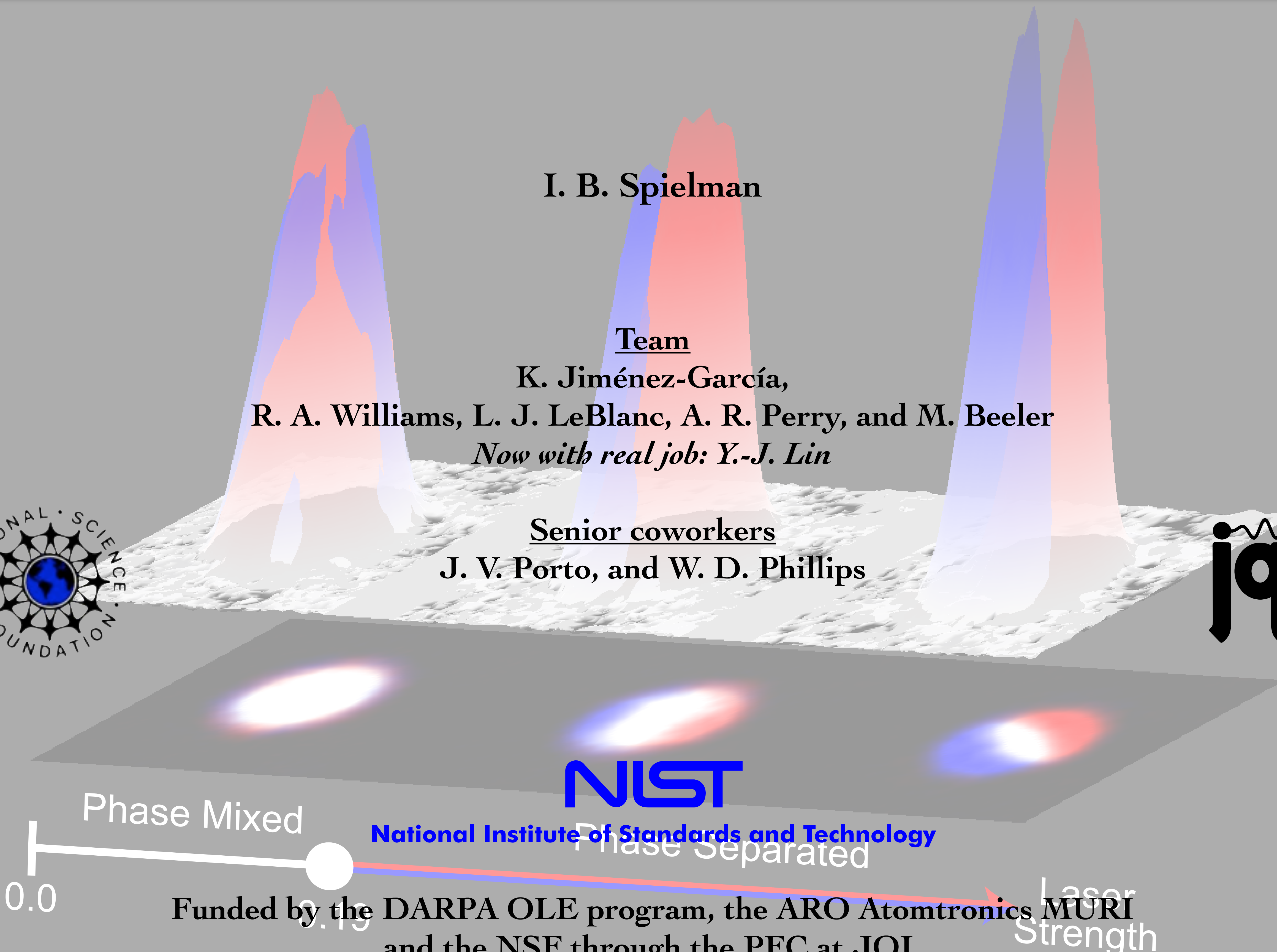
# Spins with neutral atoms

I. B. Spielman




**Team**

K. Jiménez-García,
R. A. Williams, L. J. LeBlanc, A. R. Perry, and M. Beeler
*Now with real job: Y.-J. Lin*

**Senior coworkers**

J. V. Porto, and W. D. Phillips

NIST
National Institute of Standards and Technology

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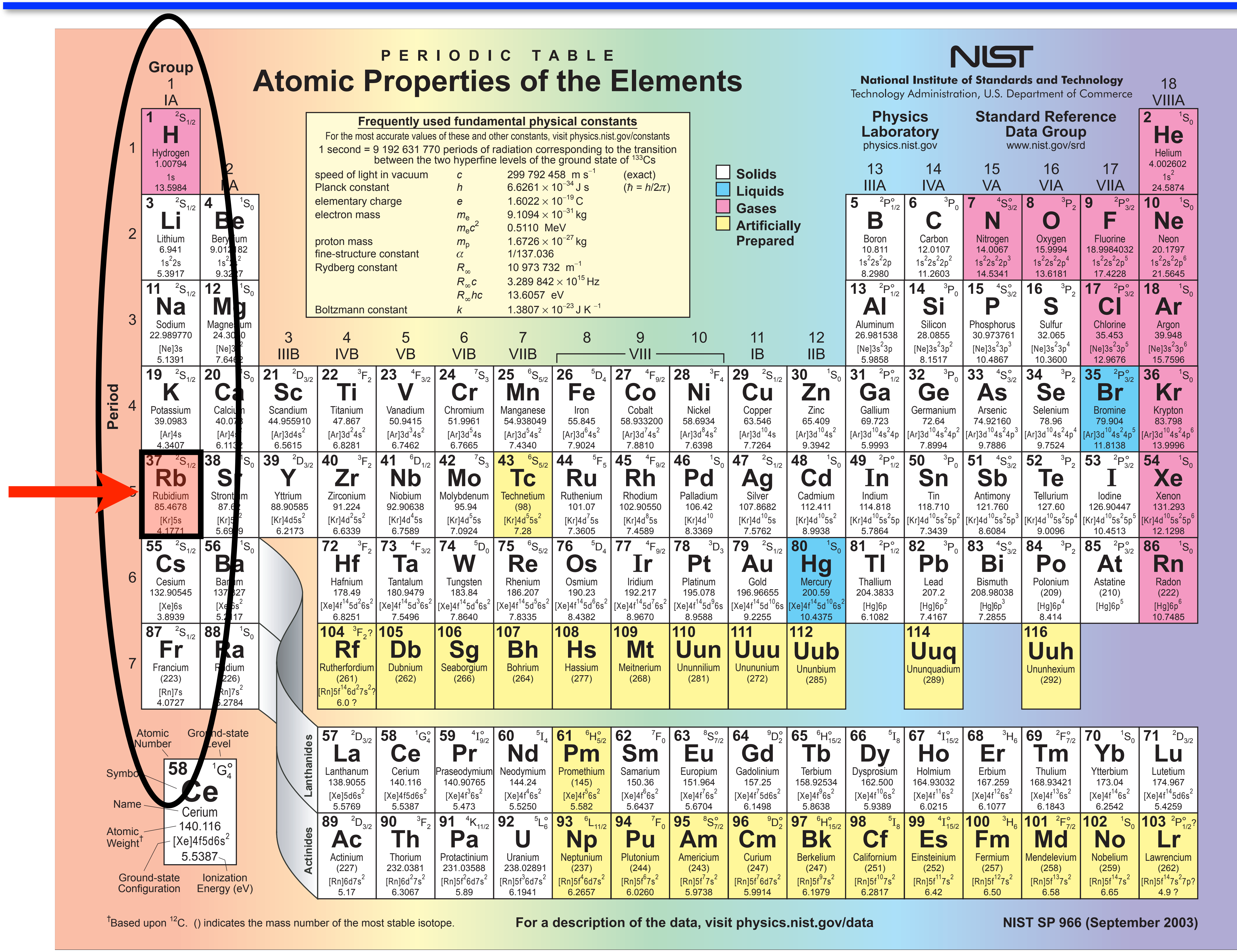
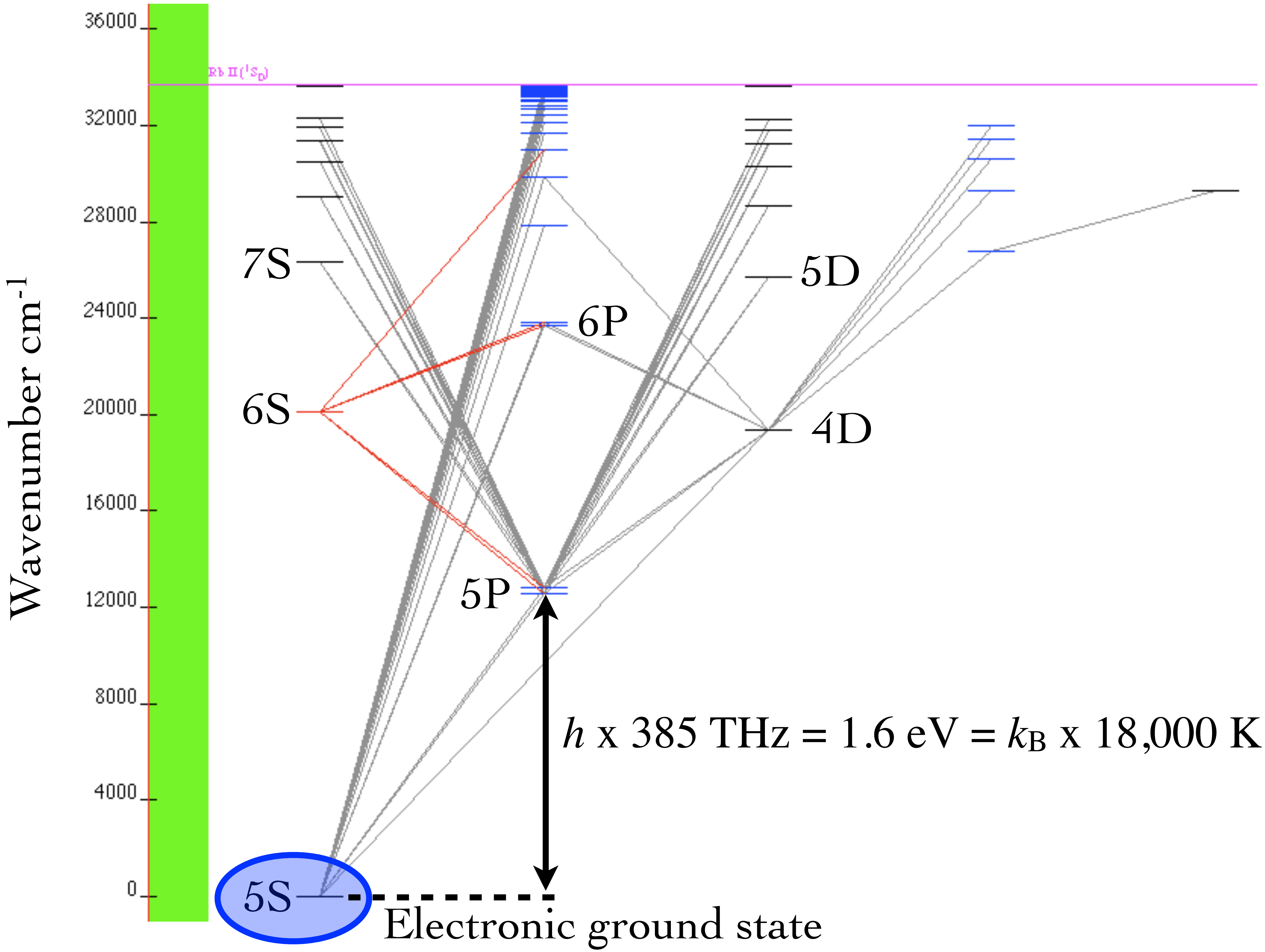
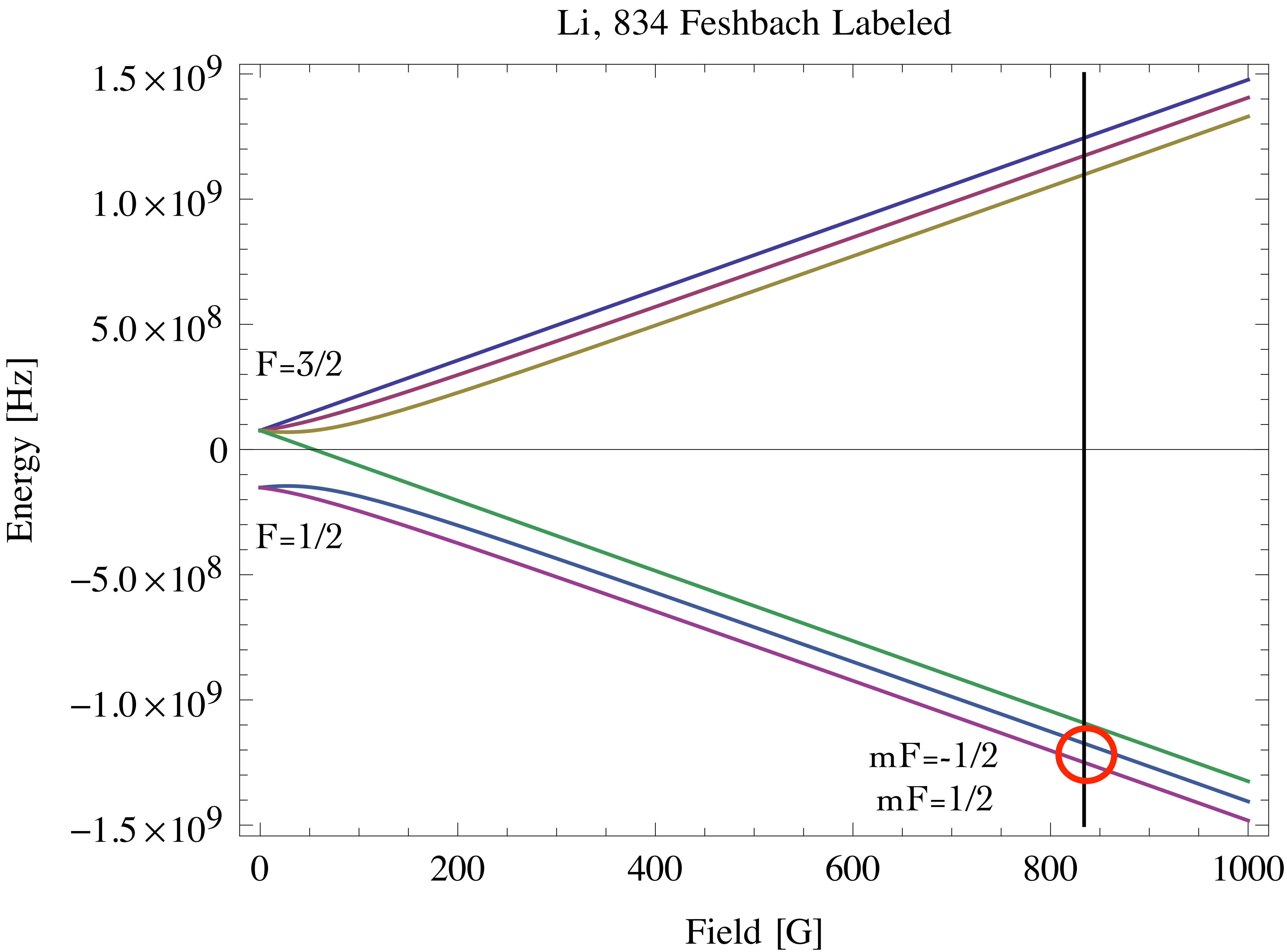
Dec. 2011

