

Spins with neutral atoms

I. B. Spielman

Team

K. Jiménez-García,

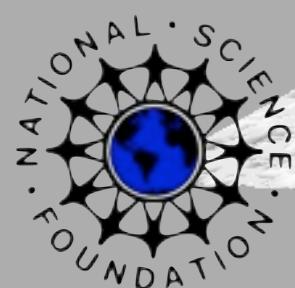
R. A. Williams, L. J. LeBlanc, A. R. Perry, and M. Beeler

Now with real job: Y.-J. Lin

jqi

Senior coworkers

J. V. Porto, and W. D. Phillips



NIST

National Institute of Standards and Technology

Phase Mixed

0.0

Phase Separated

Funded by the DARPA OLE program, the ARO Atomtronics MURI
and the NSF through the PFC at JQI.

Laser Strength

Dec. 2011

Rubidium 87

The Periodic Table is a tabular arrangement of chemical elements. It consists of 18 groups (vertical columns) and 7 periods (horizontal rows). The elements are color-coded by state of matter: Solids (light blue), Liquids (medium blue), Gases (light green), and Artificially Prepared (yellow). The table includes atomic number, symbol, name, atomic weight, and ground-state configuration for each element. A red arrow points to Rubidium (Rb) in the fifth period.

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/Constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s $^{-1}$	(exact)
Planck constant	h	6.6261×10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022×10^{-19} C	
electron mass	m_e	9.1094×10^{-31} kg	
proton mass	m_p	0.5110 MeV	
fine-structure constant	α	$1/137.036$	
Rydberg constant	R_{∞}	10 973 732 m $^{-1}$	
	$R_{\infty}c$	$3.289 842 \times 10^{15}$ Hz	
	$R_{\infty}hc$	13.6057 eV	
Boltzmann constant	k	1.3807×10^{-23} J K $^{-1}$	

Period

Group 1 IA

Group 2 IA

Group 3 IIIB

Group 4 IVB

Group 5 VB

Group 6 VIB

Group 7 VIIB

Group 8 VIII

Group 9 IB

Group 10 IIB

Group 11 IIIB

Group 12 IIB

Group 13 IIIA

Group 14 IVA

Group 15 VA

Group 16 VIA

Group 17 VIIA

Group 18 VIIIA

NIST
National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Physics Laboratory
physics.nist.gov

Standard Reference Data Group
www.nist.gov/srd

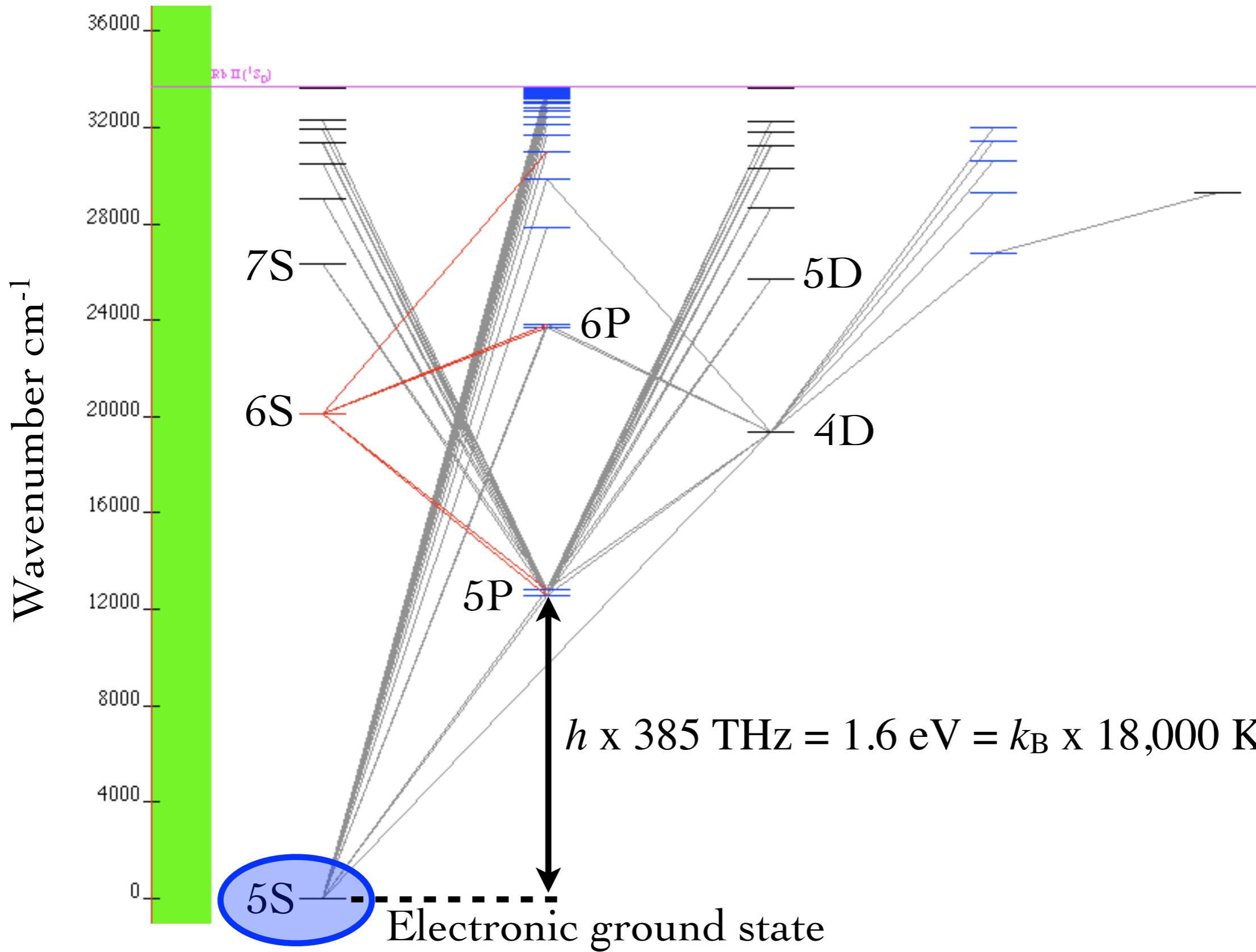
13	14	15	16	17	18
III A	IV A	V A	VI A	VII A	VII A
5 B Boron 10.811 $1s^2 2s^2 2p$ 8.2980	6 C Carbon 12.017 $1s^2 2s^2 2p^2$ 11.2603	7 N Nitrogen 14.0067 $1s^2 2s^2 2p^3$ 14.5341	8 O Oxygen 15.9994 $1s^2 2s^2 2p^4$ 13.6181	9 F Fluorine 18.9984032 $1s^2 2s^2 2p^5$ 17.4228	10 Ne Neon 20.1797 $1s^2 2s^2 2p^6$ 21.5645
13 Al Aluminum 26.981538 $[Ne]3s^2 3p^1$ 5.9858	14 Si Silicon 28.0855 $[Ne]3s^2 3p^2$ 8.1517	15 P Phosphorus 30.973761 $[Ne]3s^2 3p^3$ 10.4867	16 S Sulfur 32.065 $[Ne]3s^2 3p^4$ 10.3600	17 Cl Chlorine 35.453 $[Ne]3s^2 3p^5$ 12.9676	18 Ar Argon 39.948 $[Ne]3s^2 3p^6$ 15.7596
31 P Gallium 69.723 $[Ar]3d^1 4s^2 4p$ 5.9993	32 Ge Germanium 72.64 $[Ar]3d^1 4s^2 4p^2$ 7.8994	33 As Arsenic 78.96 $[Ar]3d^1 4s^2 4p^3$ 9.7524	34 Se Selenium 79.904 $[Ar]3d^1 4s^2 4p^5$ 11.8138	35 Br Bromine 83.798 $[Ar]3d^1 4s^2 4p^6$ 13.9996	36 Kr Krypton 83.798 $[Ar]3d^1 4s^2 4p^6$ 13.9996
49 Pd Palladium 106.42 $[Kr]4d^10 5s^2$ 8.3369	50 Ag Silver 107.8682 $[Kr]4d^10 5s^2$ 7.5762	47 S Cadmium 112.411 $[Kr]4d^10 5s^2 5p^1$ 8.9938	48 Cd Cadmium 114.818 $[Kr]4d^10 5s^2 5p^2$ 7.3439	51 In Indium 118.710 $[Kr]4d^10 5s^2 5p^3$ 8.6084	52 Sn Tin 121.760 $[Kr]4d^10 5s^2 5p^4$ 9.0096
53 I Iodine 126.90447 $[Kr]4d^10 5s^2 5p^5$ 10.4513	54 Xe Xenon 131.293 $[Kr]4d^10 5s^2 5p^6$ 12.1298				
81 Tl Thallium 204.3833 $[Hg]6p$ 6.1082	82 Pb Lead 207.2 $[Hg]6p^2$ 7.4167	83 Bi Bismuth 208.98038 $[Hg]6p^3$ 7.2855	84 Po Polonium (209) $[Hg]6p^4$ 8.414	85 At Astatine (210) $[Hg]6p^5$ 10.7485	86 Rn Radon (222) $[Hg]6p^6$ 10.7485
114 Uuq Ununquadium (289)	116 Uuh Ununhexium (292)				

For a description of the data, visit physics.nist.gov/data

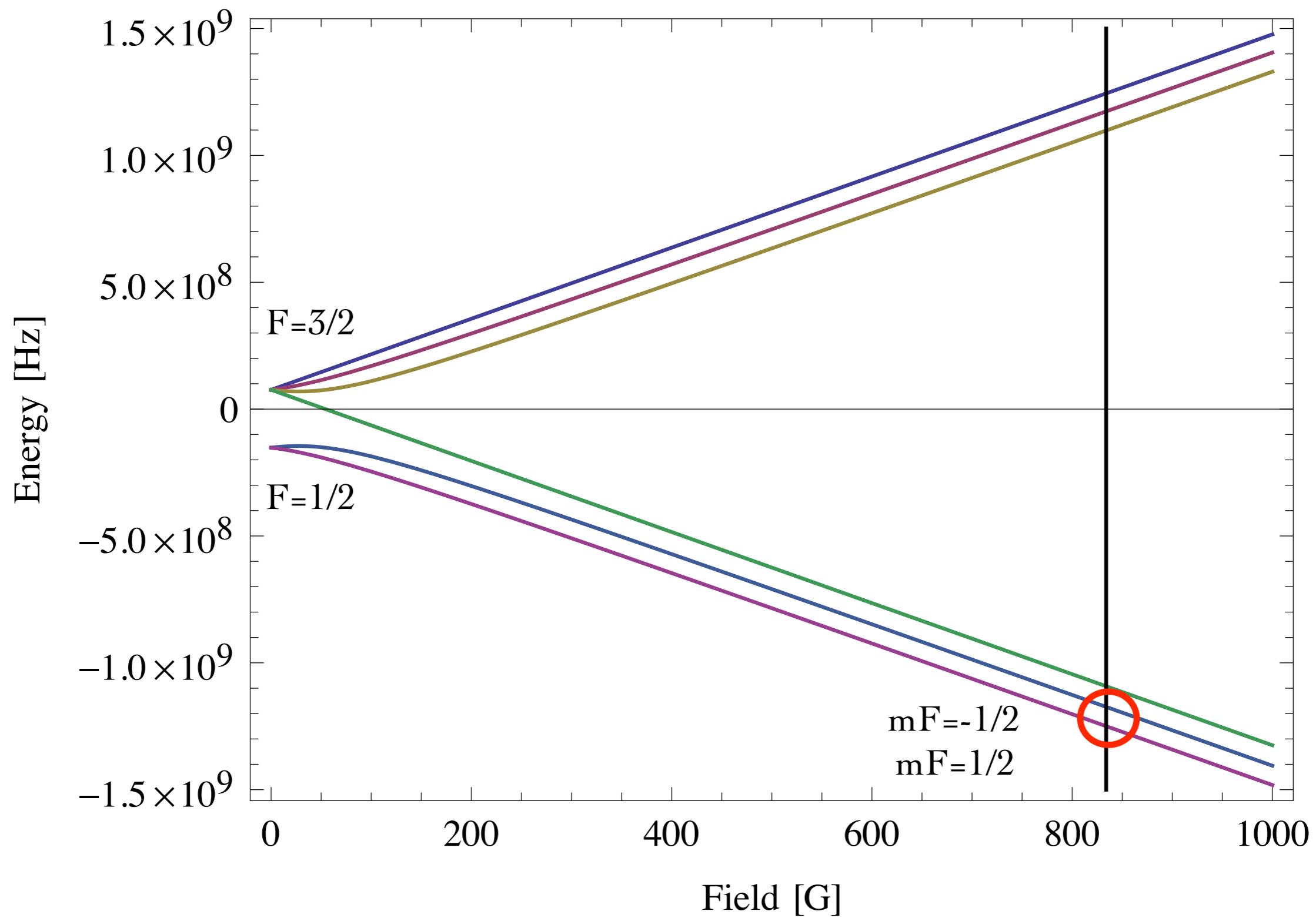
NIST SP 966 (September 2003)

[†]Based upon ^{12}C . () indicates the mass number of the most stable isotope.

