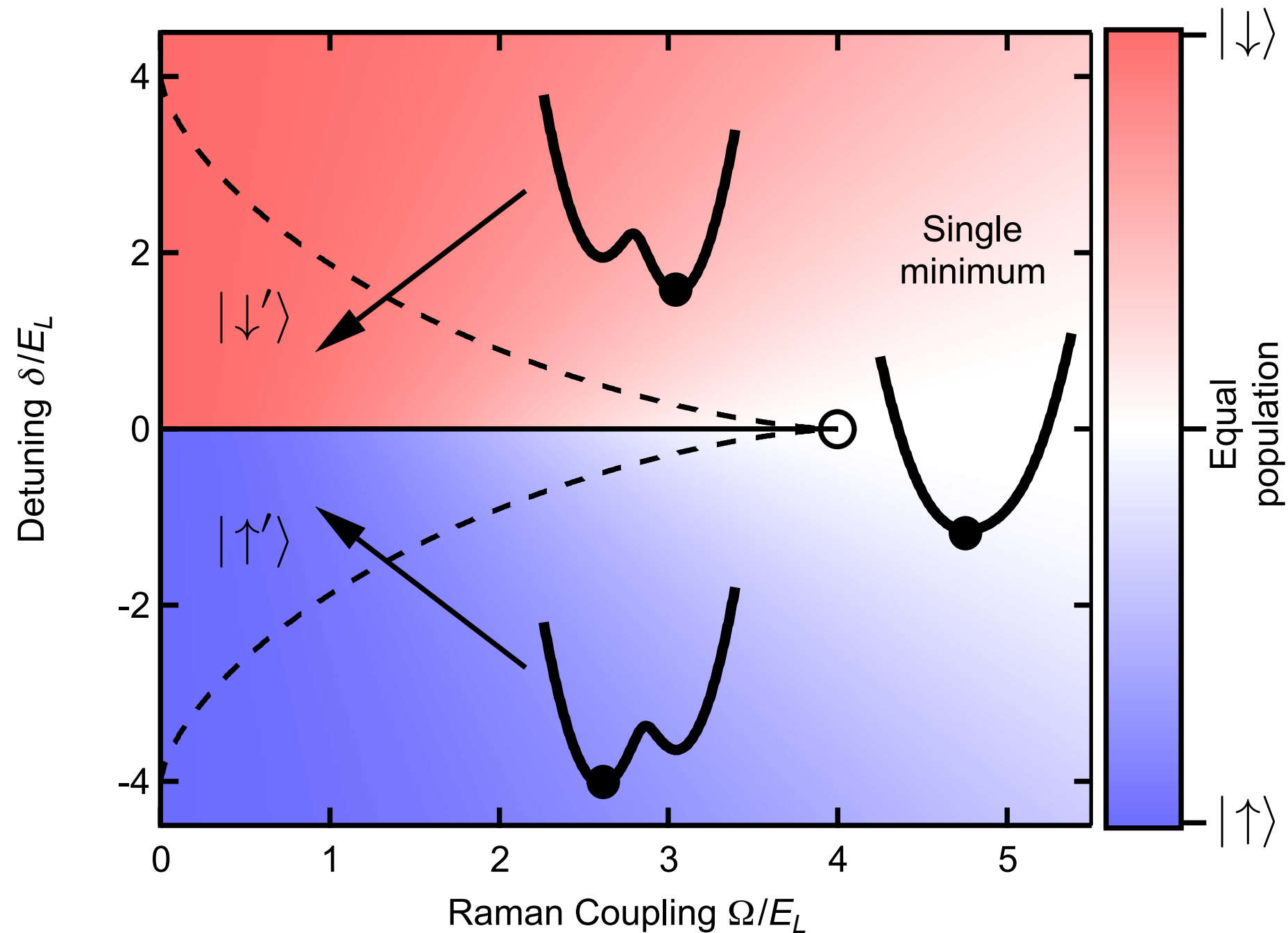
Double Well
“Spin-orbit limit”

Single minimum
“vector potential limit”