# Rubidium 87


PERIODIC TABLE
Atomic Properties of the Elements
NIST
National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce
Physics Laboratory
physics.nist.gov
Standard Reference Data Group
www.nist.gov/srd
Frequently used fundamental physical constants
For the most accurate values of these and other constants, visit physics.nist.gov/constants
1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of 133Cs
speed of light in vacuum c 299 792 458 m s−1 (exact)
Planck constant h 6.6261 × 10−34 J s (ħ = h/2π)
elementary charge e 1.6022 × 10−19 C
electron mass me 9.1094 × 10−31 kg
mec2 0.5110 MeV
proton mass mp 1.6726 × 10−27 kg
fine-structure constant α 1/137.036
Rydberg constant R∞ 10 973 732 m−1
R∞c 3.289 842 × 1015 Hz
R∞hc 13.6057 eV
Boltzmann constant k 1.3807 × 10−23 J K−1
Solids
Liquids
Gases
Artificially Prepared
Group
Period
37 Rb Rubidium 85.4678 [Kr]5s 4.1771
Atomic Number
Ground-state Level
Symbol
Name
Atomic Weight†
Ground-state Configuration
Ionization Energy (eV)
Lanthanides
Actinides
†Based upon 12C. () indicates the mass number of the most stable isotope.
For a description of the data, visit physics.nist.gov/data
NIST SP 966 (September 2003)

# Rubidium 87: Level structure

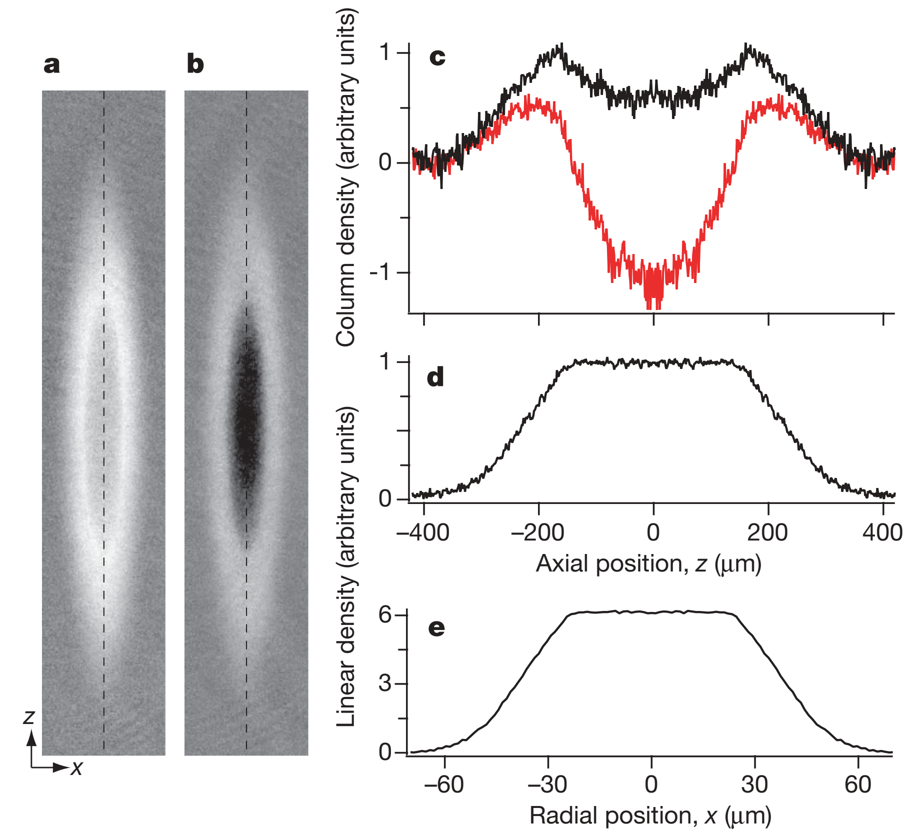




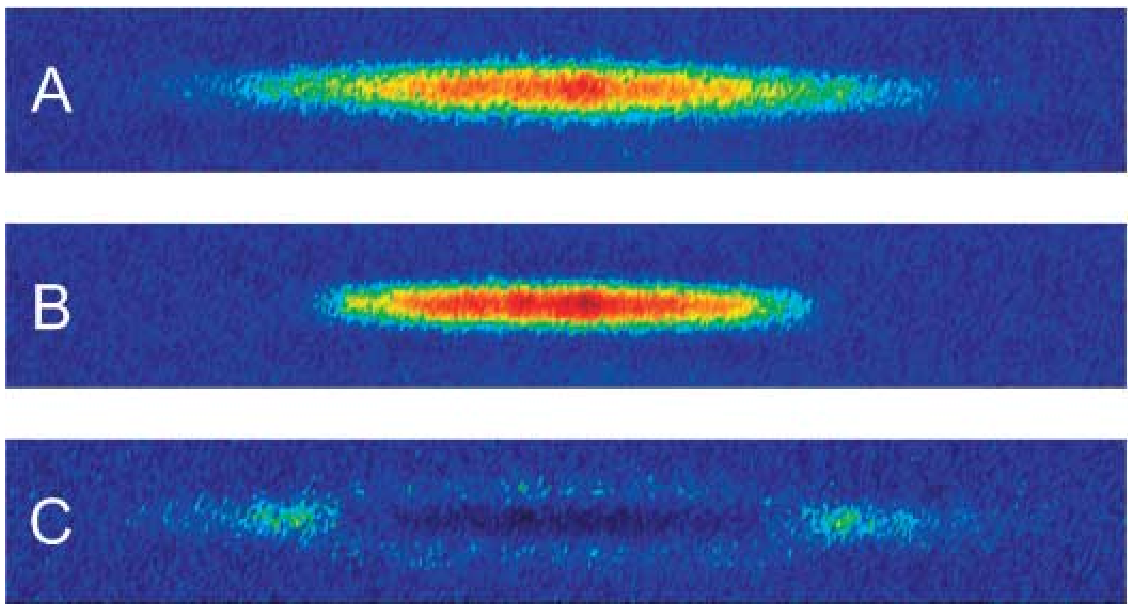
Li, 834 Feshbach Labeled
Energy [Hz]
1.5×10^9
1.0×10^9
5.0×10^8
0
−5.0×10^8
−1.0×10^9
−1.5×10^9
F=3/2
F=1/2
mF=-1/2
mF=1/2
0
200
400
600
800
1000
Field [G]

# BEC- BCS Crossover physics

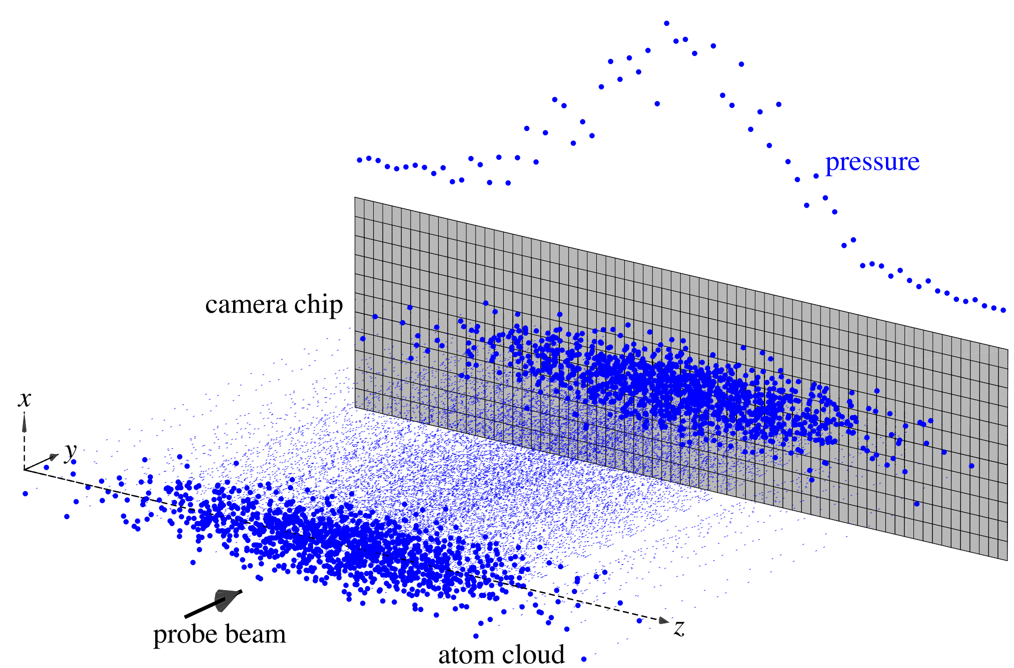
## Ketterle (MIT)