Rubidium 87: Level structure



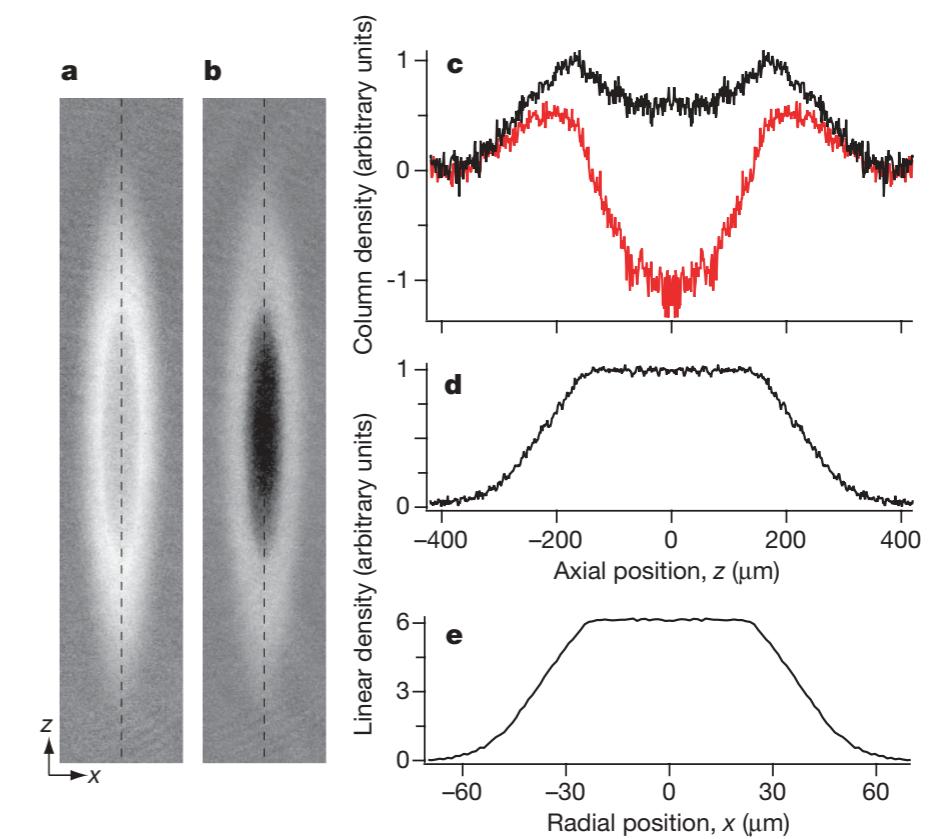
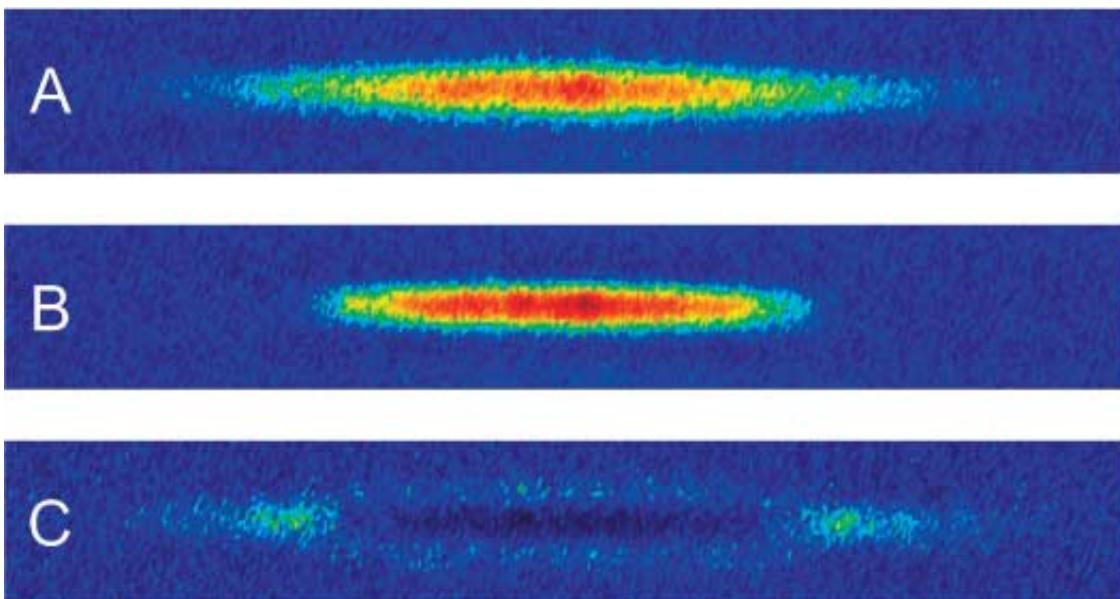
Li, 834 Feshbach Labeled



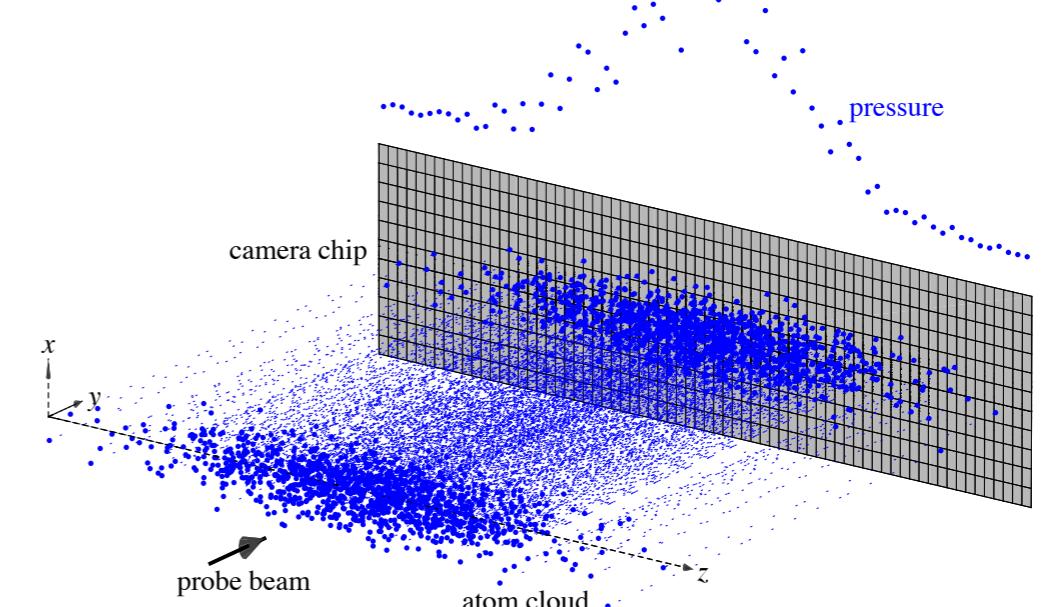
BEC- BCS Crossover physics

Ketterle (MIT)

Hulet (Rice)

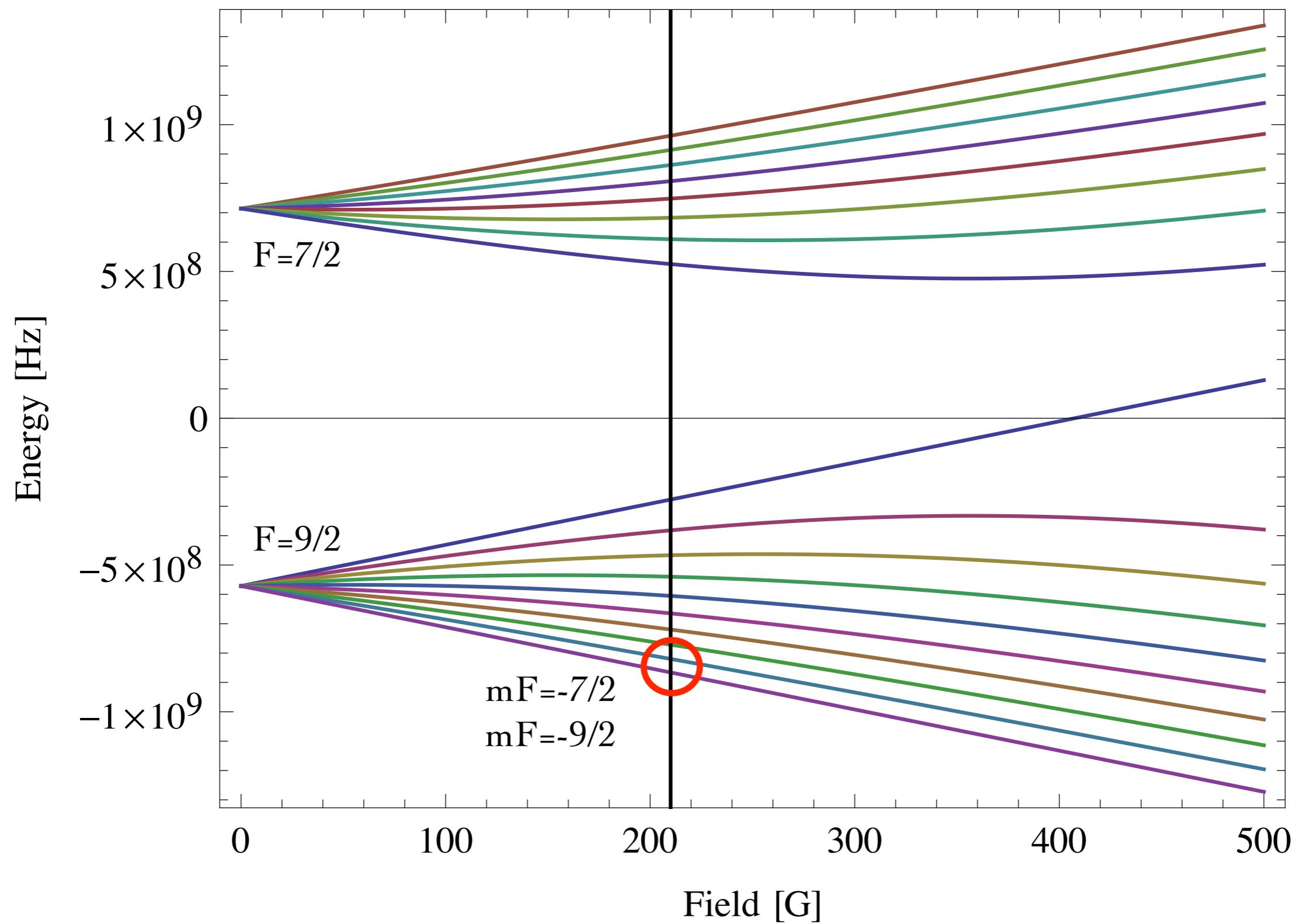


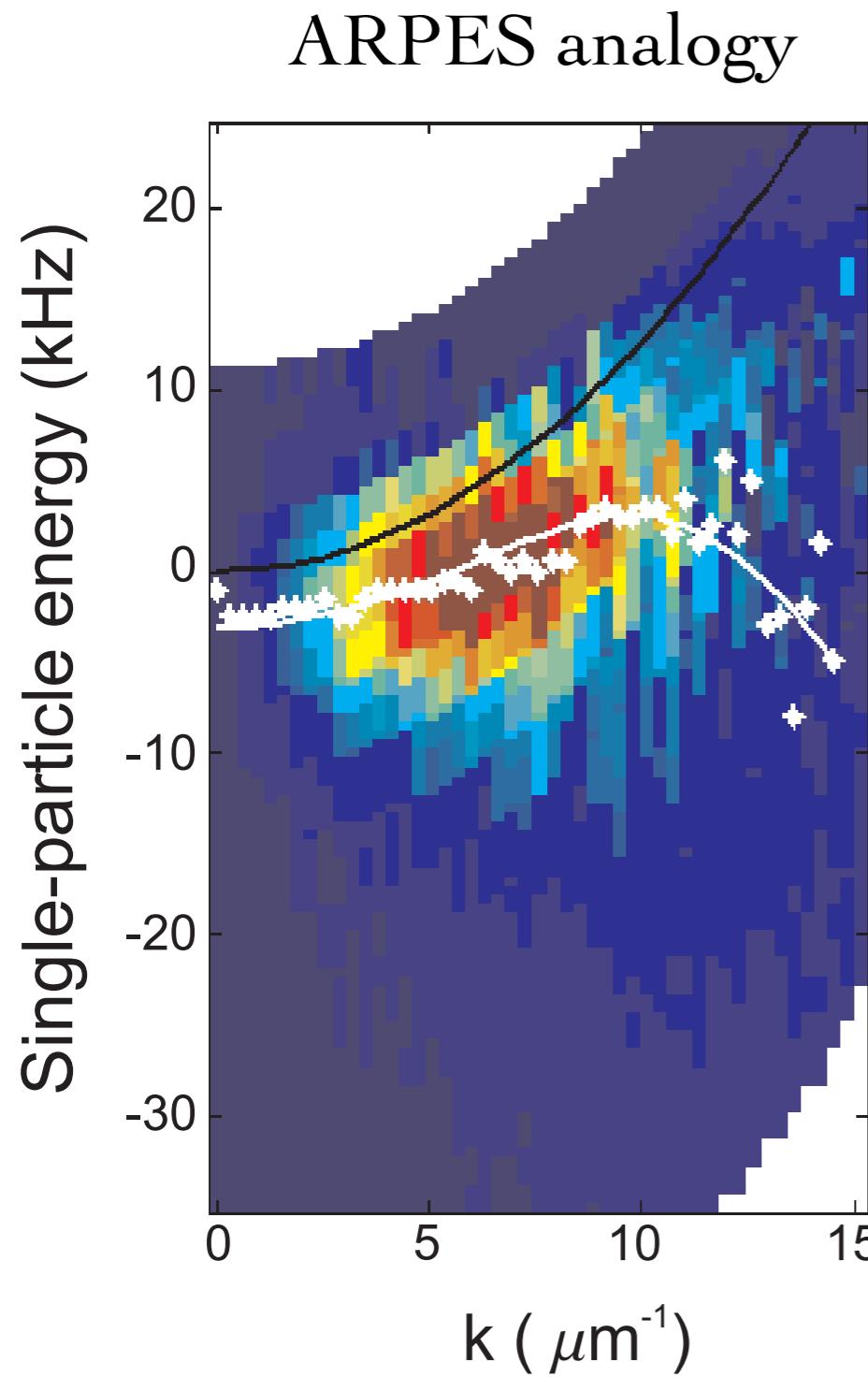
Solomon (ENS)