Phase diagram I: Large single particle energy

Minimize single particle energy

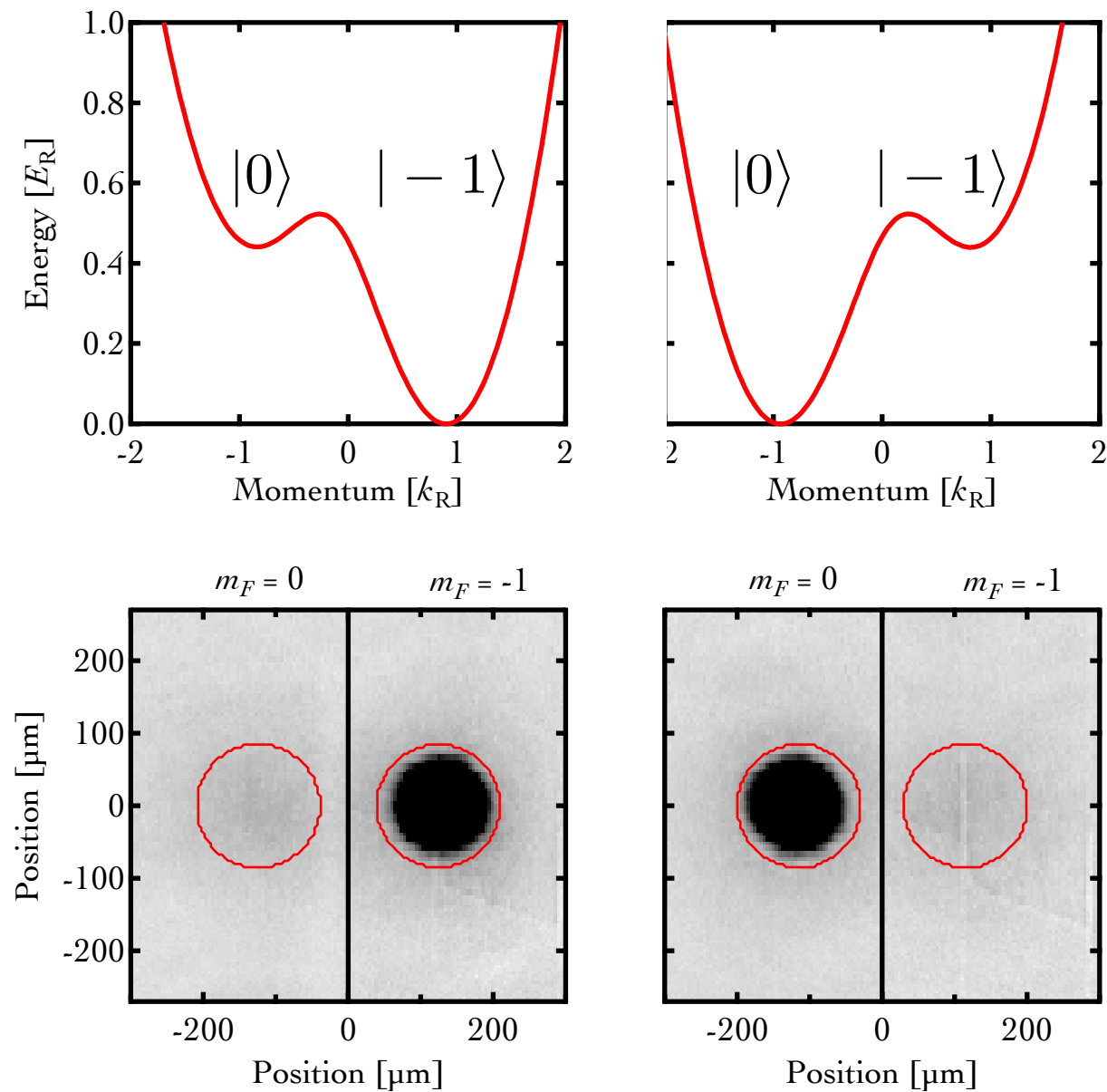


Refs.

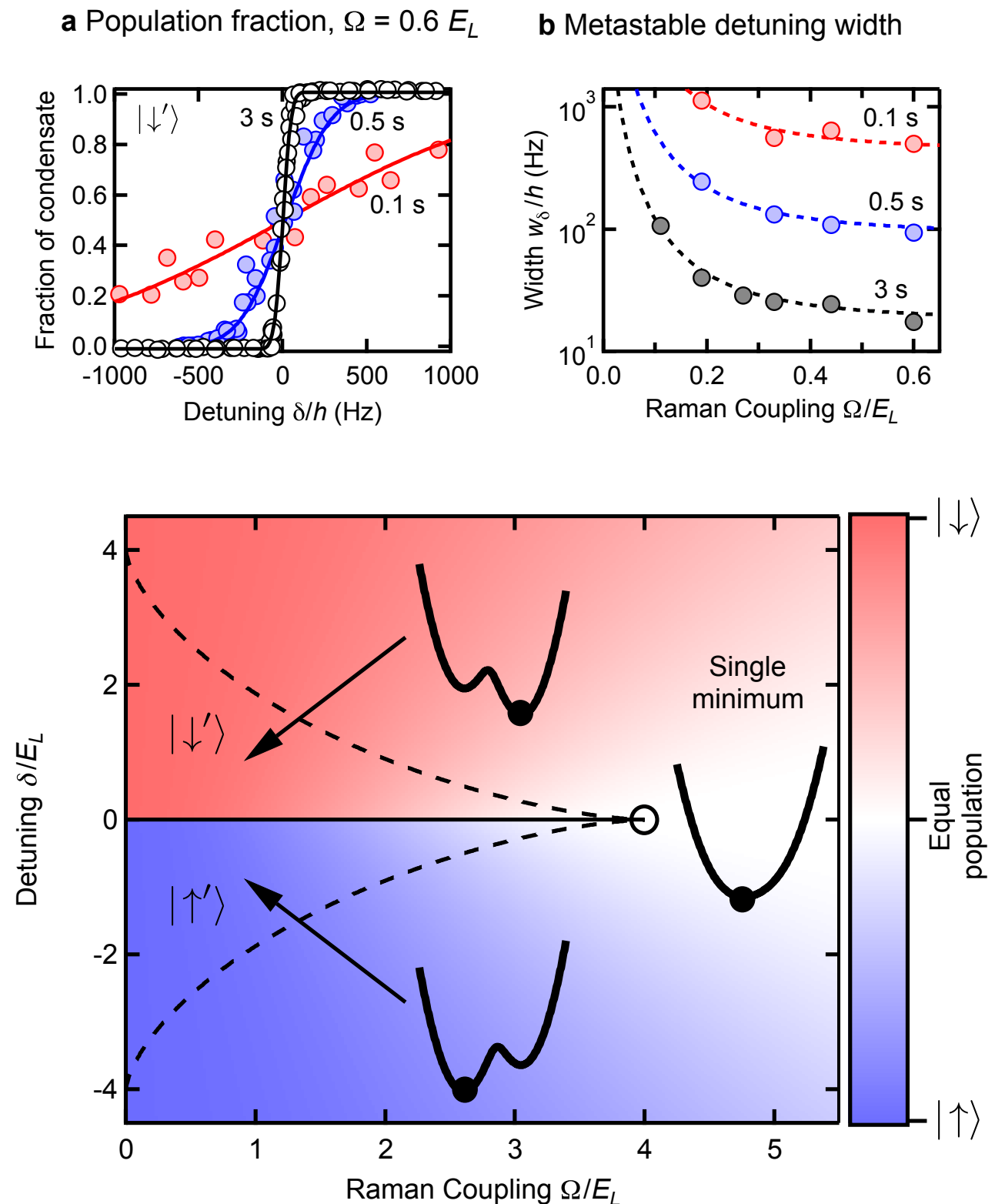
Y.-J. Lin et al, Nature (2011)