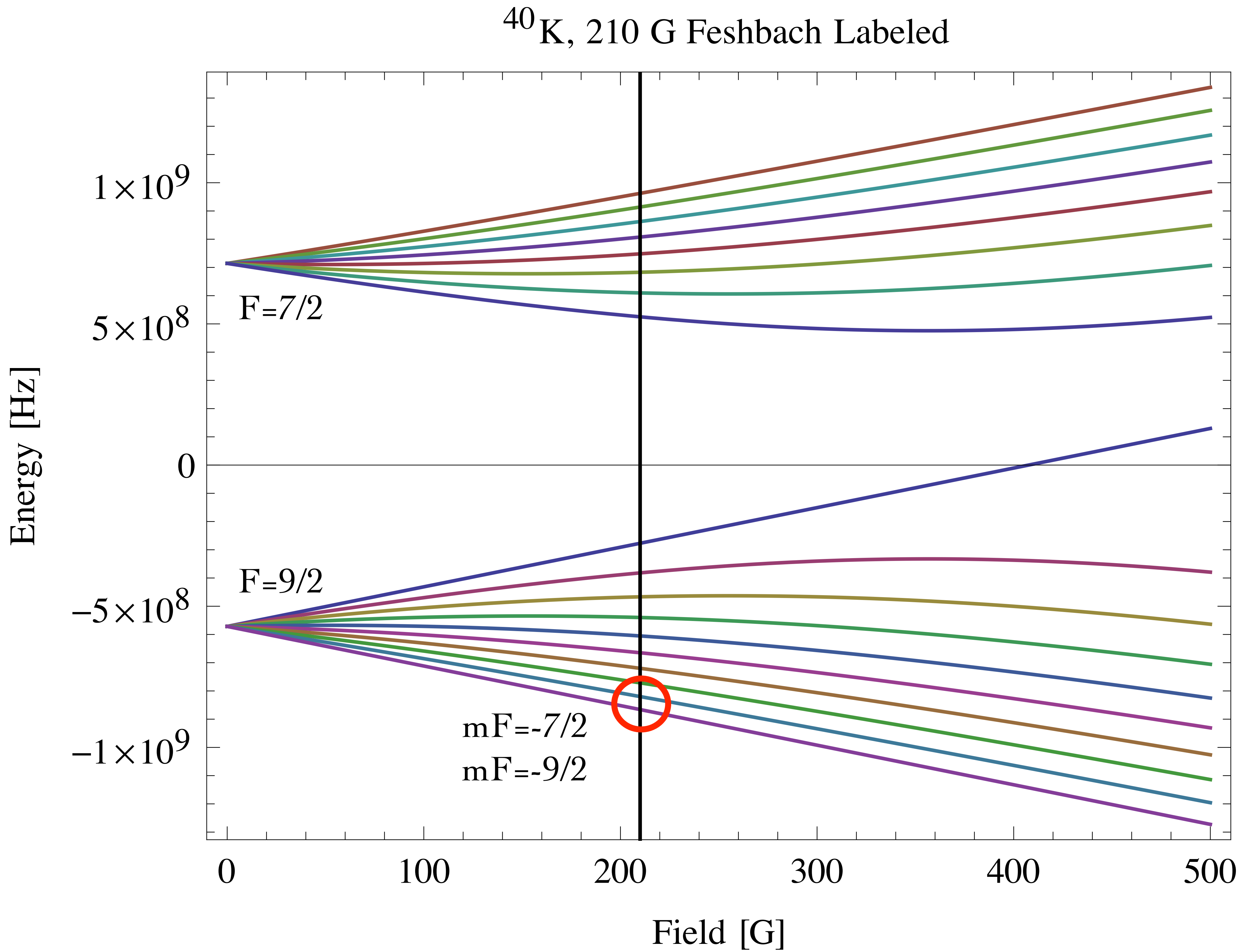

## Hulet (Rice)




## Solomon (ENS)

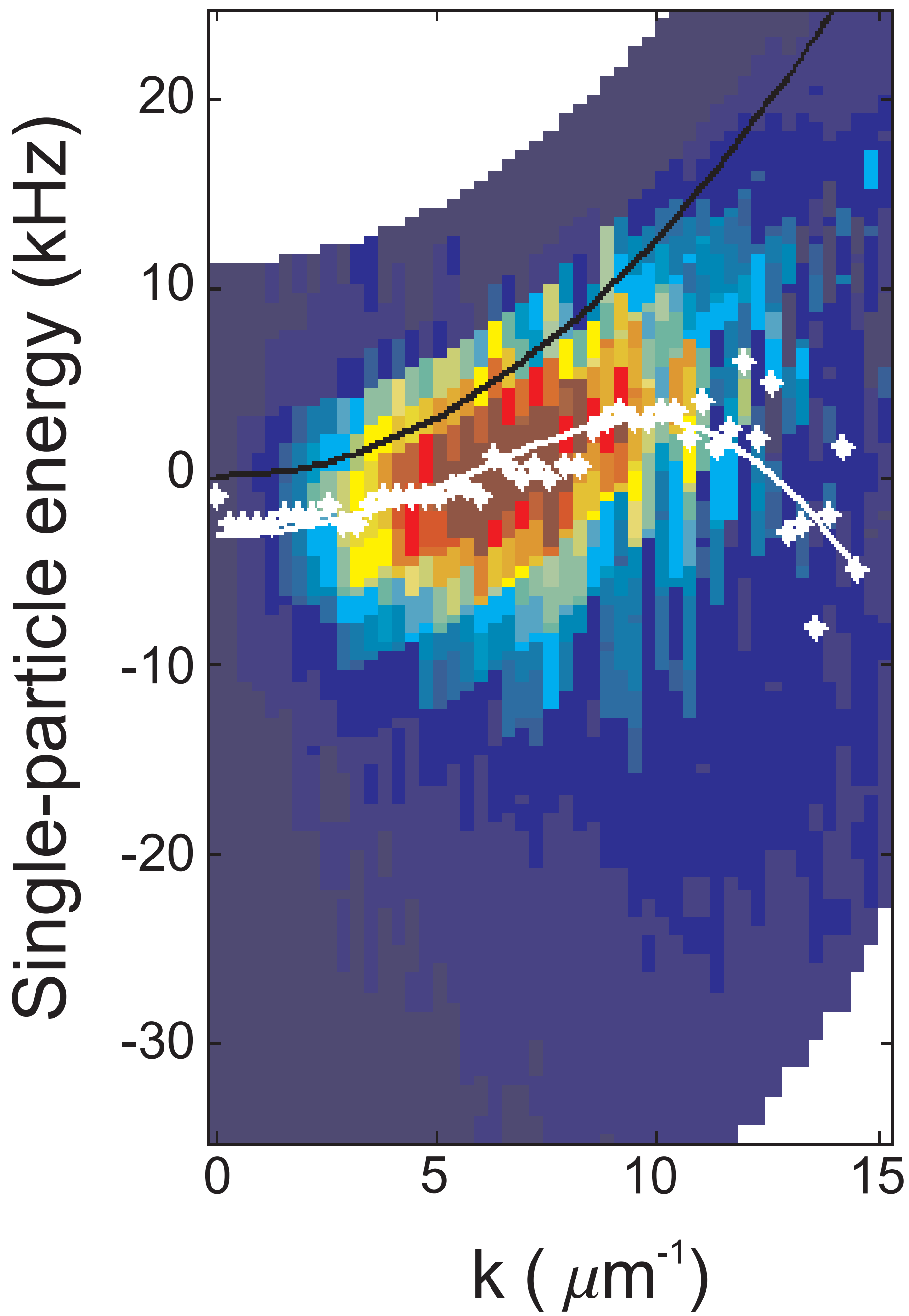


# $^{40}$K has inverted hyperfine

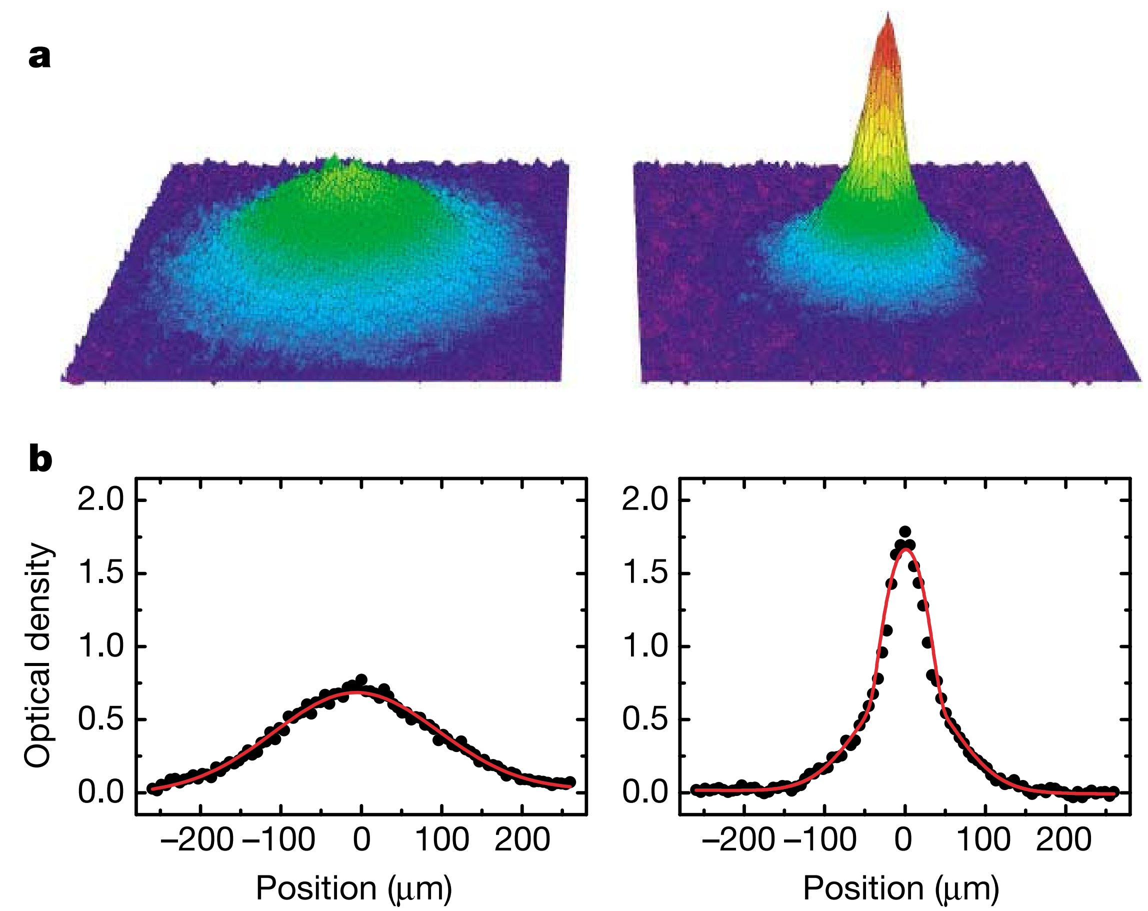


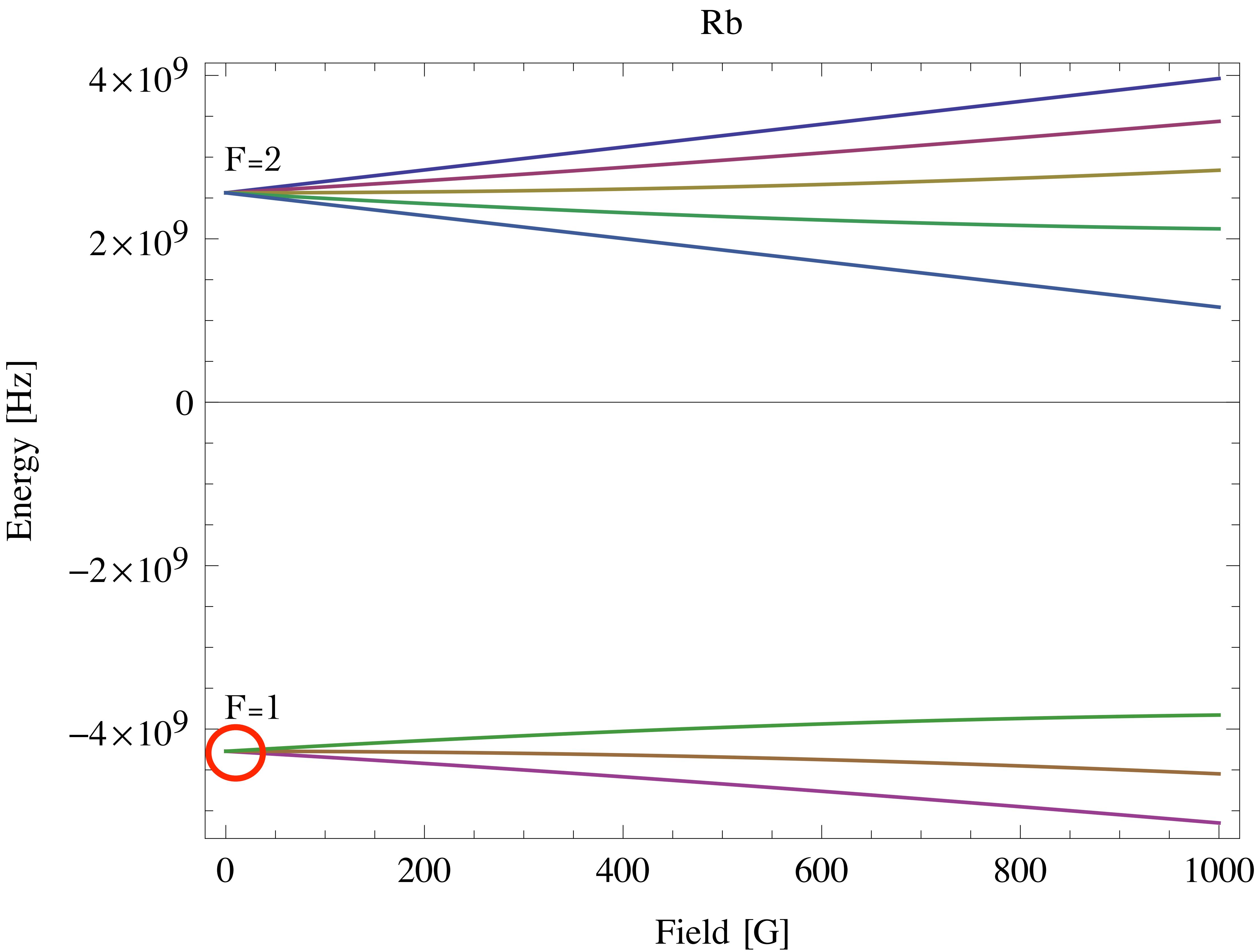
# Jin group (JILA)