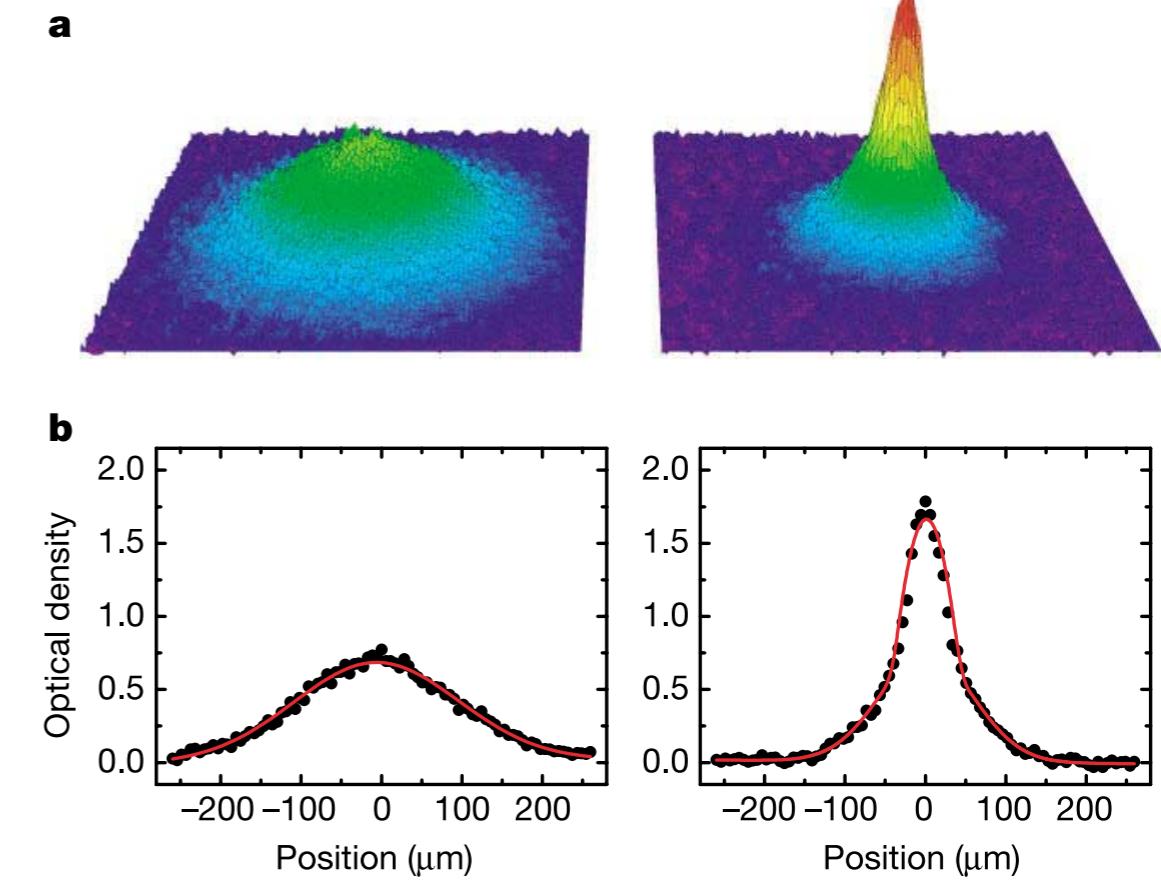
^{40}K has inverted hyperfine

^{40}K , 210 G Feshbach Labeled

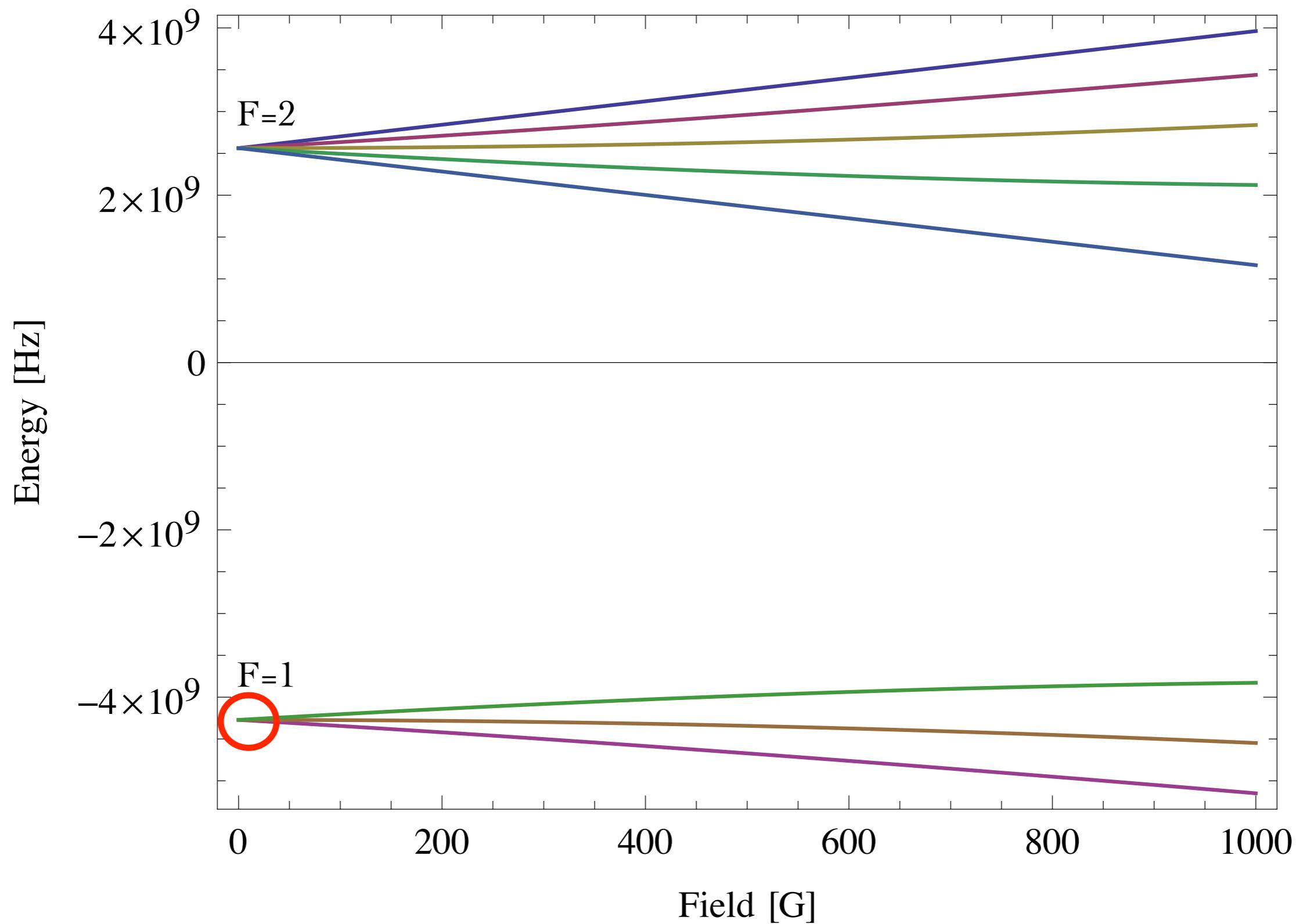




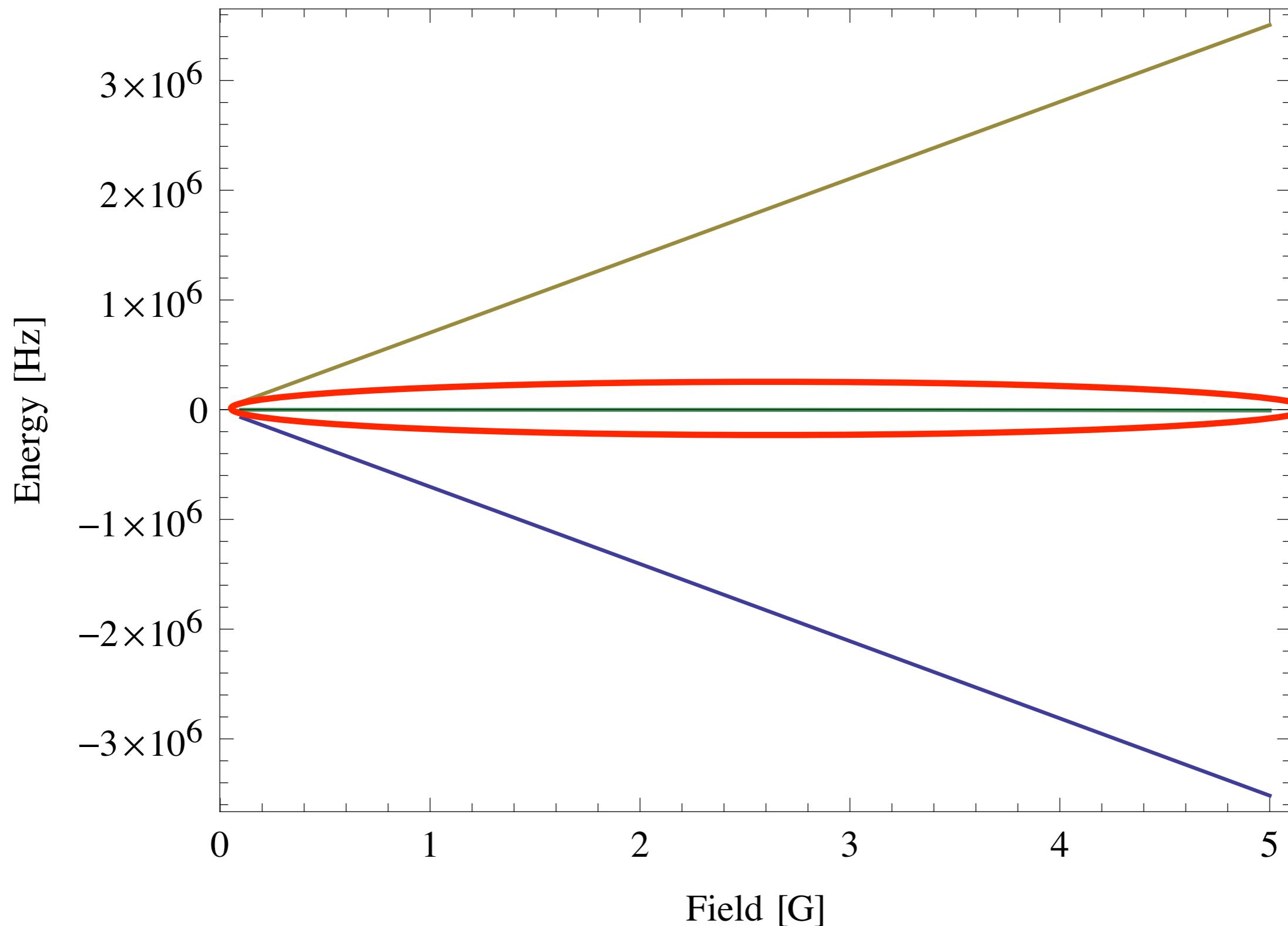
BEC of molecules



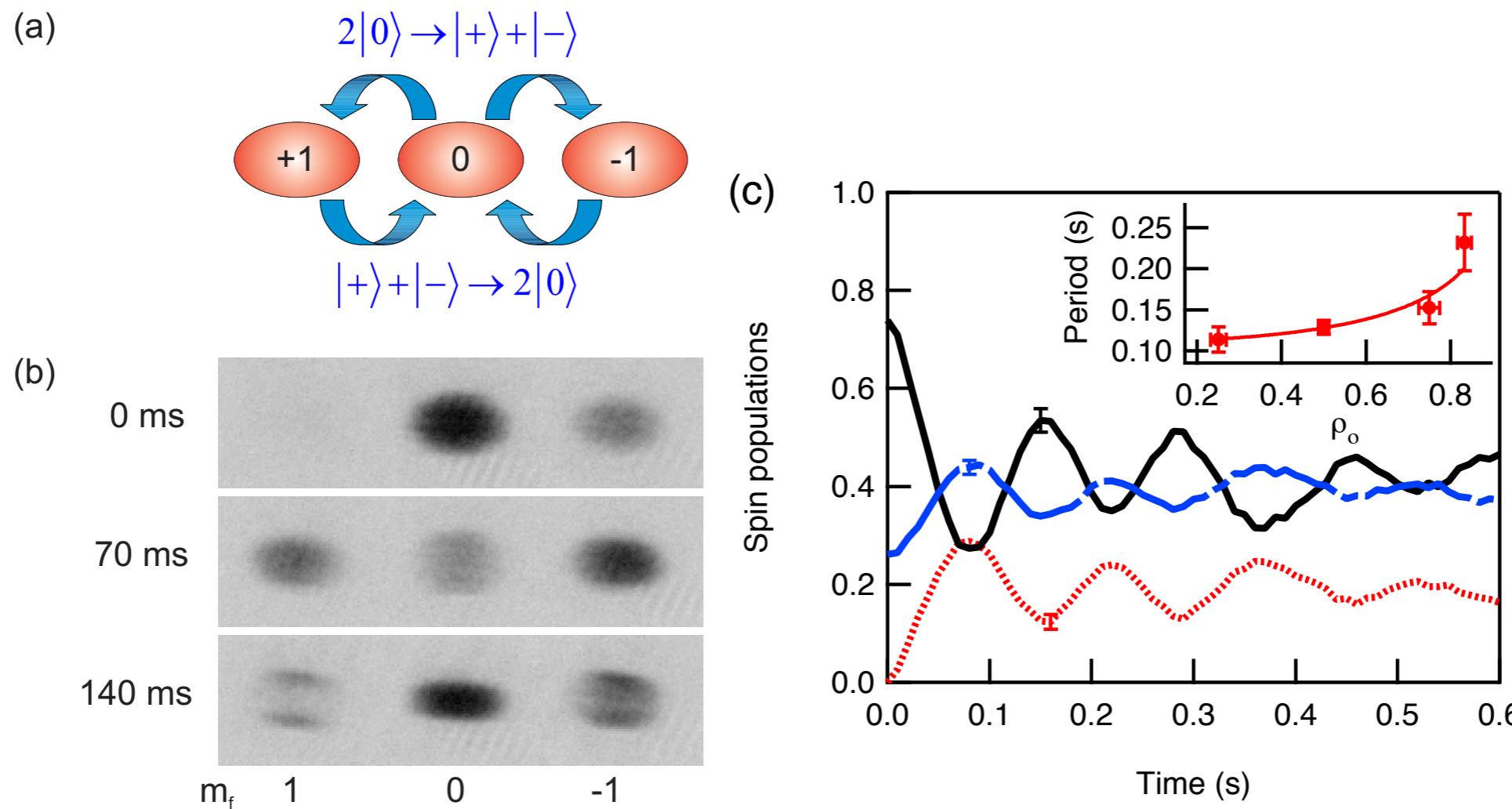
Rb



^{87}Rb F=1 at low field

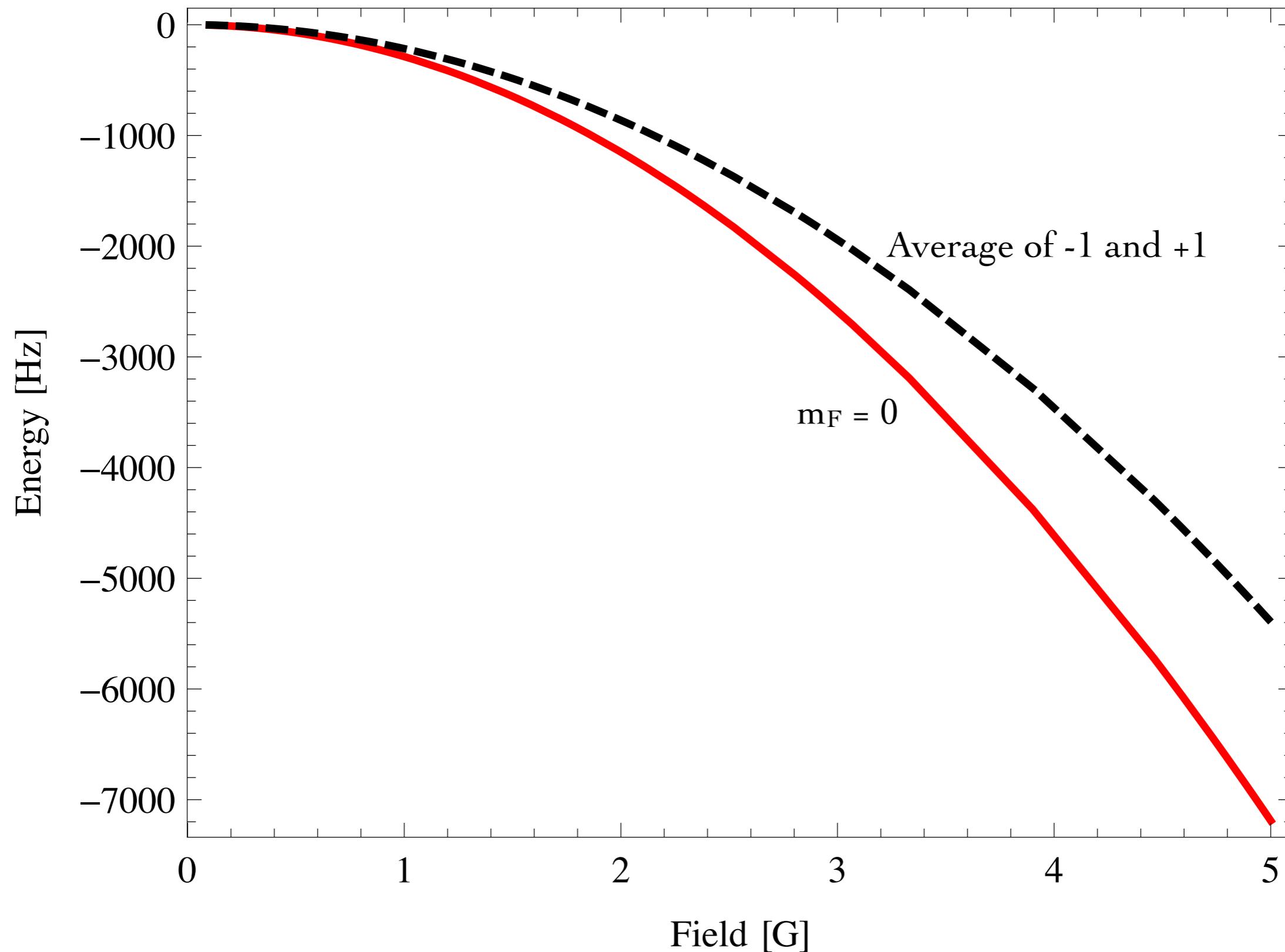


Spinor BEC's (Chapman group) for example

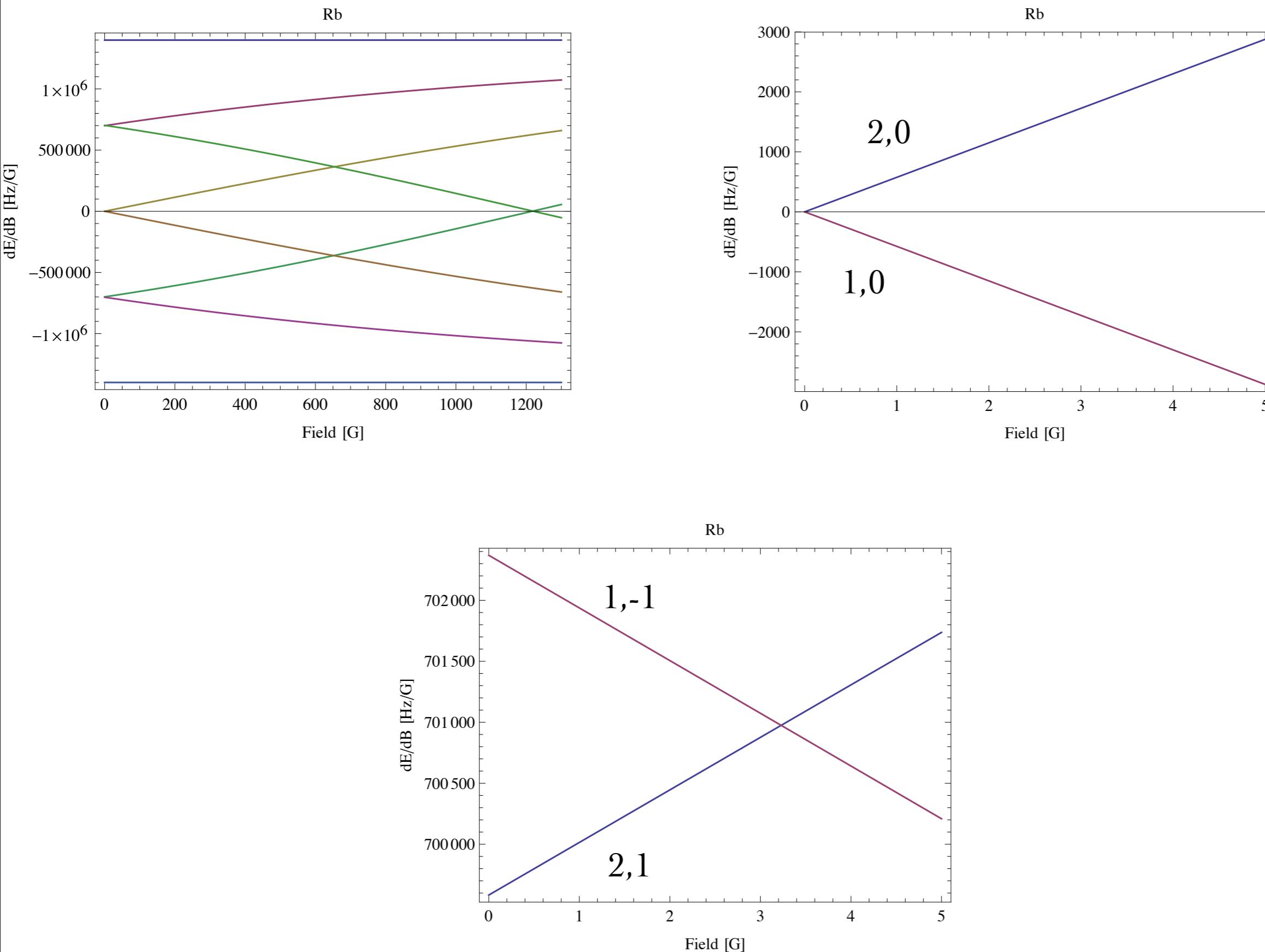


Three spins! How often do you see THAT with electrons...

^{87}Rb F=1 quadratic effects



^{87}Rb Clock states

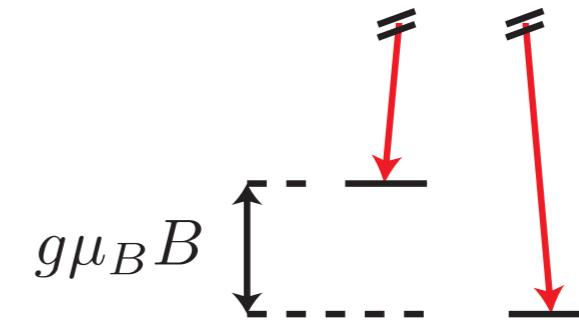
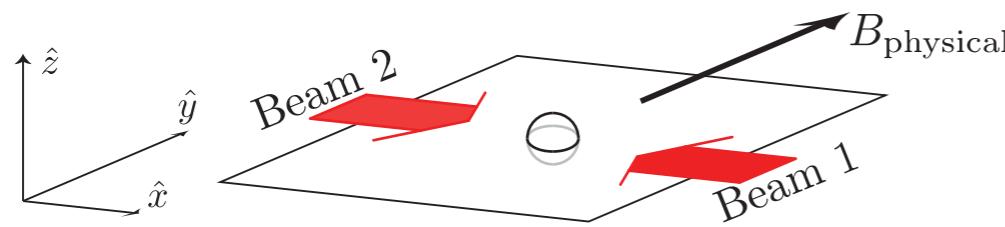


Standard picture

Space dependent coupling

Now we have no time-dependence, but instead space-dependence.

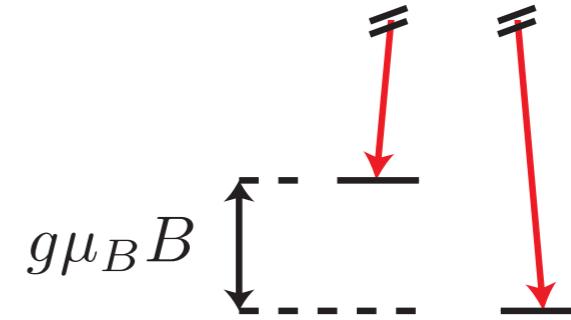
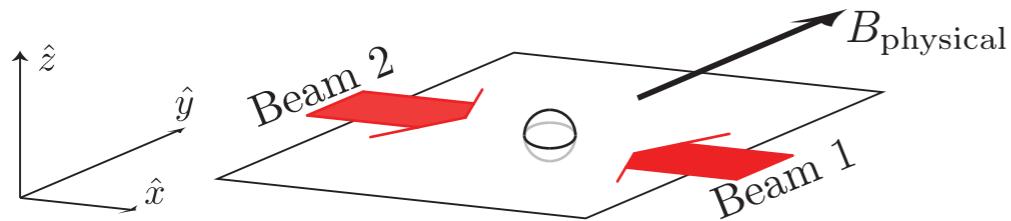
Geometry and initial Hamiltonian



$$\begin{aligned}\hat{H} &= \frac{\hbar^2 \hat{k}^2}{2m} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\Omega_R}{2} \begin{pmatrix} 0 & \exp(i2k_R x) \\ \exp(-i2k_R x) & 0 \end{pmatrix} + \begin{pmatrix} \delta/2 & 0 \\ 0 & -\delta/2 \end{pmatrix} \\ &= \frac{\hbar^2 \hat{k}^2}{2m} \mathbf{1} + \frac{\Omega}{2} [\cos(2k_R x) \check{\sigma}_x - \sin(2k_R x) \check{\sigma}_y] + \frac{\delta}{2} \check{\sigma}_z\end{aligned}$$

Exact picture

Geometry and initial Hamiltonian



Work in the momentum basis!

$$\begin{aligned}\hat{U}_1 \hat{H} \hat{U}_1^\dagger &= (\hat{k}\mathbf{1} - \check{\sigma}_z)^2 + \frac{\Omega}{2}\check{\sigma}_x + \frac{\delta}{2}\check{\sigma}_z \\ &= \begin{pmatrix} (k-1)^2 + \delta/2 & \Omega/2 \\ \Omega/2 & (k+1)^2 - \delta/2 \end{pmatrix}\end{aligned}$$

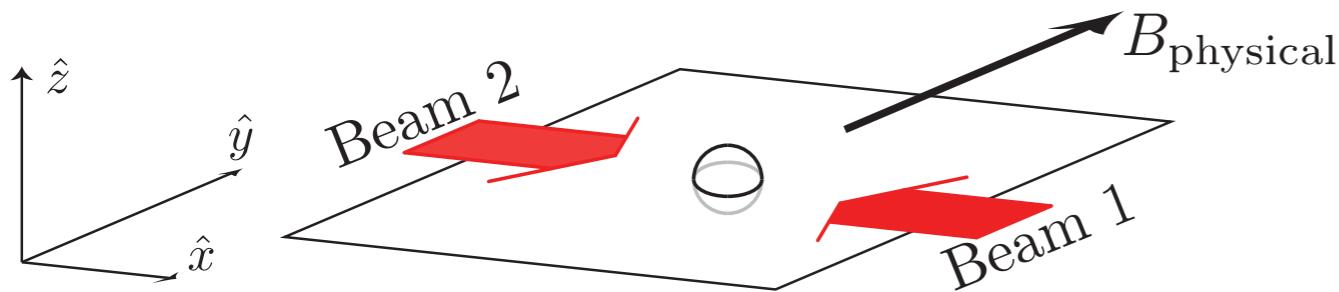
Numerically solve without approximation

Spin orbit coupling: origin

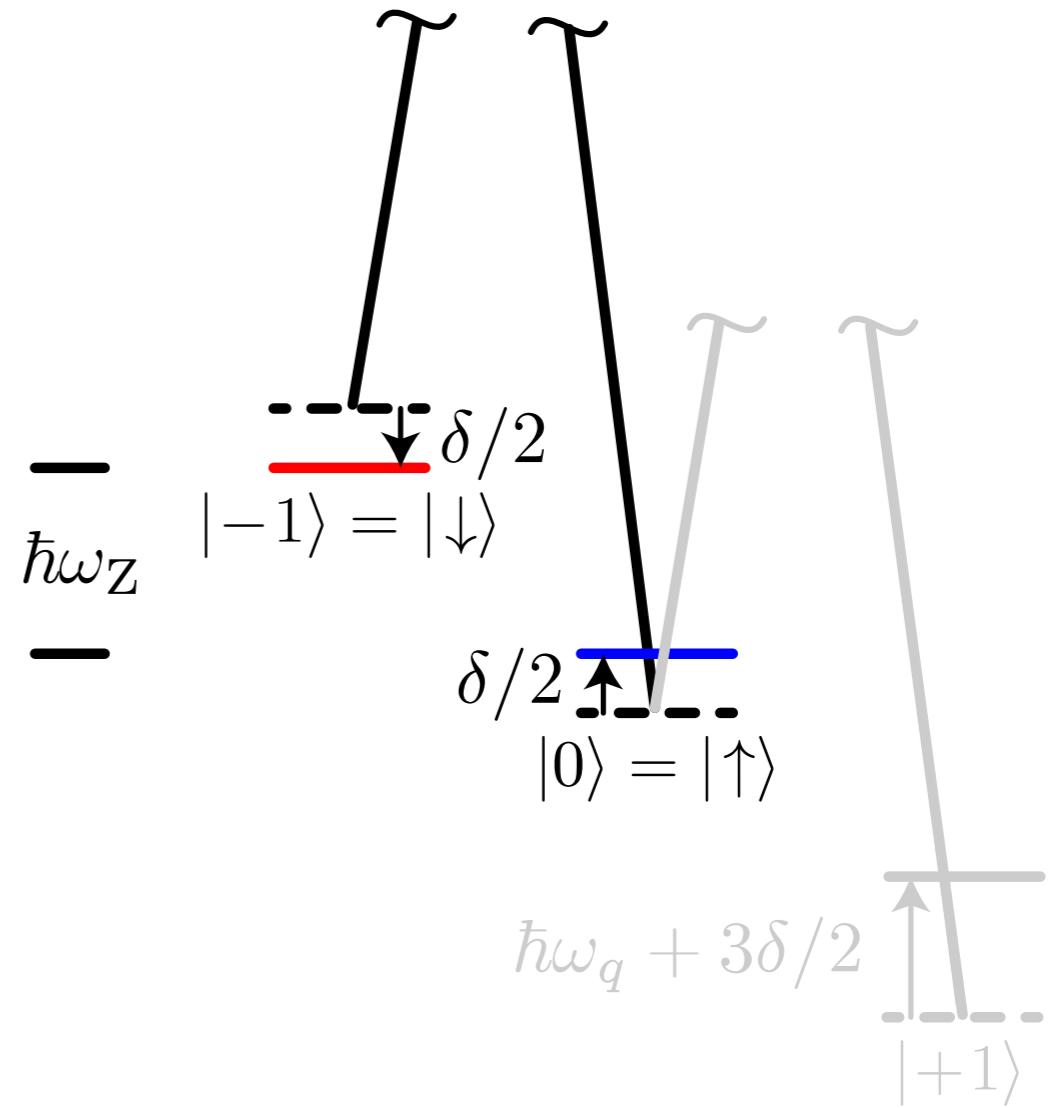
Momentum representation

$$H = \sum_k \left\{ \left(\langle k-1, \uparrow | \quad \langle k+1, \downarrow | \right) \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \left(|k-1, \uparrow \rangle \quad |k+1, \downarrow \rangle \right) \right\}$$