Equilibrium: slow

Minimize single particle energy



Metastable populations



References

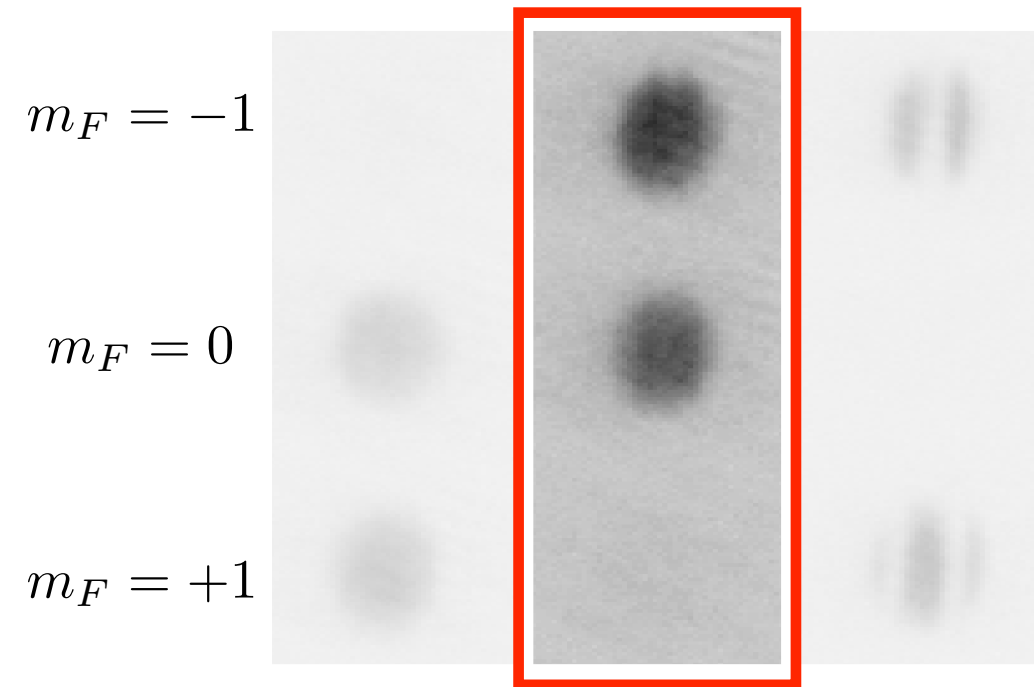
Y.-J. Lin *et al* Nature (2011)

Effective Hamiltonian for dressed spins

Two Level contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow} + \hat{\rho}_{\uparrow})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow}^2 - \hat{\rho}_{\uparrow}^2) + c_2 \hat{\rho}_{\downarrow} \hat{\rho}_{\uparrow} \right] :$$