## ARPES analogy

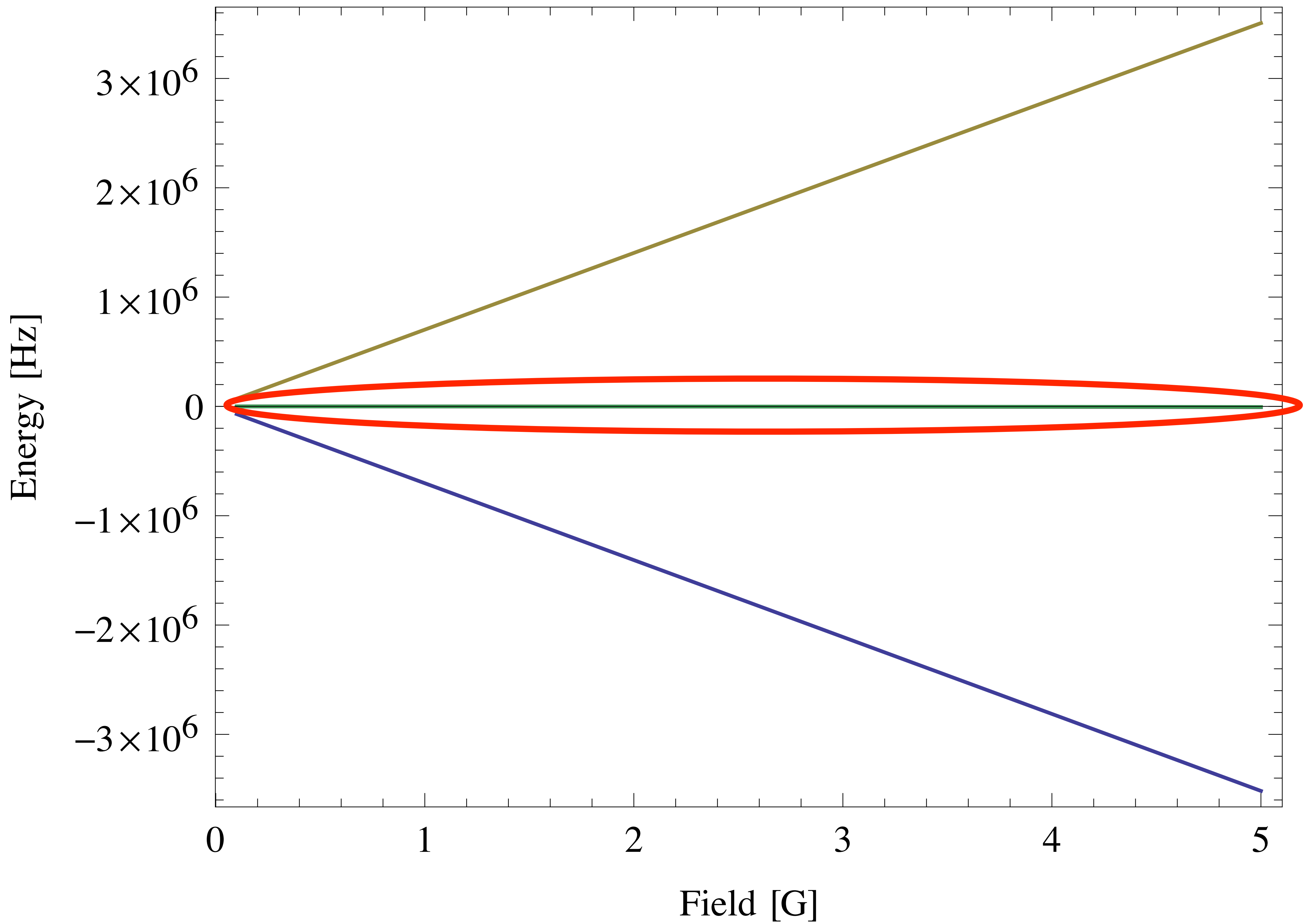



## BEC of molecules

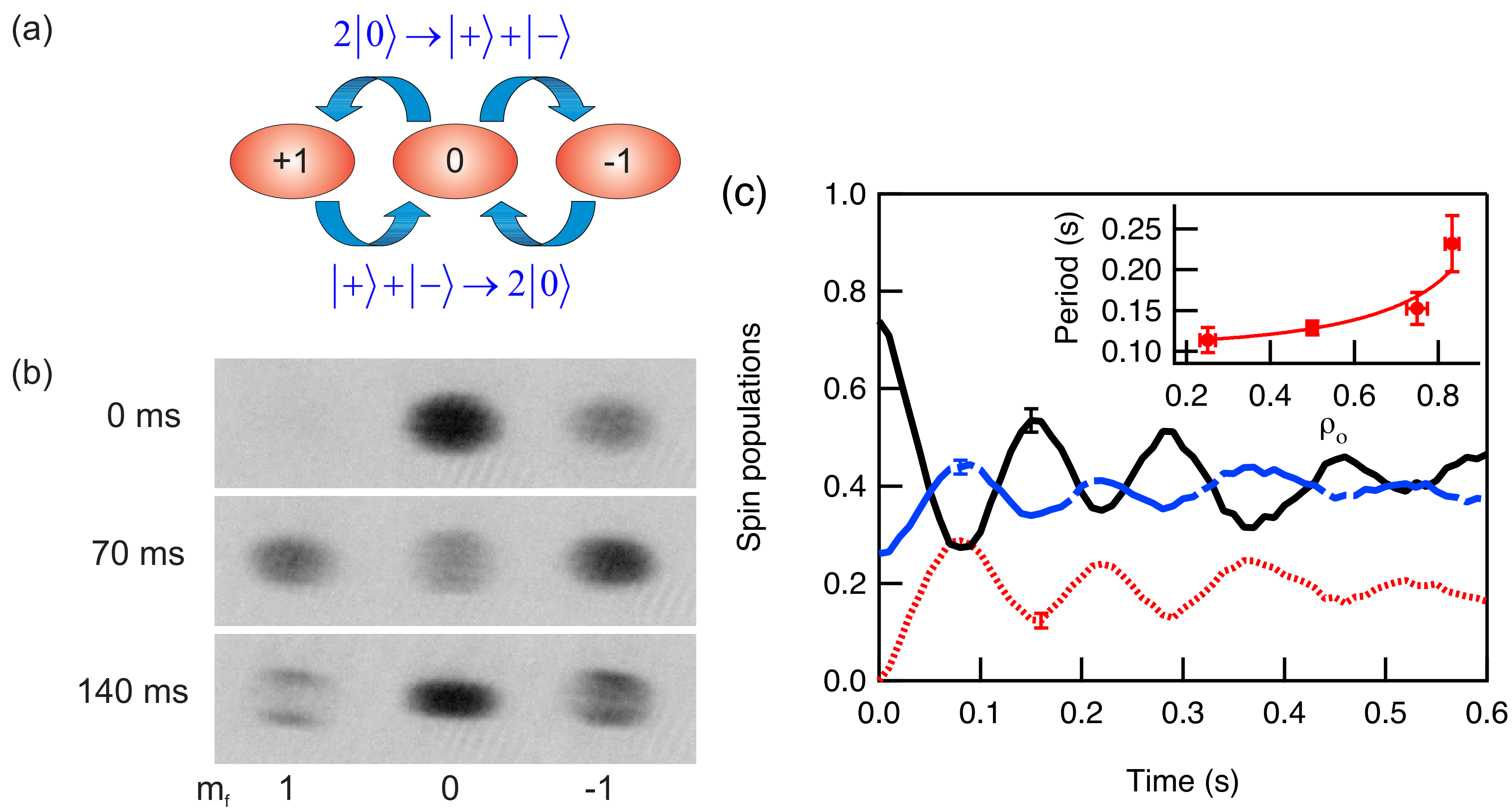



Rb
F=2
F=1
4×10^9
2×10^9
0
−2×10^9
−4×10^9
Energy [Hz]
0
200
400
600
800
1000
Field [G]

# $^{87}$Rb F=1 at low field

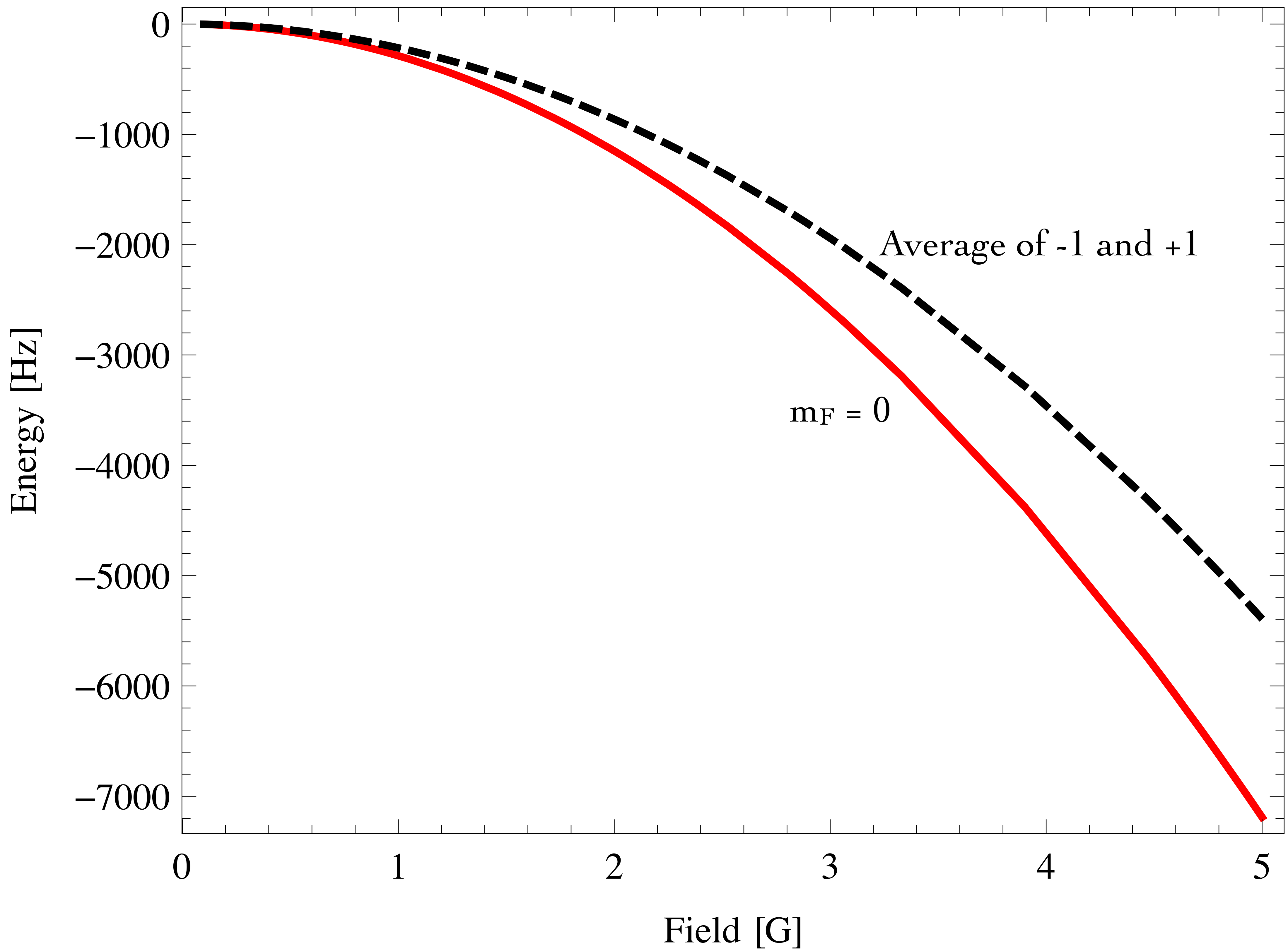


# Spinor BEC's (Chapman group) for example

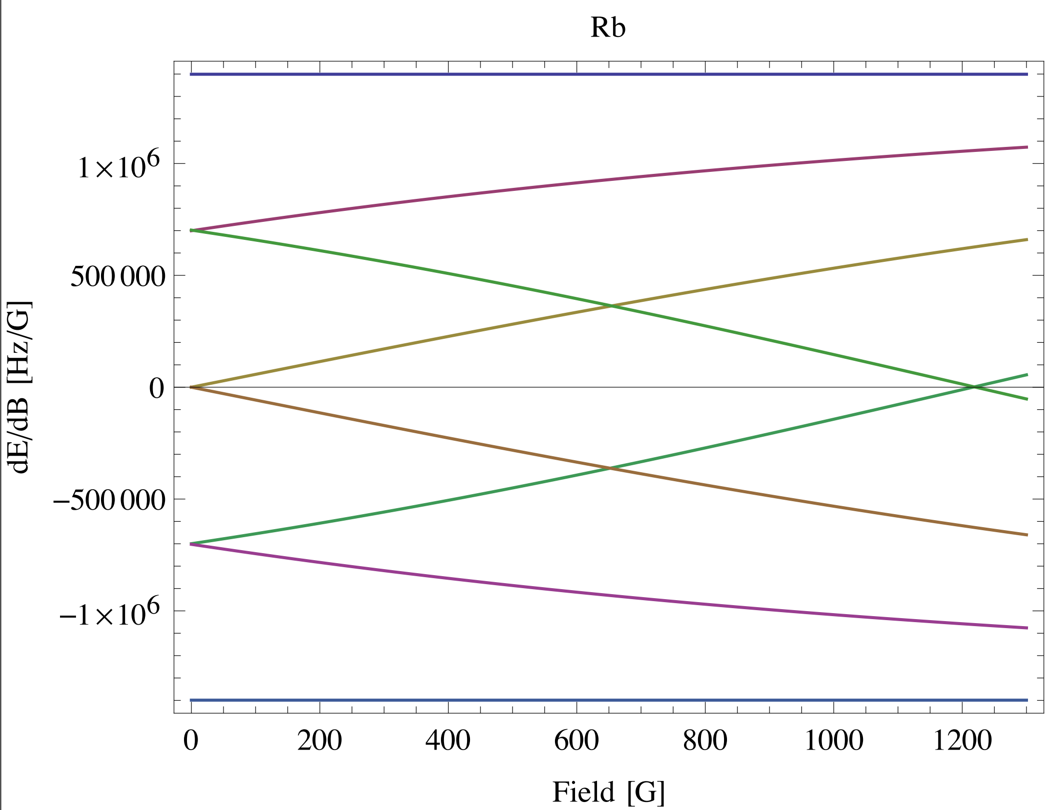


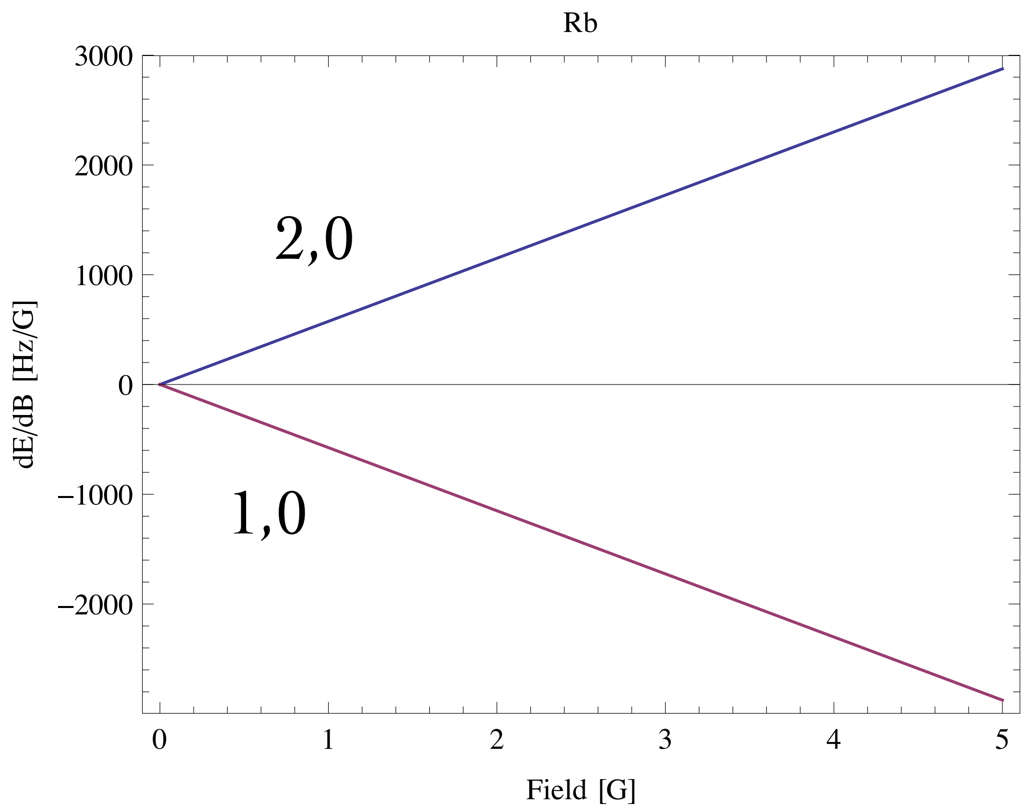

Three spins! How often do you see THAT with electrons...