Geometry



Levels



References

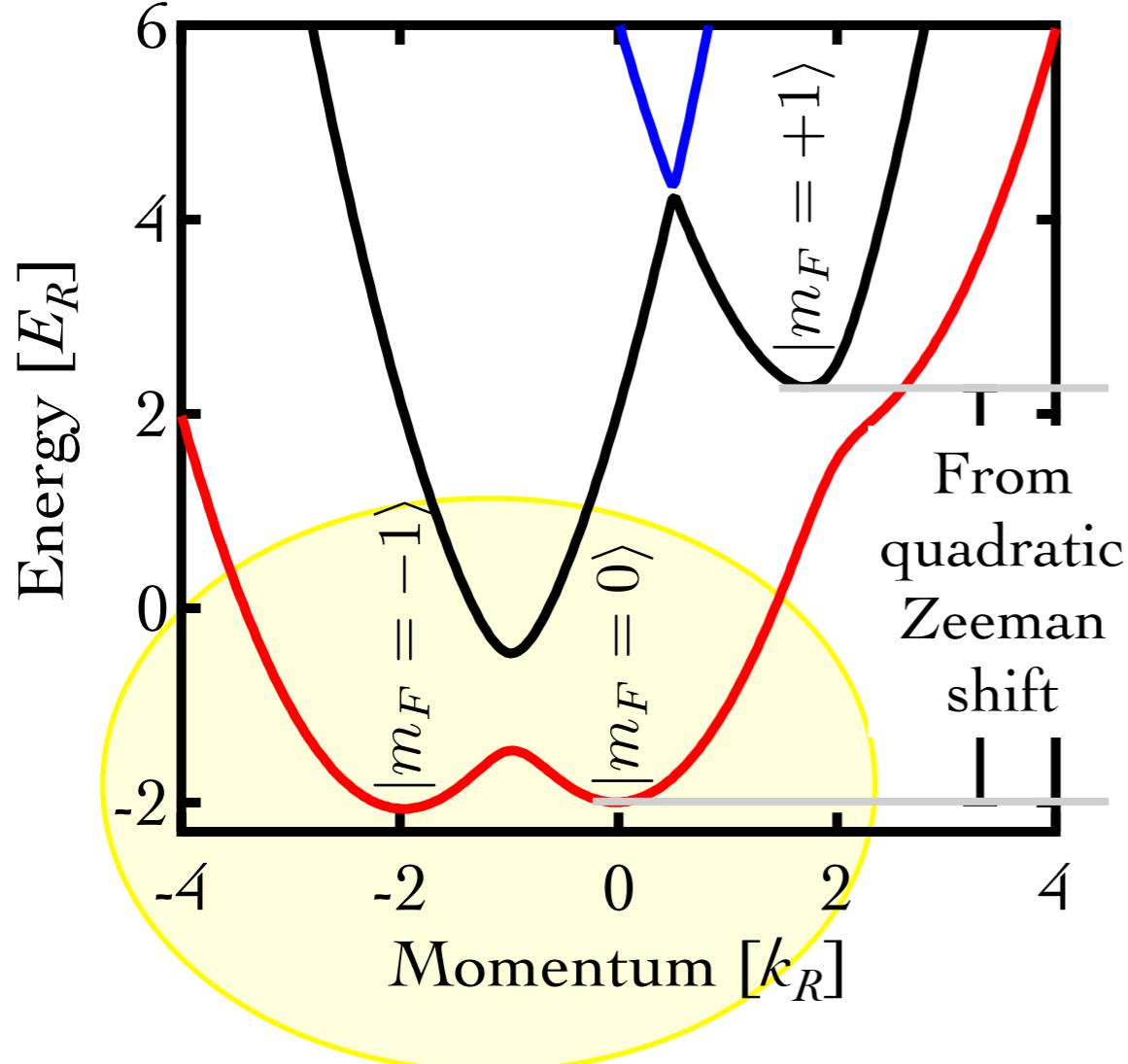
Y.-J. Lin *et al* Nature (2011)

Spin orbit coupling: origin

Momentum representation

$$H = \sum_k \left\{ \left(\langle k-1, \uparrow | \quad \langle k+1, \downarrow | \right) \left(\begin{array}{cc} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{array} \right) \left(\begin{array}{c} |k-1, \uparrow \rangle \\ |k+1, \downarrow \rangle \end{array} \right) \right\}$$

Spin 1/2 bosons????



Transform to

$$\begin{aligned} \hat{H} &= \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} \hat{1} + \left(\frac{\delta}{2} + \frac{\hbar^2 k_R}{m} \hat{k}_x \right) \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z + \Delta E \check{1} \\ &= \frac{\hbar^2}{2m} \left[\left(\hat{k}_x \check{1} + k_R \check{\sigma}_y \right)^2 + \left(\hat{k}_y \check{1} - 0 \right)^2 \right] + \frac{\delta}{2} \check{\sigma}_y + \frac{\Omega}{2} \check{\sigma}_z \end{aligned}$$

NOTICE

Written as a “2x2” vector potential, this S-O coupling is *NOT* non-Abelian (it is A_x)

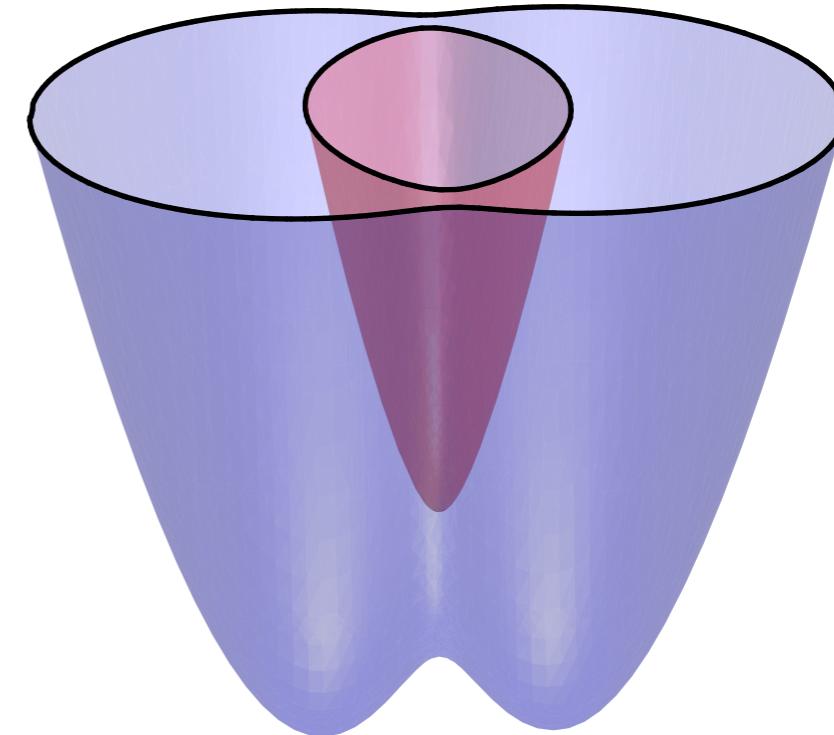
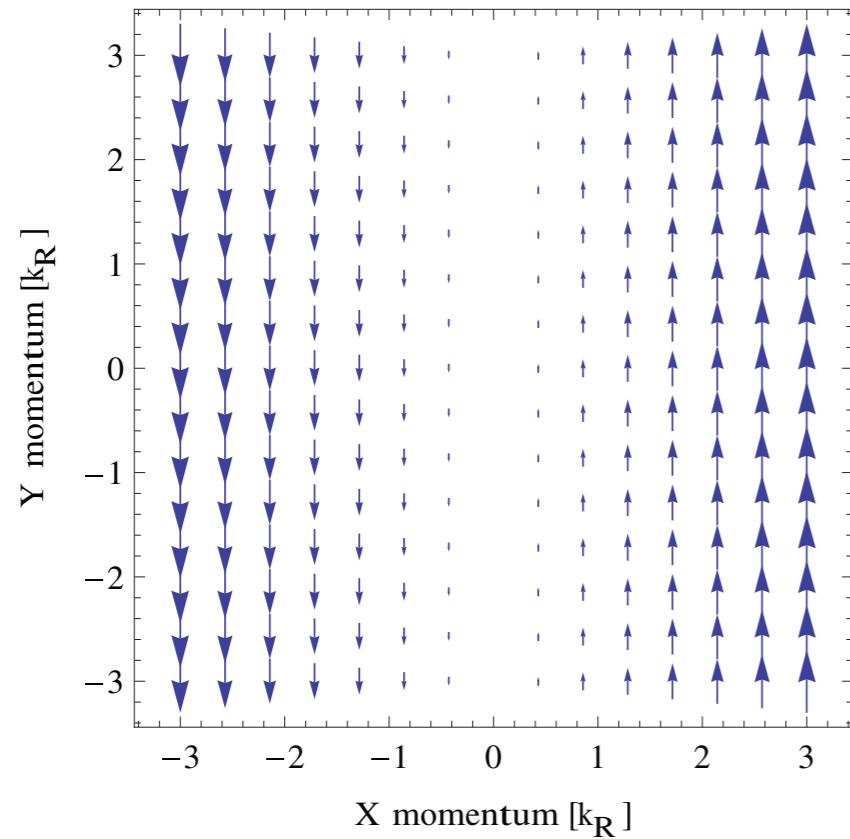
However, the Hamiltonian *is* non-trivial owing to the Zeeman field along z .

Spin-orbit coupling

Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m} \mathbf{\hat{1}} + \frac{\delta}{2} \check{\sigma}_z + \alpha (k_x \check{\sigma}_y - k_y \check{\sigma}_x) + \beta (k_x \check{\sigma}_x - k_y \check{\sigma}_y).$$