$m_F = -1, m_F = 0$ mixture: miscible for ^{87}Rb



Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

Effective Hamiltonian for dressed spins

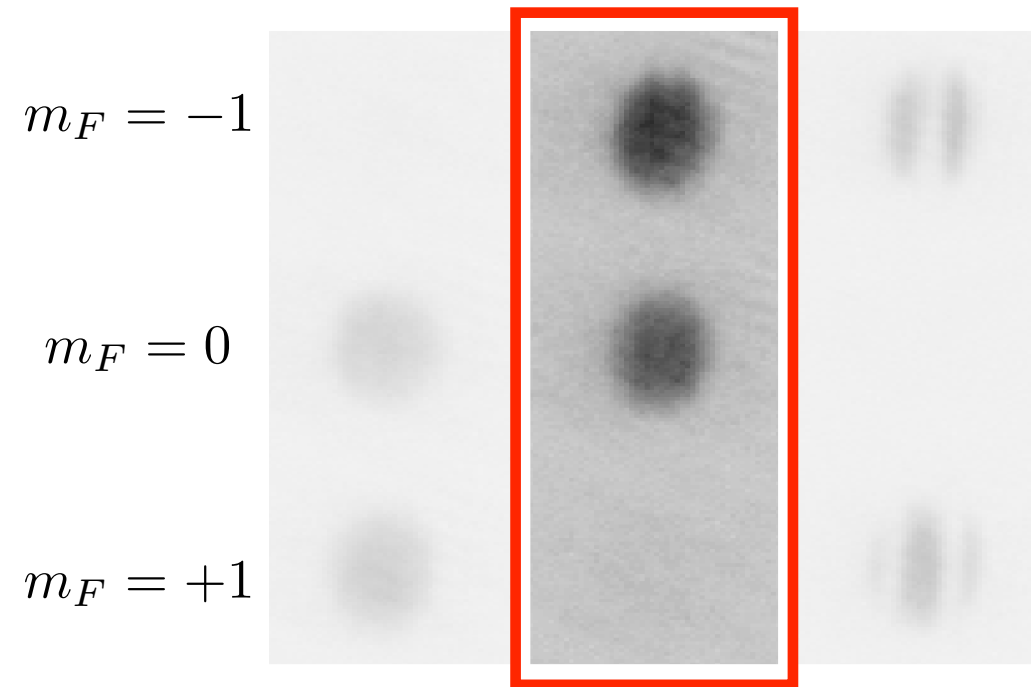
Two Level contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow} + \hat{\rho}_{\uparrow})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow}^2 - \hat{\rho}_{\uparrow}^2) + c_2 \hat{\rho}_{\downarrow} \hat{\rho}_{\uparrow} \right] :$$
$$\rightarrow \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow'} + \hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow'}^2 - \hat{\rho}_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \hat{\rho}_{\downarrow'} \hat{\rho}_{\uparrow'} \right] :$$

Spin-orbit term

$$c'_{\uparrow,\downarrow} \approx c_0 \frac{\Omega^2}{8}$$

$m_F = -1, m_F = 0$ mixture: miscible for ^{87}Rb



Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

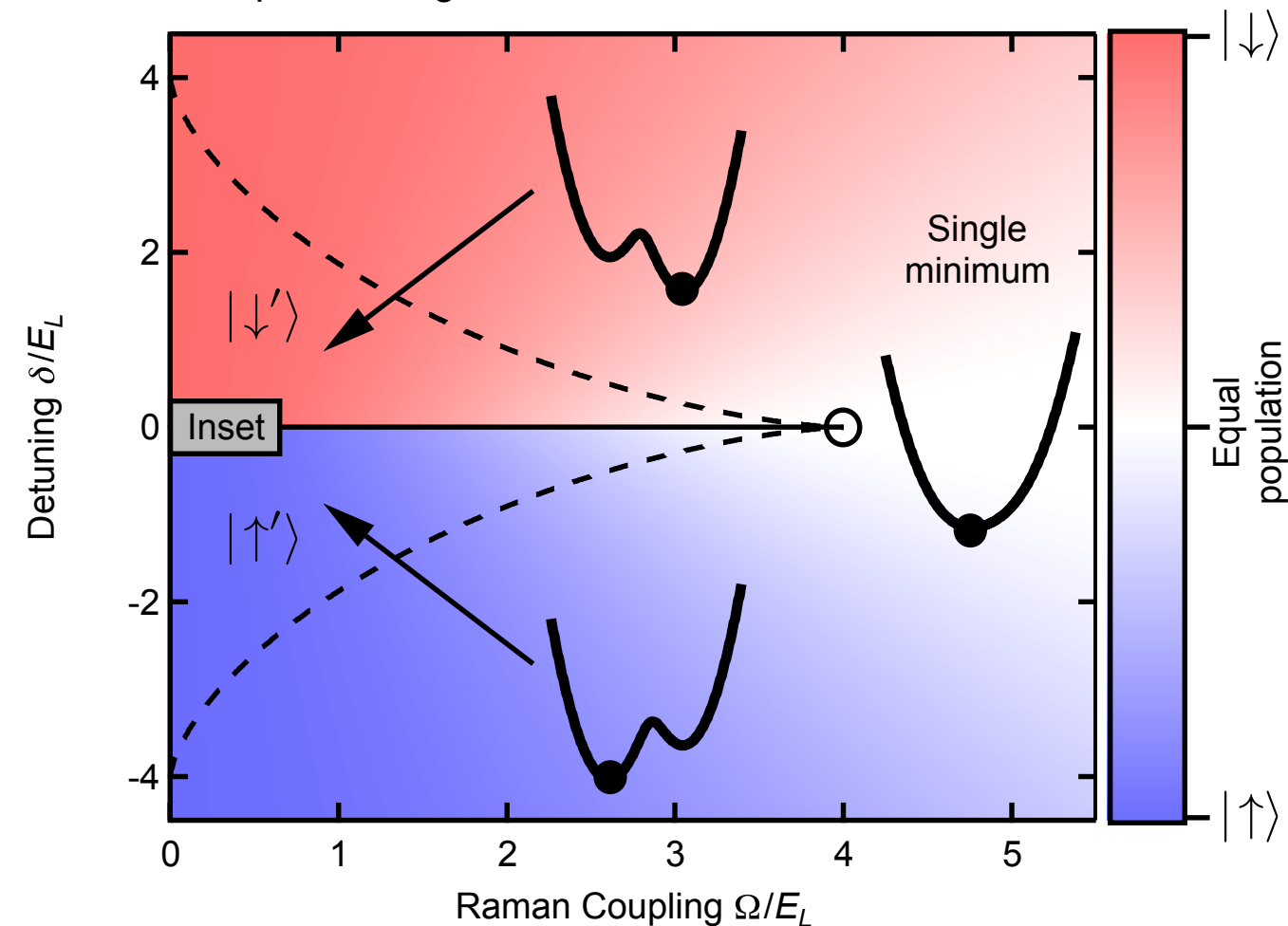
Mean field phase diagram with SO coupling