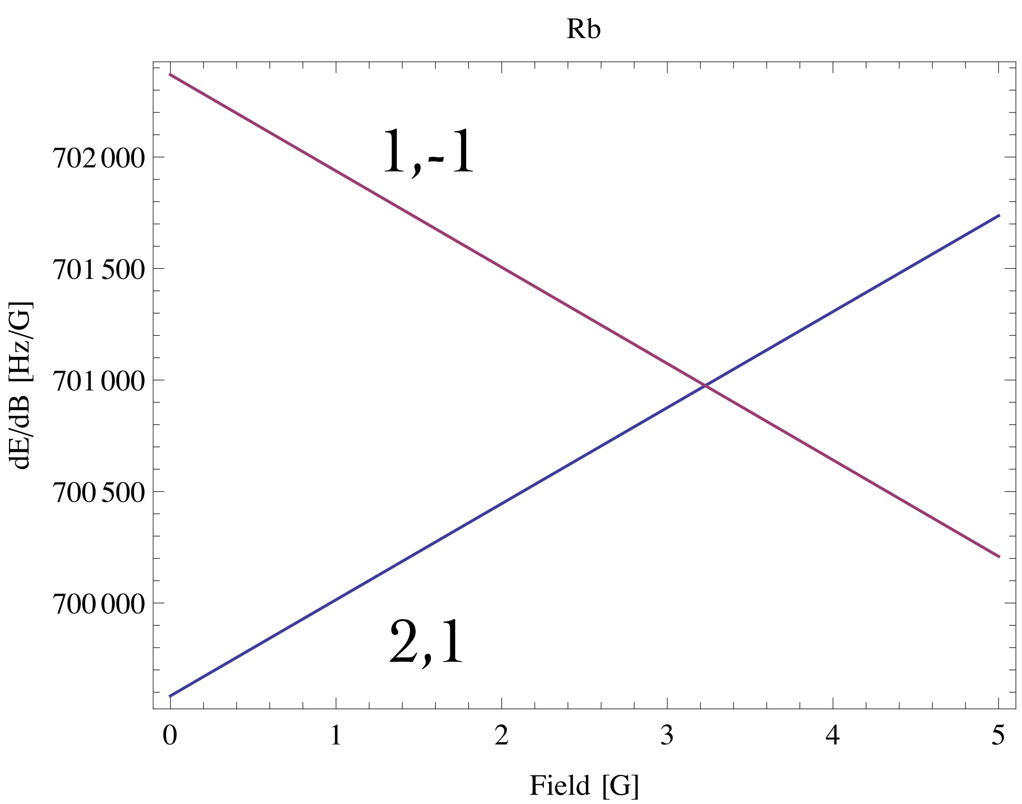
# $^{87}$Rb F=1 quadratic effects



# $^{87}$Rb Clock states



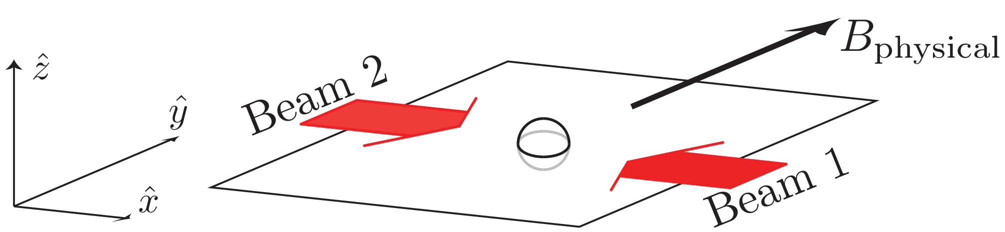



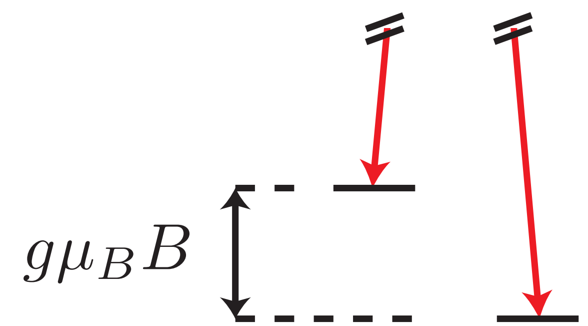
# Standard picture

## Space dependent coupling

Now we have no time-dependence, but instead space-dependence.

## Geometry and initial Hamiltonian



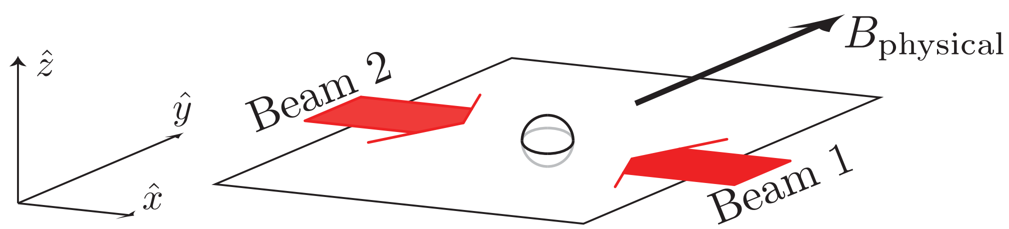



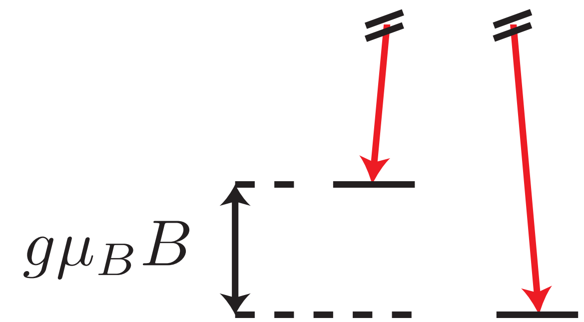

$$\hat{H} = \frac{\hbar^2 \hat{k}^2}{2m} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{\Omega_{\rm R}}{2} \begin{pmatrix} 0 & \exp(i2k_R x) \\ \exp(-i2k_R x) & 0 \end{pmatrix} + \begin{pmatrix} \delta/2 & 0 \\ 0 & -\delta/2 \end{pmatrix}$$

$$= \frac{\hbar^2 \hat{k}^2}{2m} \check{1} + \frac{\Omega}{2} \left[\cos(2k_R x)\check{\sigma}_x - \sin(2k_R x)\check{\sigma}_y\right] + \frac{\delta}{2}\check{\sigma}_z$$

# Exact picture

## Geometry and initial Hamiltonian




## Work in the momentum basis!

$$\hat{U}_1 \hat{H} \hat{U}_1^\dagger = (\hat{k}\check{1} - \check{\sigma}_z)^2 + \frac{\Omega}{2}\check{\sigma}_x + \frac{\delta}{2}\check{\sigma}_z$$
$$= \begin{pmatrix} (k-1)^2 + \delta/2 & \Omega/2 \\ \Omega/2 & (k+1)^2 - \delta/2 \end{pmatrix}$$

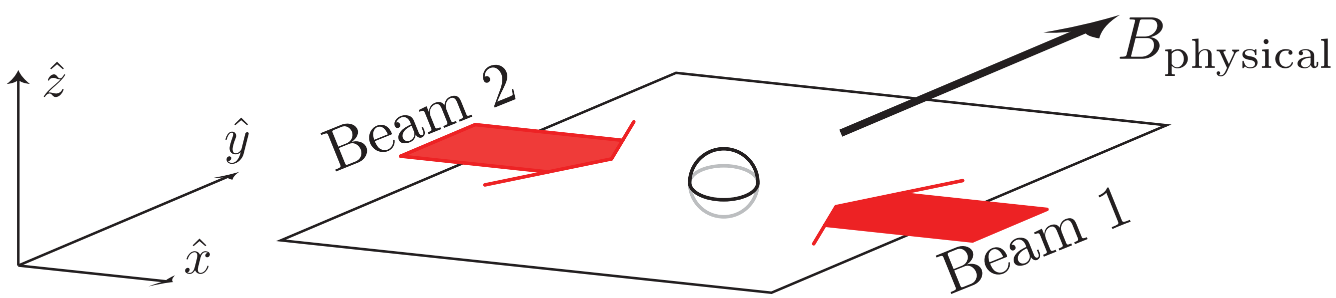
Numerically solve without approximation

# Spin orbit coupling: origin

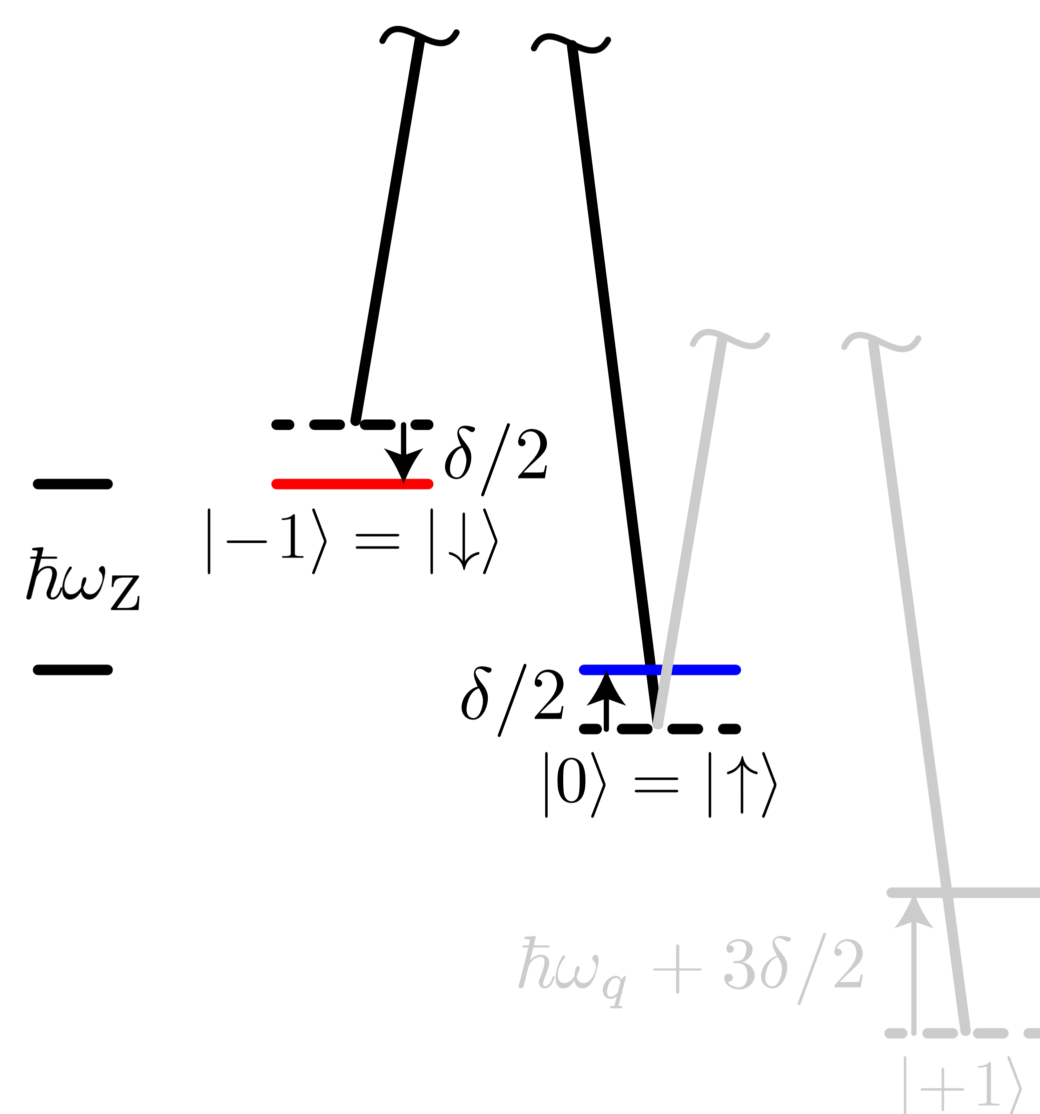
## Momentum representation

$$H = \sum_k \left\{ \begin{pmatrix} \langle k-1,\uparrow| & \langle k+1,\downarrow| \end{pmatrix} \begin{pmatrix} (\tilde{k}_x - 1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k}_x + 1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1,\uparrow\rangle \\ |k+1,\downarrow\rangle \end{pmatrix} \right\}$$