Equal Rashba and Dresselhaus: $\alpha = \beta$



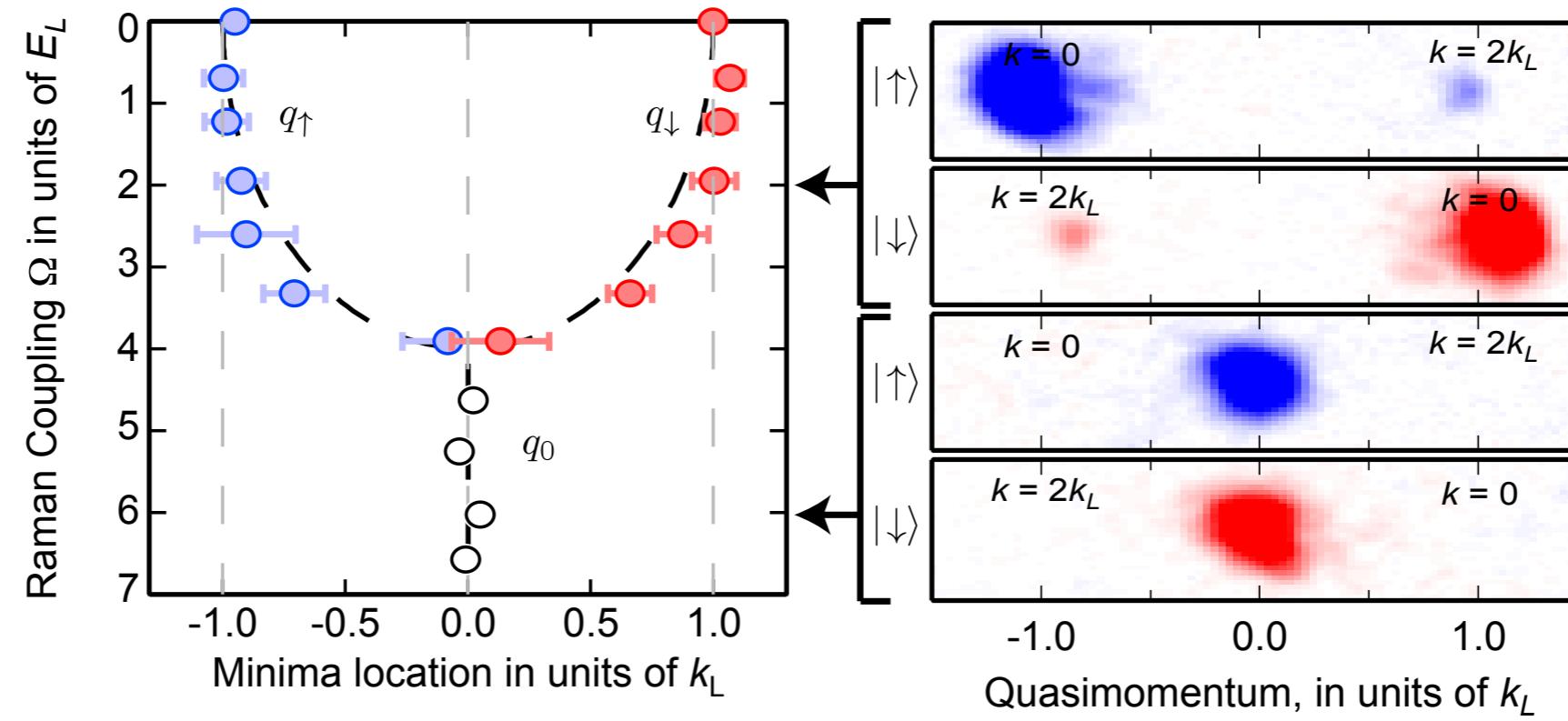
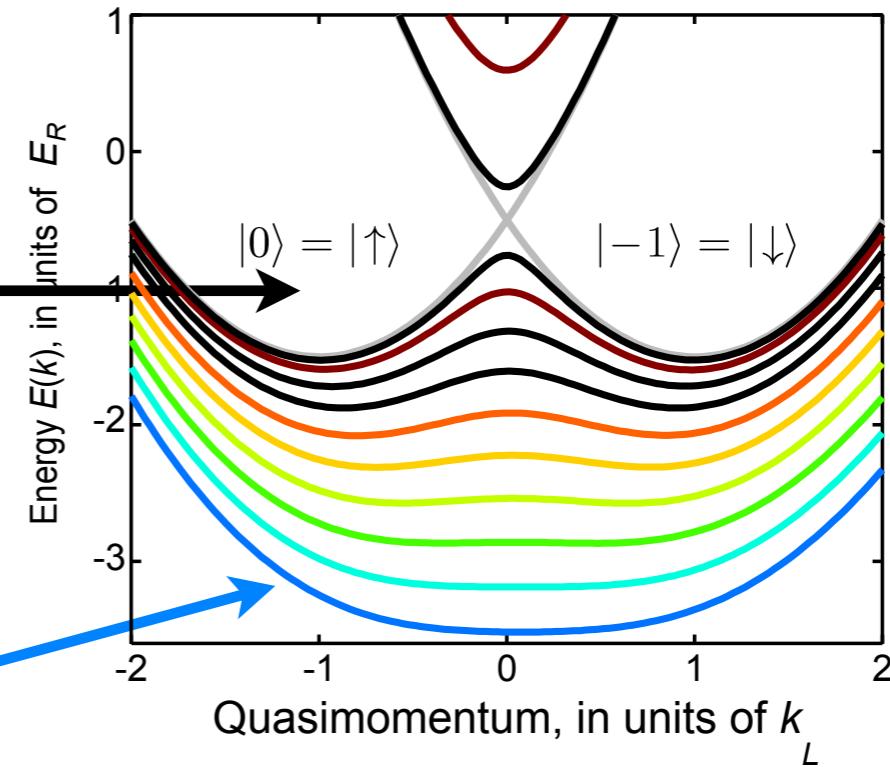
Refs.

T. D. Stanescu and B. Anderson and V. Galitski PRA (2008)

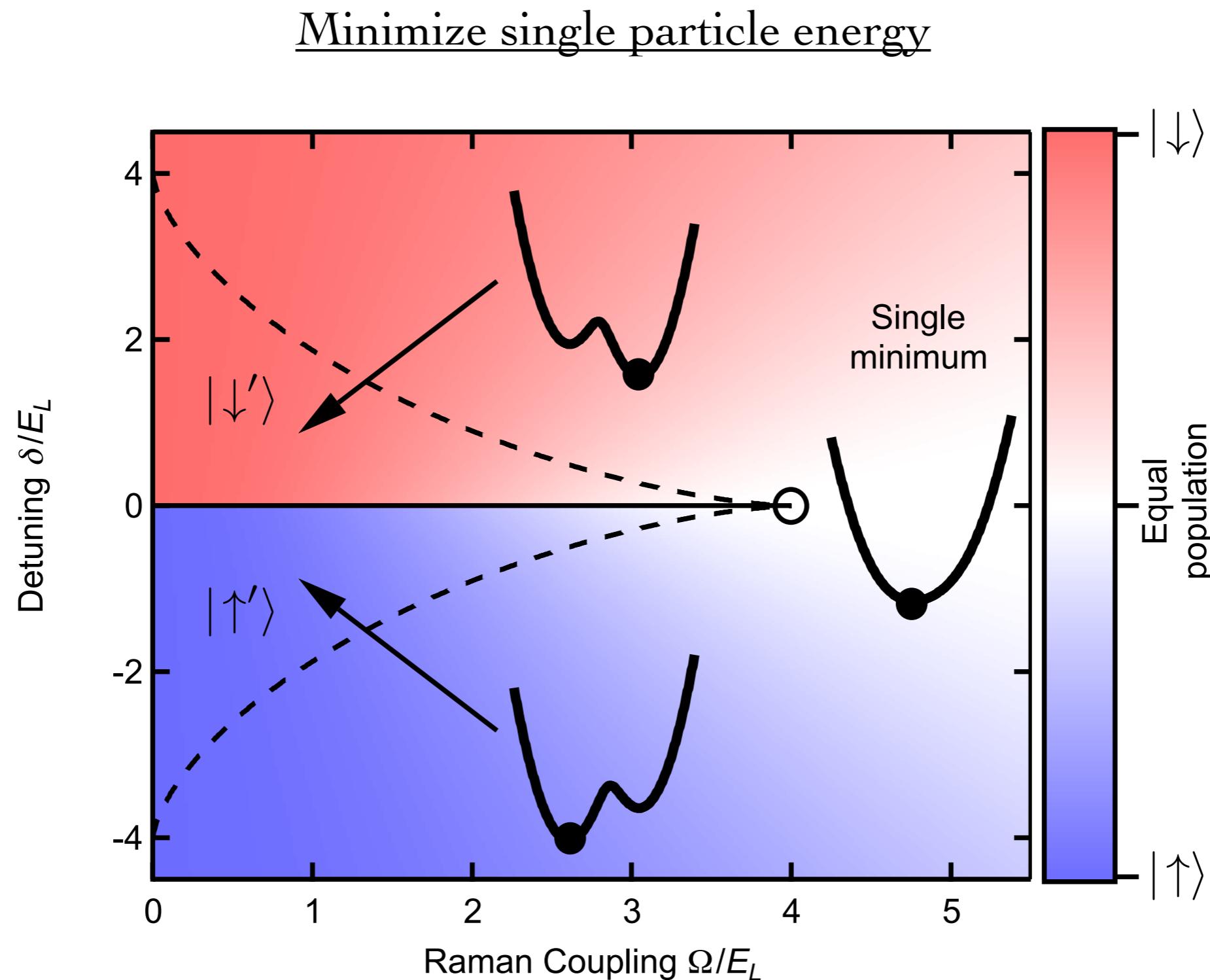
Typical data

Double Well
“Spin-orbit limit”

Single minimum
“vector potential limit”



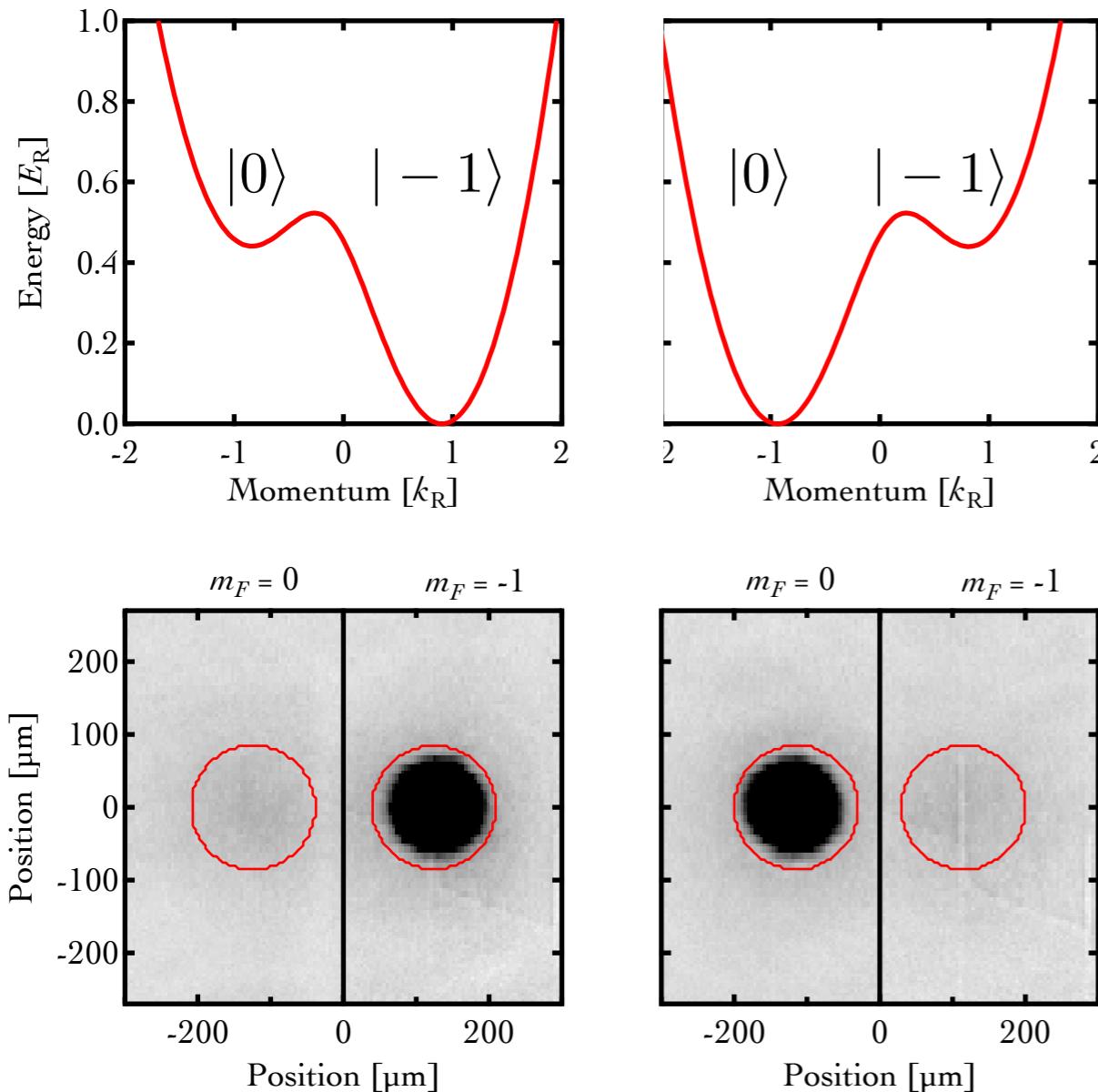
Phase diagram I: Large single particle energy



Refs.
Y.-J. Lin et al, Nature (2011)

Equilibrium: slow

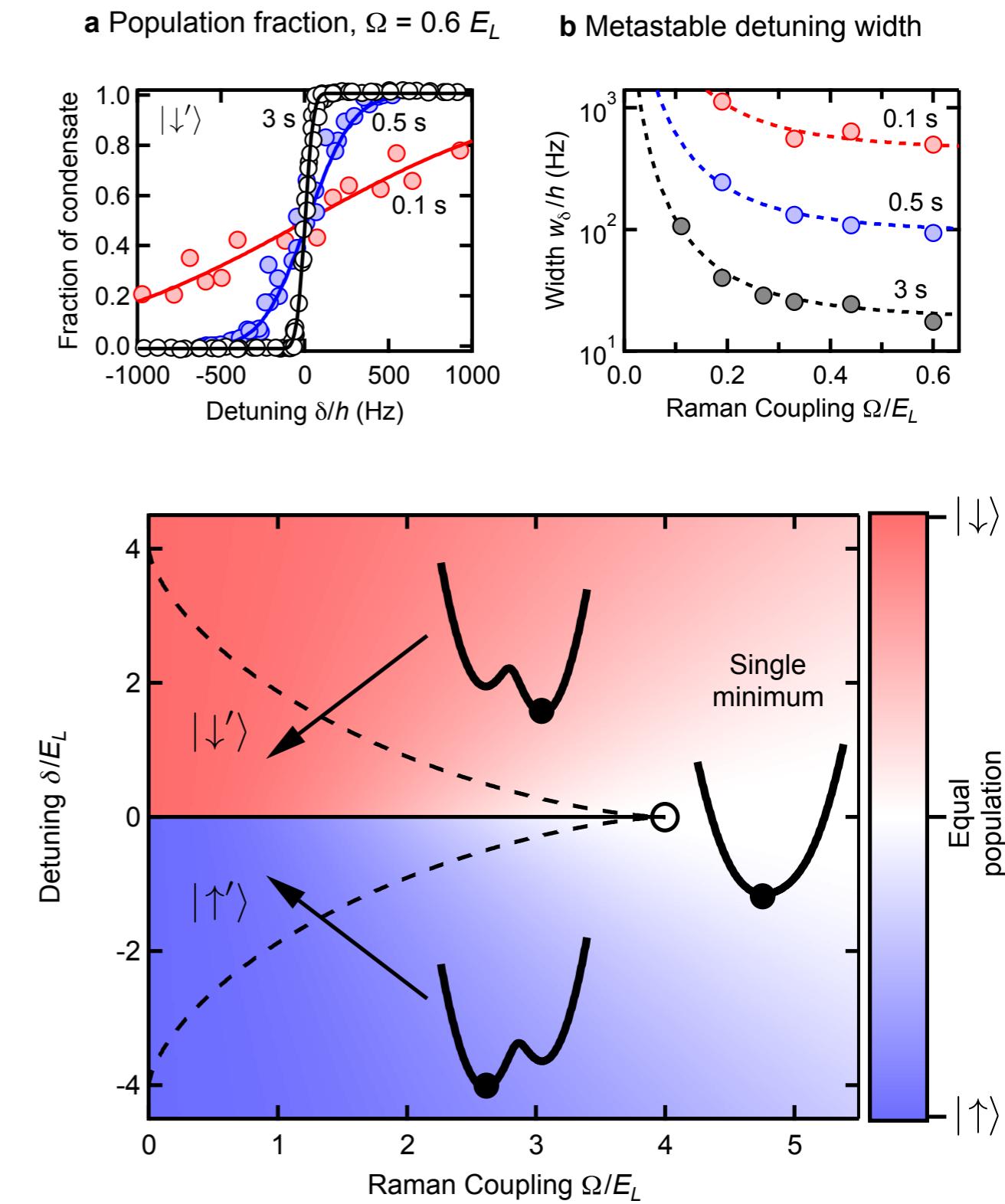
Minimize single particle energy



References

Y.-J. Lin *et al* Nature (2011)