MFT: minimize classical energy

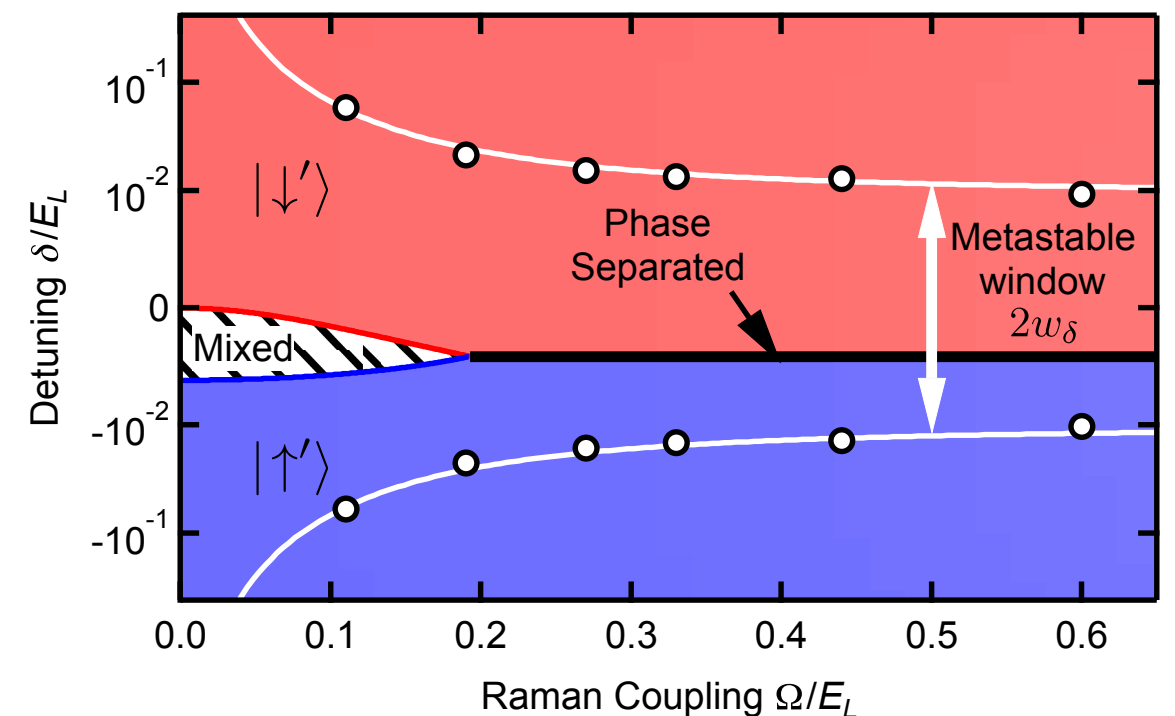
$$E_{\text{MFT}} = \frac{1}{2} \int d^3r \left[\left(c_0 + \frac{c_2}{2} \right) (\rho_{\downarrow'} + \rho_{\uparrow'})^2 + \frac{c_2}{2} (\rho_{\downarrow'}^2 - \rho_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \rho_{\downarrow'} \rho_{\uparrow'} + \delta (\rho_{\uparrow'} - \rho_{\downarrow'}) \right]$$

L

a Mean field phase diagram



b Phase diagram, inset



Refs.