## Geometry




## Levels



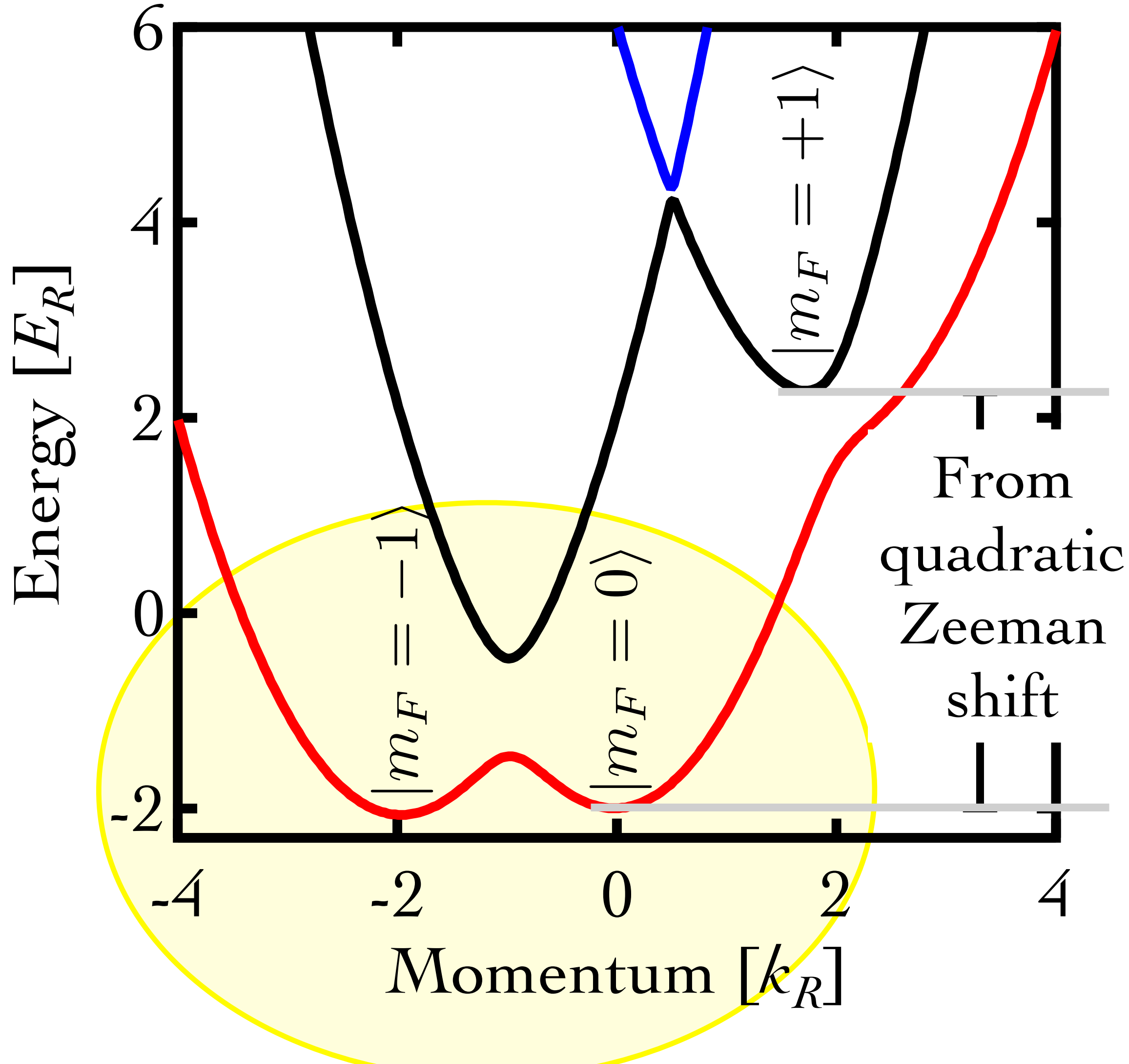

References
Y.-J. Lin *et al* Nature (2011)

# Spin orbit coupling: origin

Momentum representation

$$H = \sum_k \left\{ \begin{pmatrix} \langle k-1,\uparrow | & \langle k+1,\downarrow | \end{pmatrix} \begin{pmatrix} (\tilde{k_x}-1)^2 + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & (\tilde{k_x}+1)^2 - \delta/2 \end{pmatrix} \begin{pmatrix} |k-1,\uparrow\rangle \\ |k+1,\downarrow\rangle \end{pmatrix} \right\}$$

Spin 1/2 bosons????




Transform to

$$\begin{aligned} \hat{H} &= \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m}\check{1} + \left(\frac{\delta}{2} + \frac{\hbar^2 k_R}{m}\hat{k}_x\right)\check{\sigma}_y + \frac{\Omega}{2}\check{\sigma}_z + \Delta E \check{1} \\ &= \frac{\hbar^2}{2m}\left[\left(\hat{k}_x\check{1} + k_R\check{\sigma}_y\right)^2 + \left(\hat{k}_y\check{1} - 0\right)^2\right] + \frac{\delta}{2}\check{\sigma}_y + \frac{\Omega}{2}\check{\sigma}_z \end{aligned}$$

**NOTICE**

Written as a “2x2” vector potential, this S-O coupling is ***NOT*** non-Abelian (it is $A_x$)

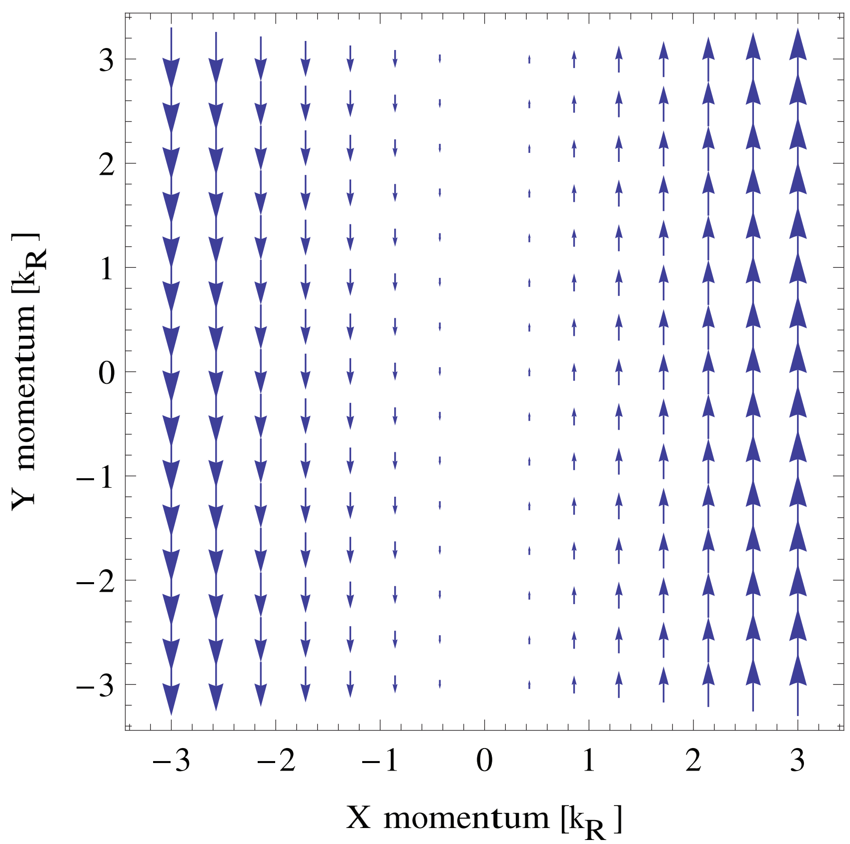
However, the Hamiltonian *is* non-trivial owing to the Zeeman field along *z*.

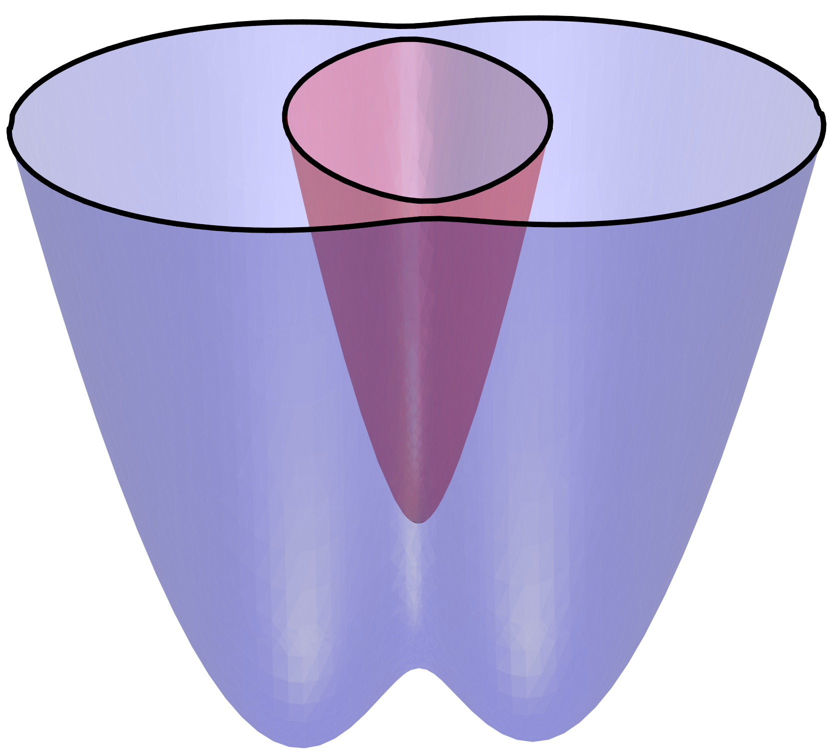
# Spin-orbit coupling

## Spin-orbit coupling

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m}\check{1} + \frac{\delta}{2}\check{\sigma}_z + \alpha\left(k_x\check{\sigma}_y - k_y\check{\sigma}_x\right) + \beta\left(k_x\check{\sigma}_x - k_y\check{\sigma}_y\right).$$

Equal Rashba and Dresselhaus: $\alpha = \beta$



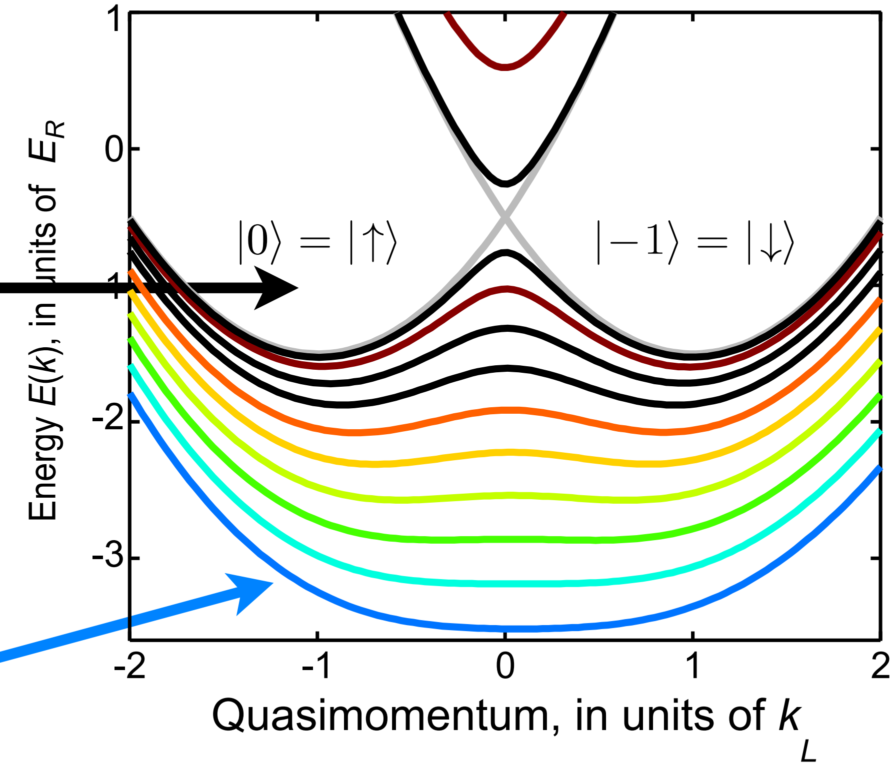

Refs.

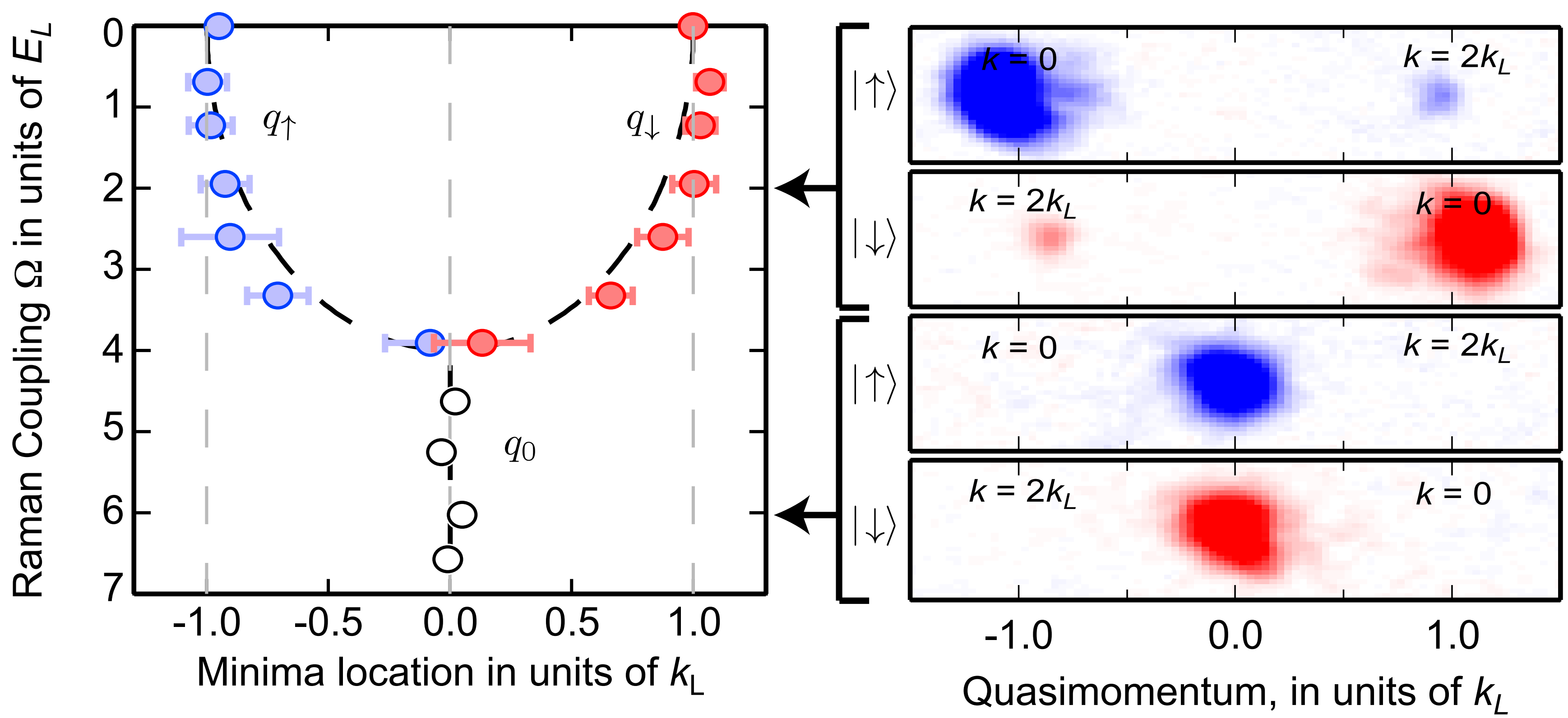
T. D. Stanescu and B. Anderson and V. Galitski PRA (2008)