Metastable populations

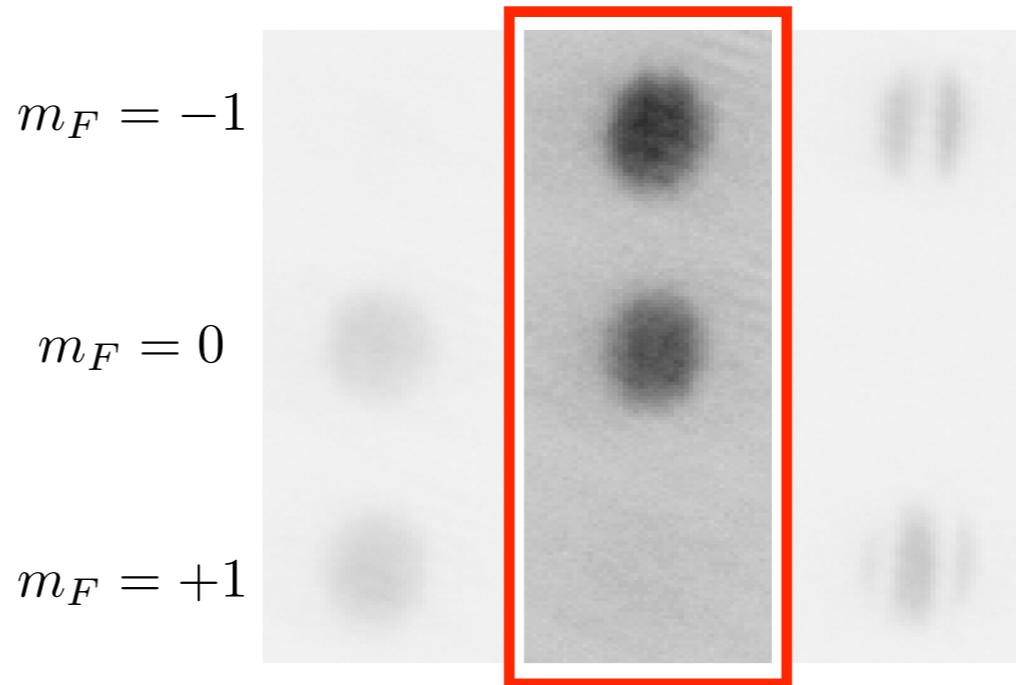


Effective Hamiltonian for dressed spins

Two Level contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2} (\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2) + c_2 \hat{\rho}_\downarrow \hat{\rho}_\uparrow \right] :$$

$m_F = -1, m_F = 0$ mixture: miscible for ^{87}Rb



Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

Effective Hamiltonian for dressed spins

Two Level contact interactions

$$\hat{H}_{\text{int}} = \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2} (\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2) + c_2 \hat{\rho}_\downarrow \hat{\rho}_\uparrow \right] : \\ \rightarrow \frac{1}{2} \int d^3r : \left[\left(c_0 + \frac{c_2}{2} \right) (\hat{\rho}_{\downarrow'} + \hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2} (\hat{\rho}_{\downarrow'}^2 - \hat{\rho}_{\uparrow'}^2) + (c_2 + c'_{\uparrow, \downarrow}) \hat{\rho}_{\downarrow'} \hat{\rho}_{\uparrow'} \right] :$$

Spin-orbit term

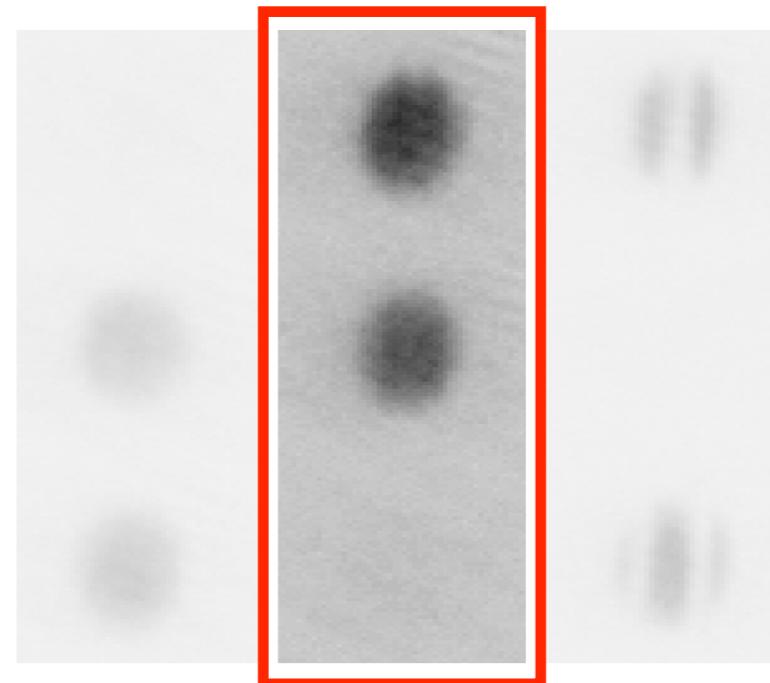
$$c'_{\uparrow, \downarrow} \approx c_0 \frac{\Omega^2}{8}$$

$m_F = -1, m_F = 0$ mixture: miscible for ^{87}Rb

$m_F = -1$

$m_F = 0$

$m_F = +1$



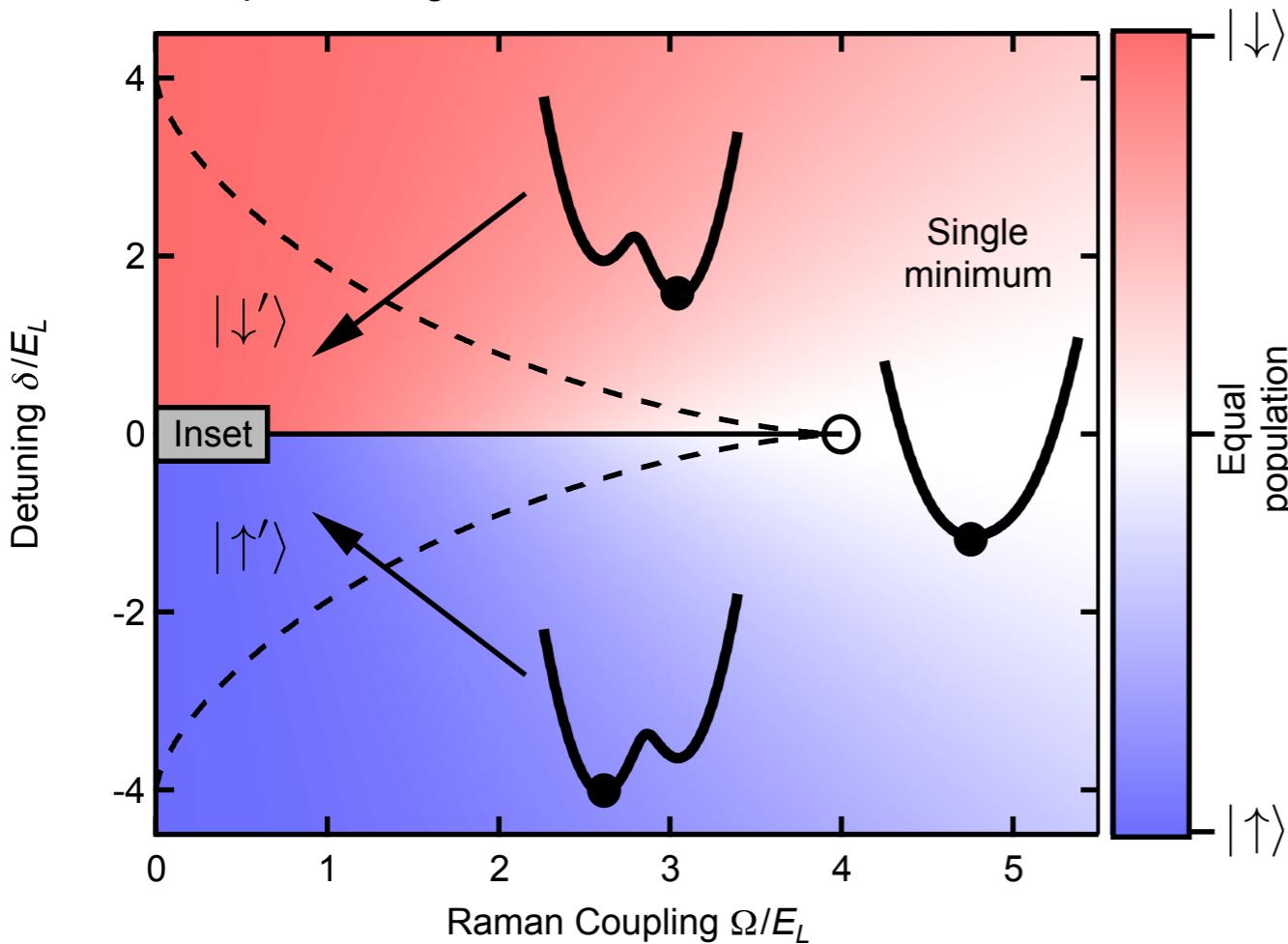
Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

Mean field phase diagram with SO coupling

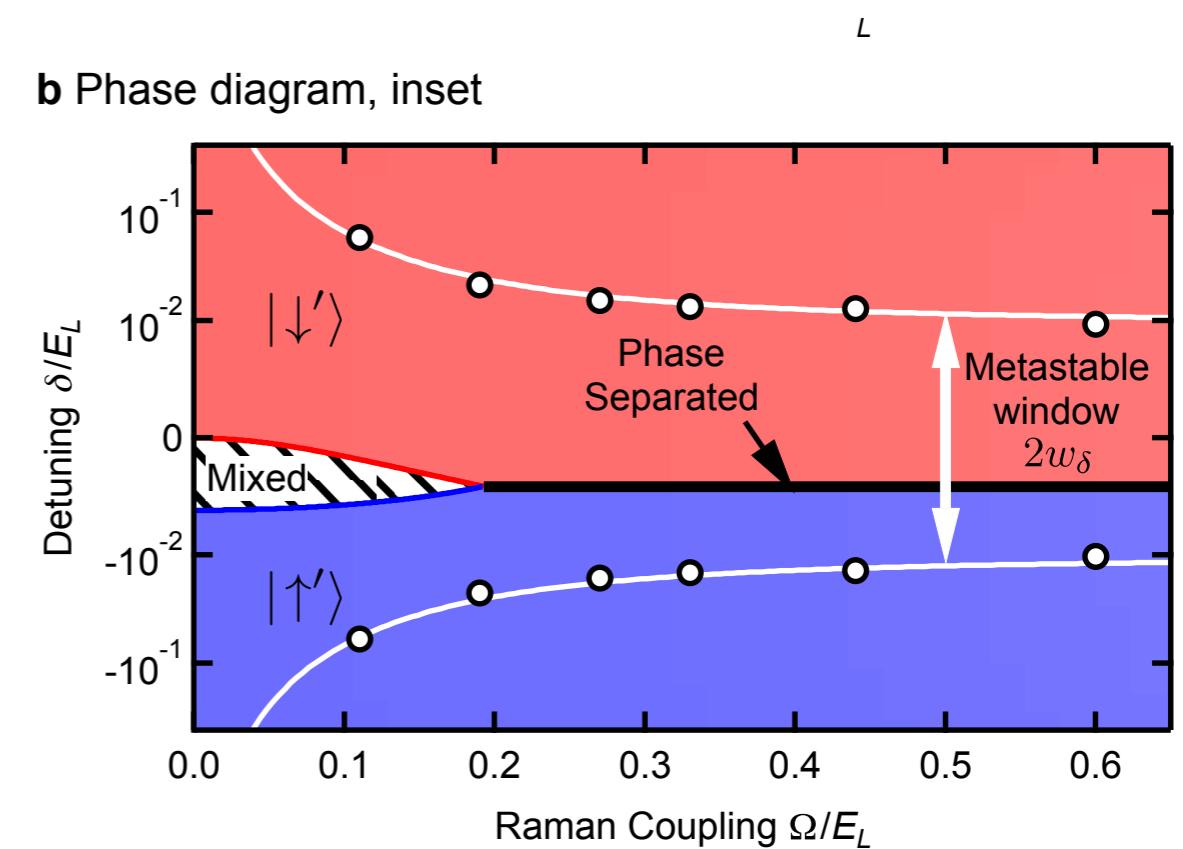
MFT: minimize classical energy

$$E_{\text{MFT}} = \frac{1}{2} \int d^3r \left[\left(c_0 + \frac{c_2}{2} \right) (\rho_{\downarrow'} + \rho_{\uparrow'})^2 + \frac{c_2}{2} (\rho_{\downarrow'}^2 - \rho_{\uparrow'}^2) + (c_2 + c'_{\uparrow,\downarrow}) \rho_{\downarrow'} \rho_{\uparrow'} + \delta (\rho_{\uparrow'} - \rho_{\downarrow'}) \right]$$

a Mean field phase diagram



b Phase diagram, inset



Refs.