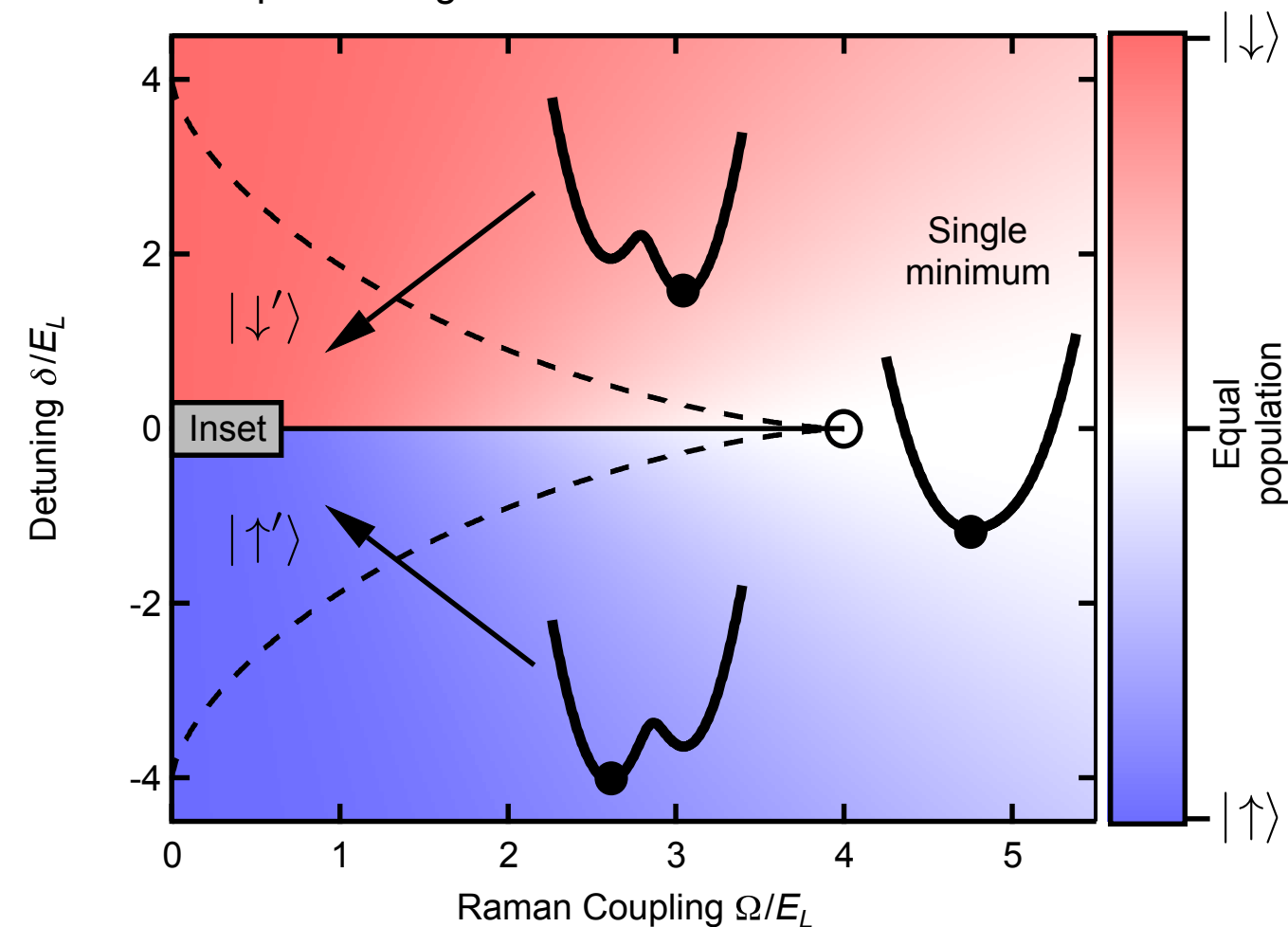
Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

Mean field phase diagram with SO coupling

MFT: minimize classical energy

$$E_{\text{MFT}} = \frac{1}{2} \int d^3r \left[\left(c_0 + \frac{c_2}{2} \right) (\rho_{\downarrow'} + \rho_{\uparrow'})^2 + \frac{c_2}{2} (\rho_{\downarrow'}^2 - \rho_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \rho_{\downarrow'} \rho_{\uparrow'} + \delta (\rho_{\uparrow'} - \rho_{\downarrow'}) \right]$$