# Typical data

Double Well
“Spin-orbit limit”

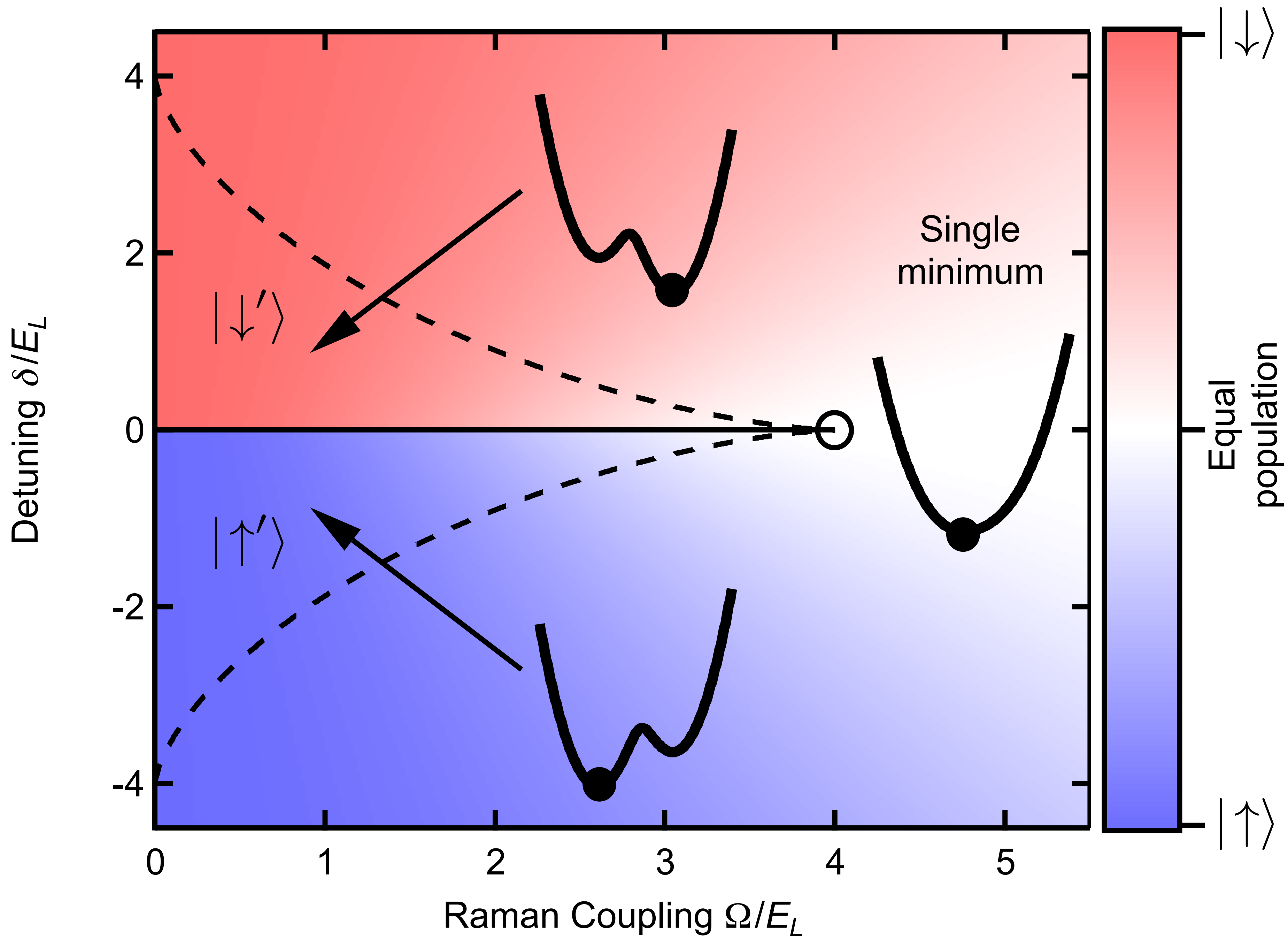
Single minimum
“vector potential limit”



# Phase diagram I: Large single particle energy

Minimize single particle energy

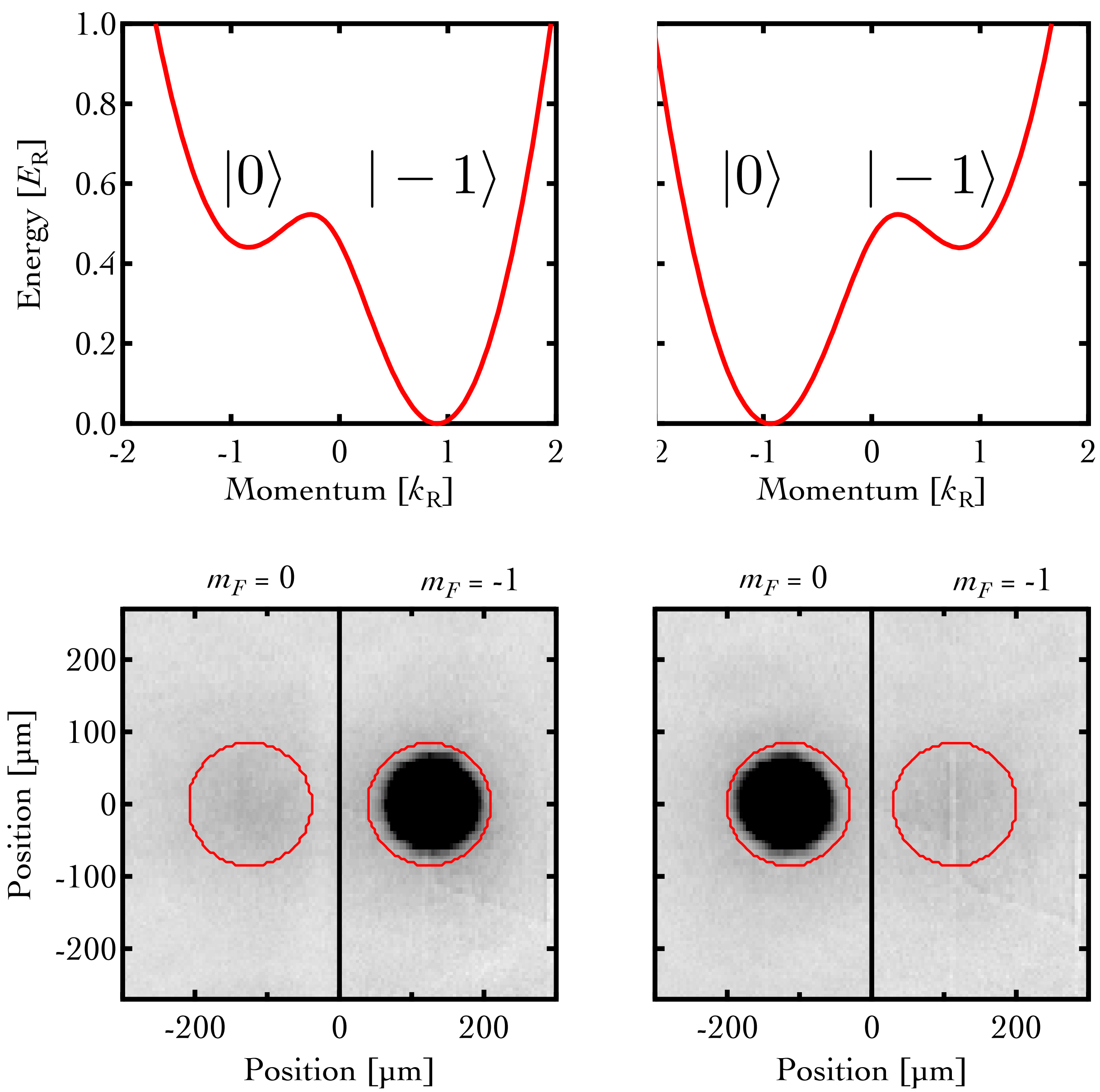



Refs.

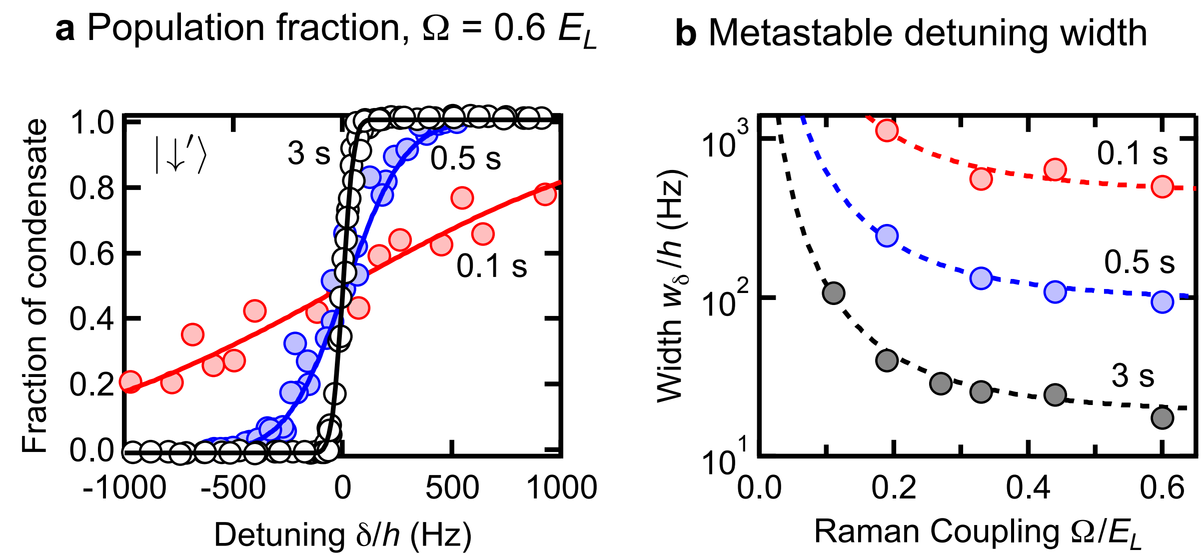
Y.-J. Lin et al, Nature (2011)