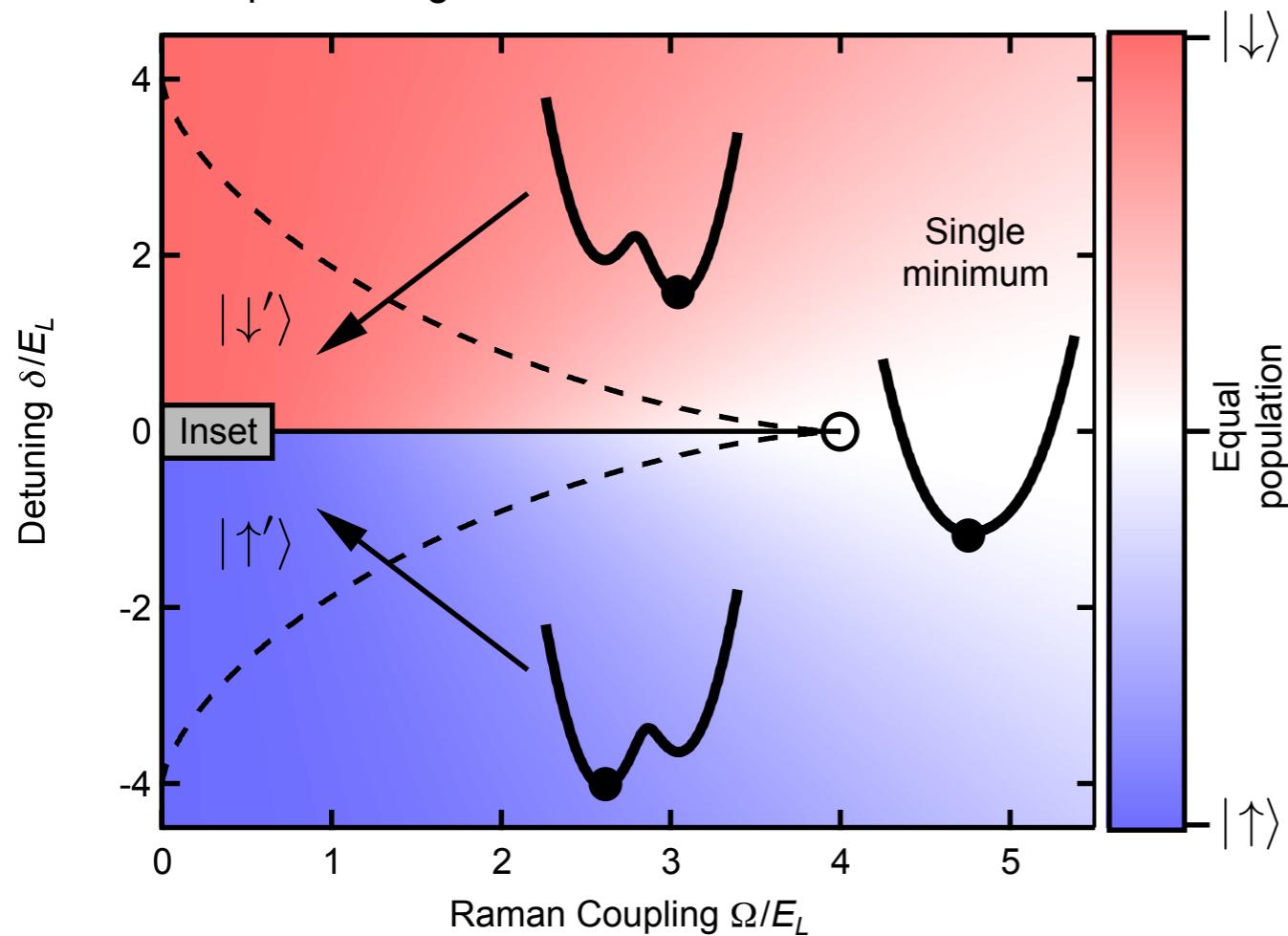
Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

Mean field phase diagram with SO coupling

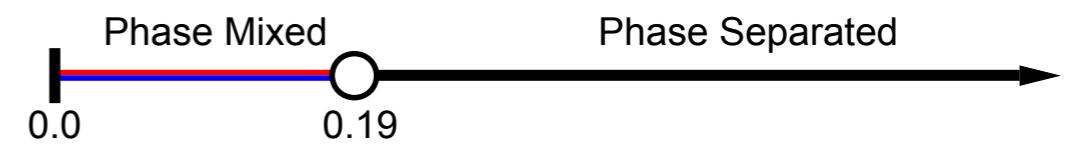
MFT: minimize classical energy

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a Mean field phase diagram



Fixed total spin or magnetization

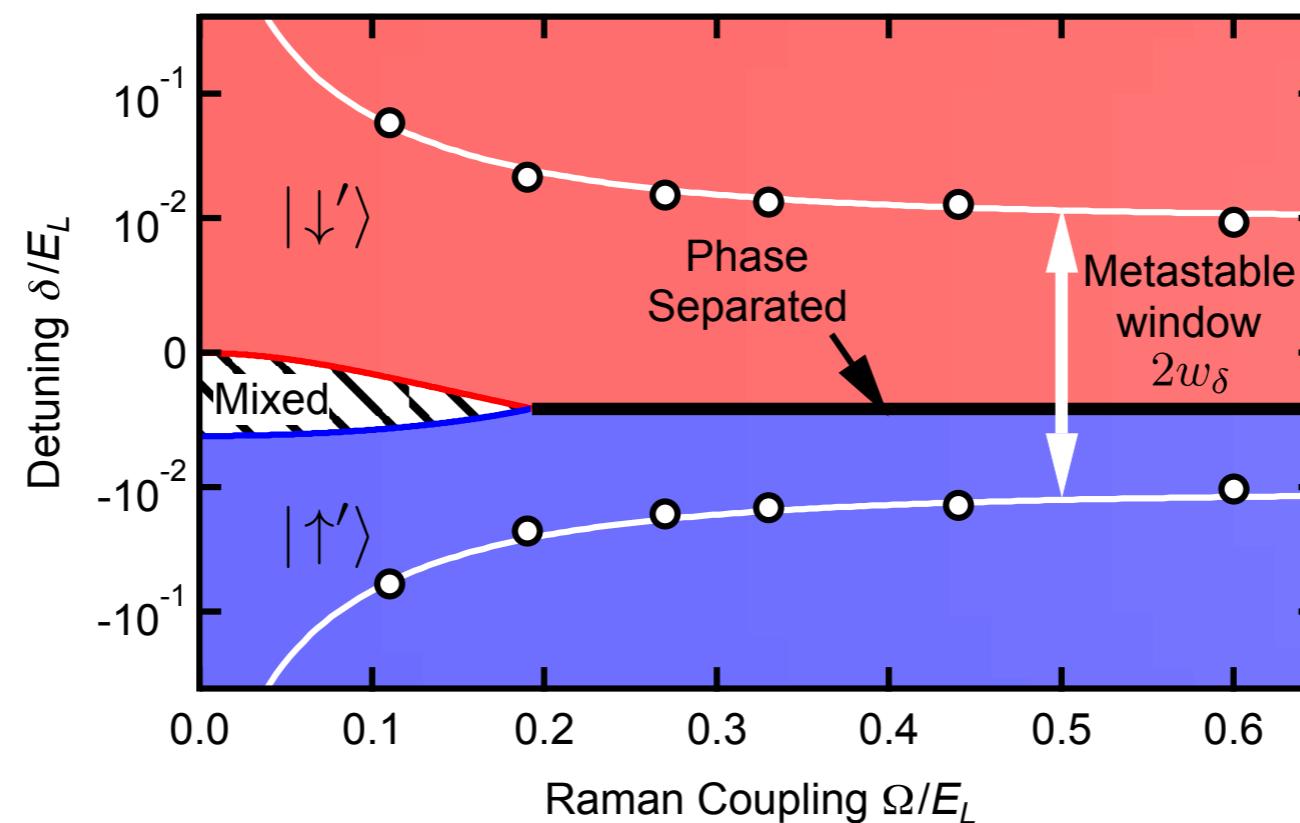


Refs.

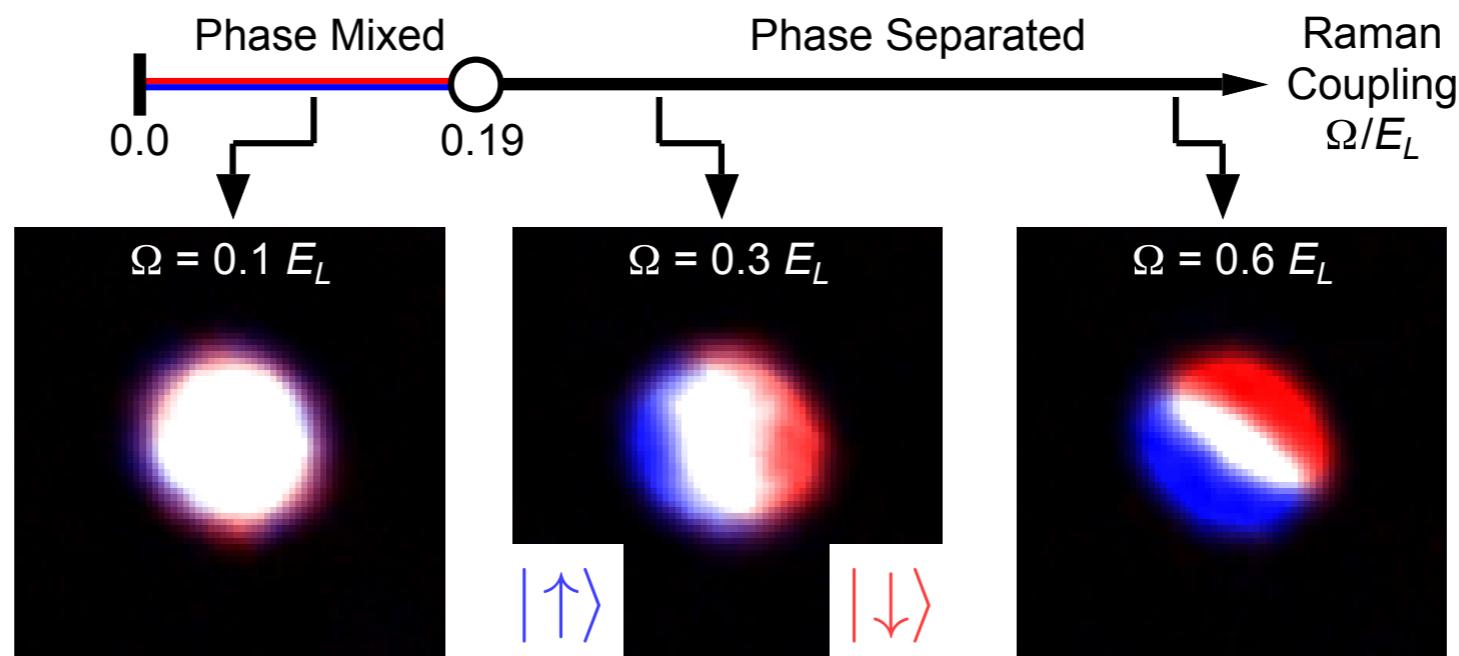
Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

Transition from miscible to immiscible

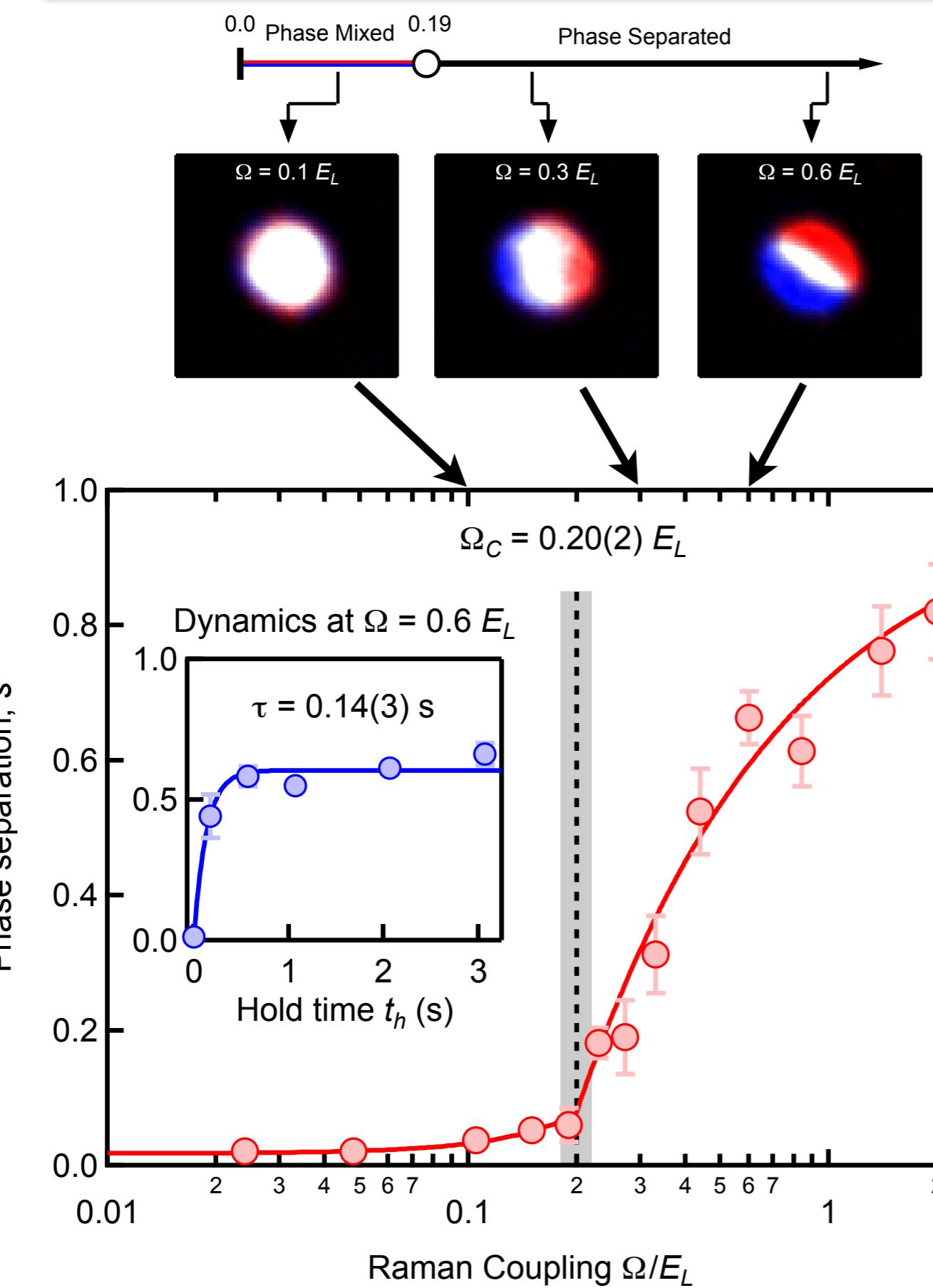
b Phase diagram, inset



c Miscible to immiscible transition



Transition from miscible to immiscible



A quantum phase transition
Previously unexpected

Our MFT prediction
Phase separation at $\Omega = 0.19 E_L$

Refs.

Y.-J. Lin et al, Nature (2011)
C. Wang et al (arXiv:1006.5148),
T.-L. Ho and S. Zhan (arXiv:1007.0650)