a Mean field phase diagram



Fixed total spin or magnetization

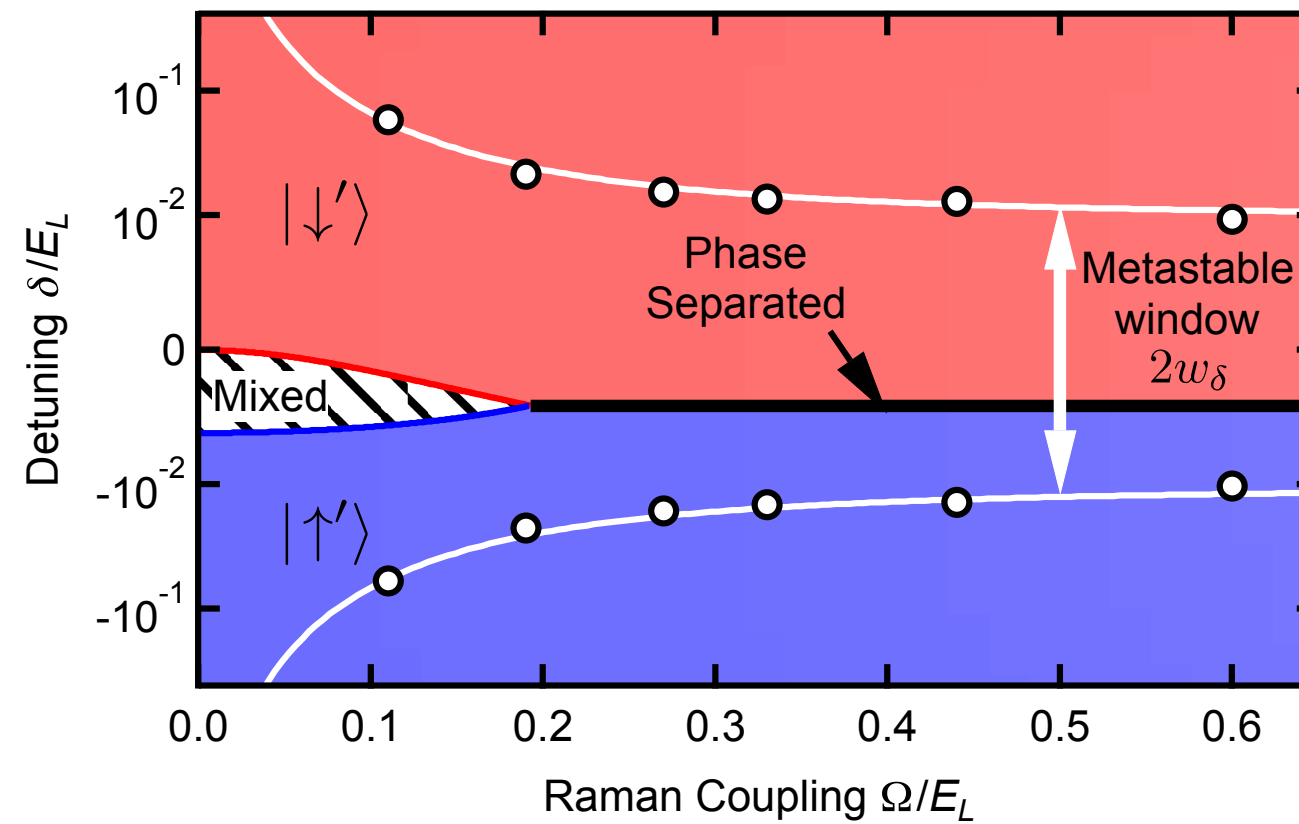


Refs.

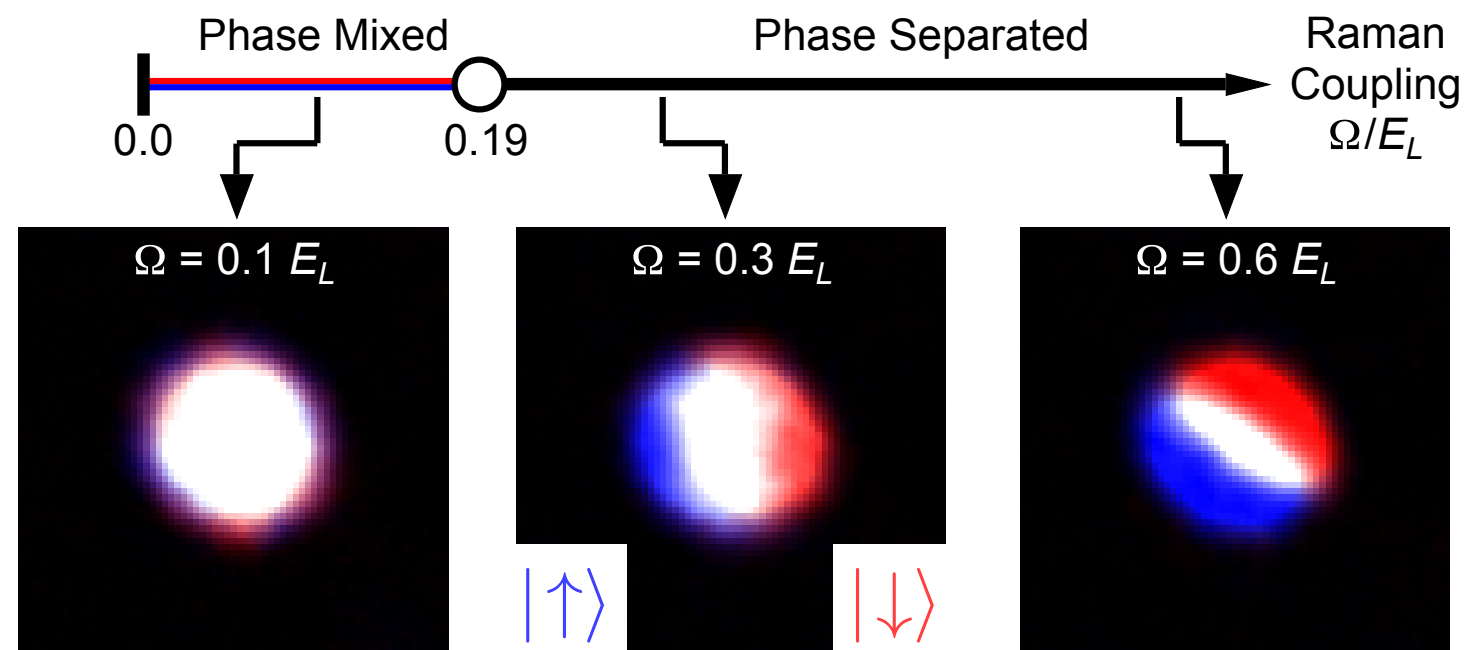
Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

Transition from miscible to immiscible

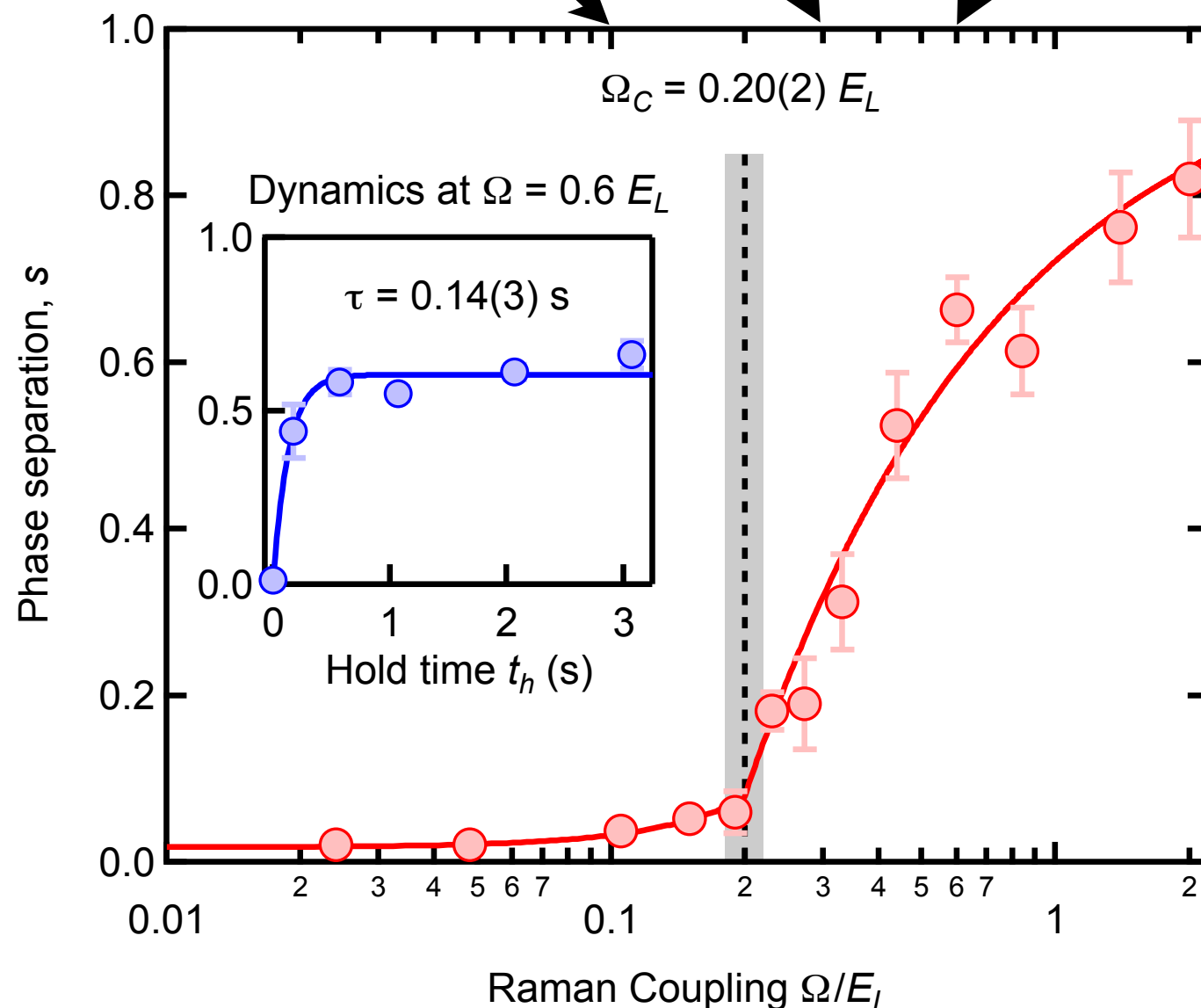
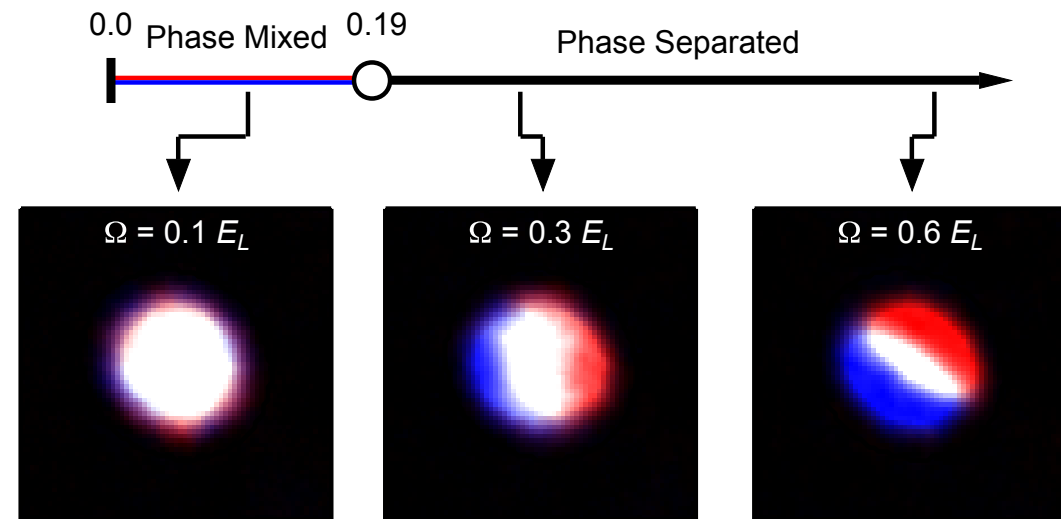
b Phase diagram, inset



c Miscible to immiscible transition



Transition from miscible to immiscible



A quantum phase transition
Previously unexpected

Our MFT prediction
Phase separation at $\Omega = 0.19 E_L$

Refs.

Y.-J. Lin et al, Nature (2011)
C. Wang et al (arXiv:1006.5148),
T.-L. Ho and S. Zhan (arXiv:1007.0650)