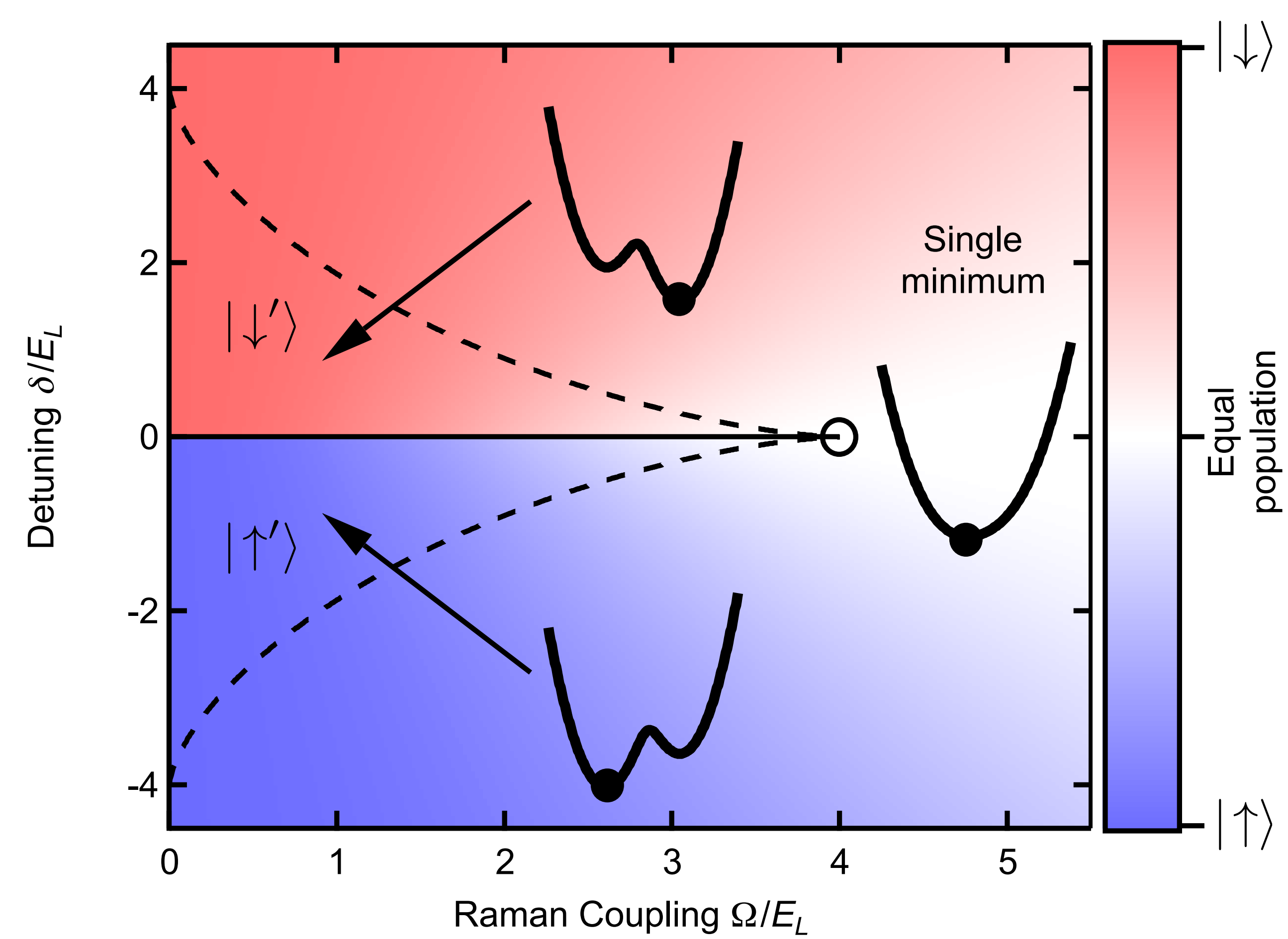
# Equilibrium: slow

## Minimize single particle energy




## Metastable populations




References

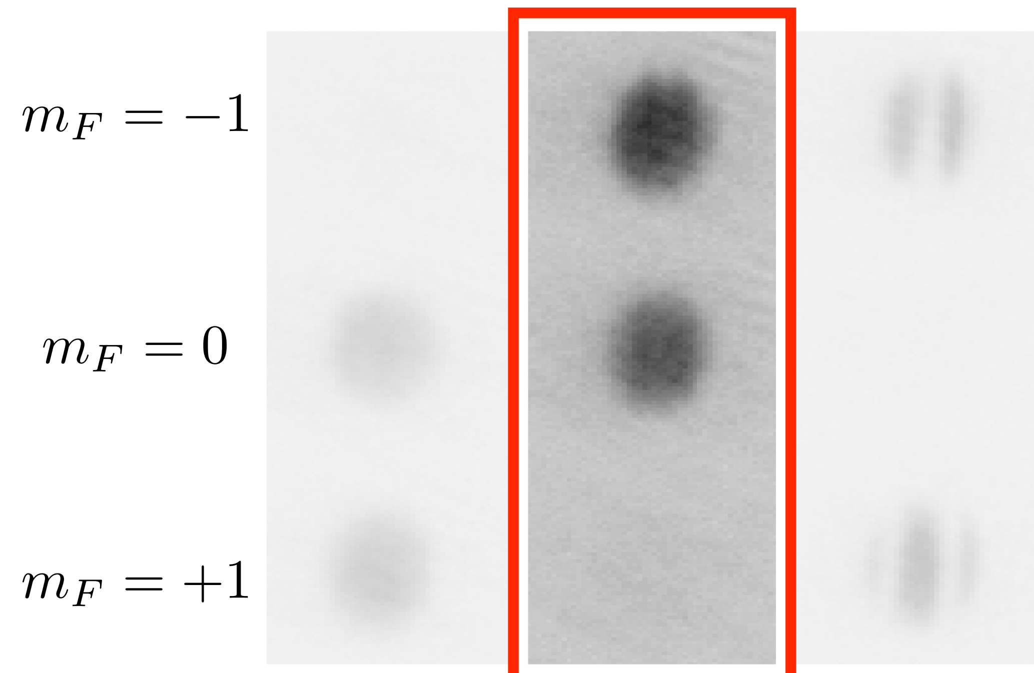
Y.-J. Lin *et al* Nature (2011)

# Effective Hamiltonian for dressed spins

## Two Level contact interactions

$$\hat{H}_{\rm int} = \frac{1}{2}\int d^3r : \left[\left(c_0 + \frac{c_2}{2}\right)\left(\hat{\rho}_\downarrow + \hat{\rho}_\uparrow\right)^2 + \frac{c_2}{2}\left(\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2\right) + c_2\hat{\rho}_\downarrow\hat{\rho}_\uparrow\right] :$$

$m_F$ = -1, $m_F$ = 0 mixture: miscible for $^{87}$Rb




Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

# Effective Hamiltonian for dressed spins

## Two Level contact interactions

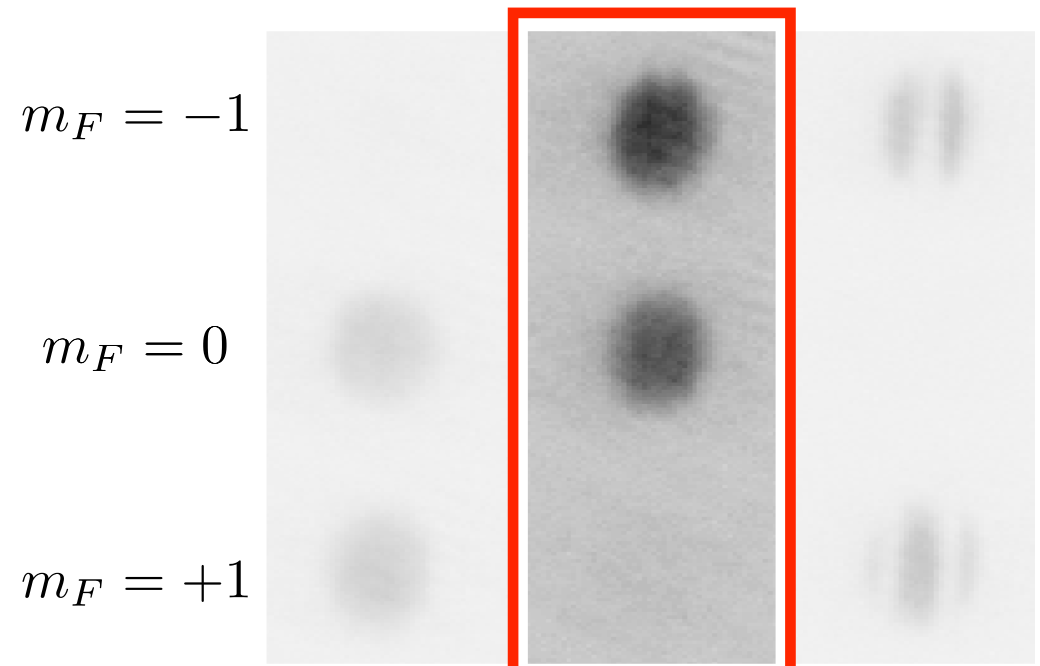
$$\hat{H}_{\text{int}} = \frac{1}{2}\int d^3r : \left[\left(c_0 + \frac{c_2}{2}\right)(\hat{\rho}_\downarrow + \hat{\rho}_\uparrow)^2 + \frac{c_2}{2}\left(\hat{\rho}_\downarrow^2 - \hat{\rho}_\uparrow^2\right) + c_2\hat{\rho}_\downarrow\hat{\rho}_\uparrow\right] :$$

$$\rightarrow \frac{1}{2}\int d^3r : \left[\left(c_0 + \frac{c_2}{2}\right)(\hat{\rho}_{\downarrow'} + \hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2}\left(\hat{\rho}_{\downarrow'}^2 - \hat{\rho}_{\uparrow'}^2\right) + \left(c_2 + c'_{\uparrow,\downarrow}\right)\hat{\rho}_{\downarrow'}\hat{\rho}_{\uparrow'}\right] :$$

## Spin-orbit term

$$c'_{\uparrow,\downarrow} \approx c_0 \frac{\Omega^2}{8}$$

## $m_F$ = -1, $m_F$ = 0 mixture: miscible for $^{87}$Rb

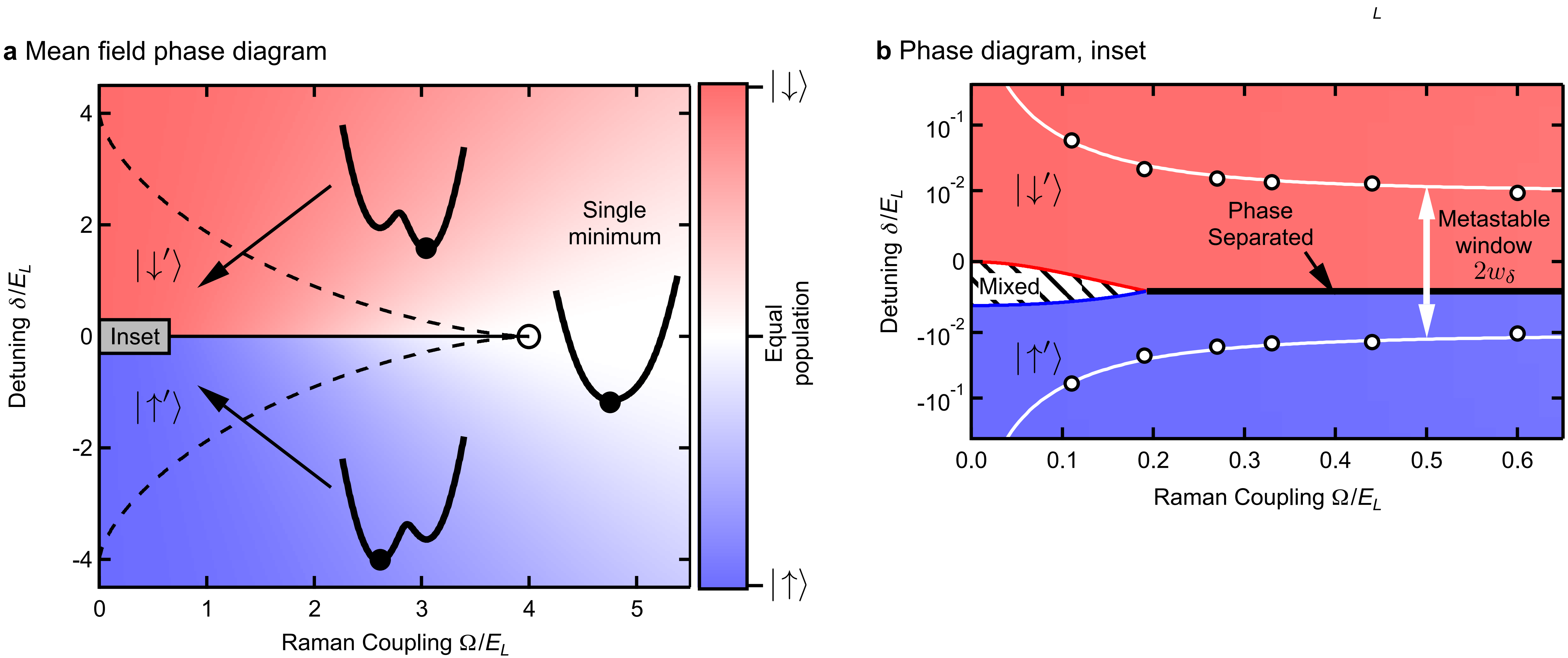



Ph.D. Thesis of Ming-Shien Chang
(Chapman group)

# Mean field phase diagram with SO coupling

## MFT: minimize classical energy

$$E_{\mathrm{MFT}} = \frac{1}{2}\int d^3r \left[\left(c_0 + \frac{c_2}{2}\right)\left(\rho_{\downarrow'} + \rho_{\uparrow'}\right)^2 + \frac{c_2}{2}\left(\rho_{\downarrow'}^2 - \rho_{\uparrow'}^2\right) + \left(c_2 + c'_{\uparrow,\downarrow}\right)\rho_{\downarrow'}\rho_{\uparrow'} + \delta\left(\rho_{\uparrow'} - \rho_{\downarrow'}\right)\right]$$





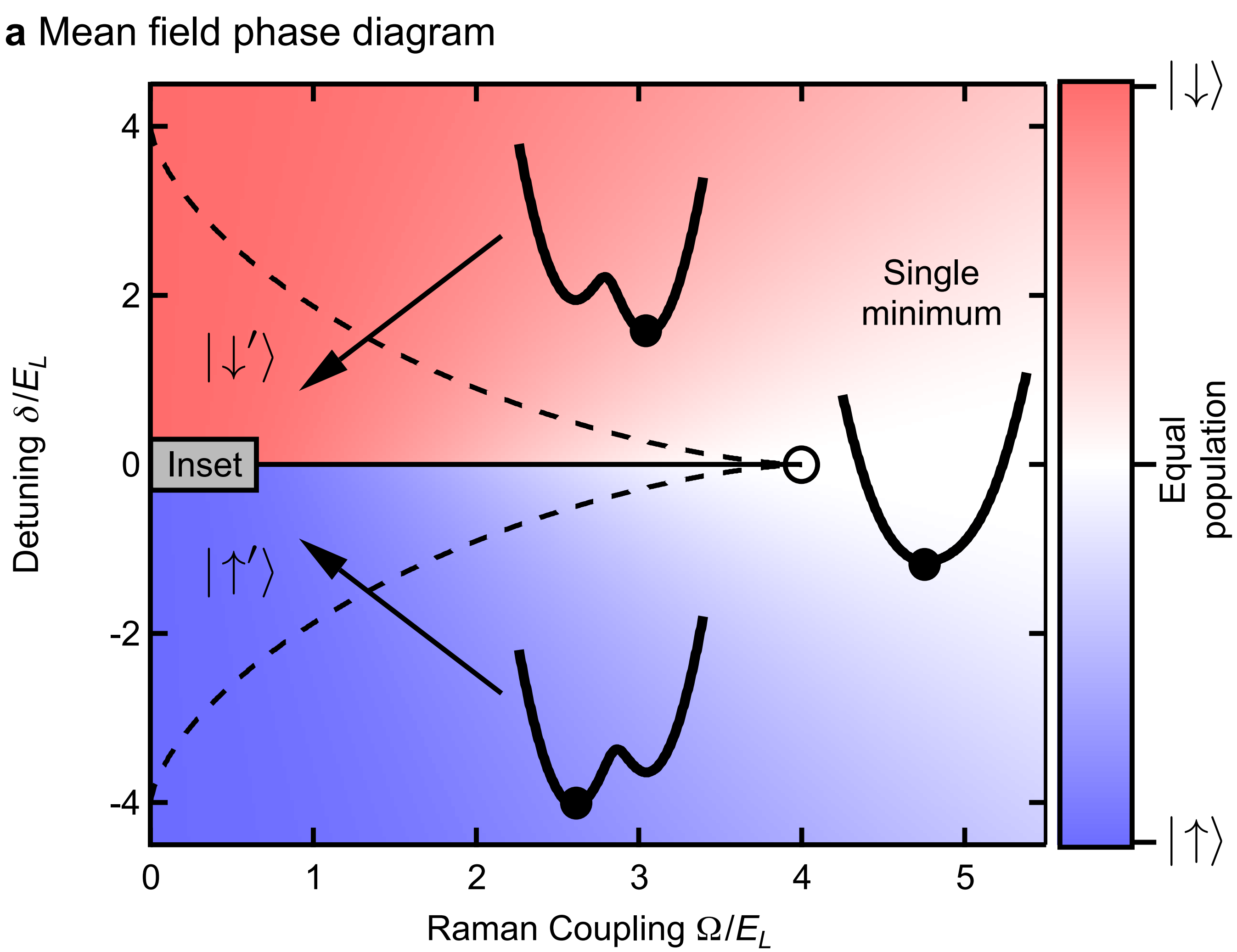
Refs.

Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

# Mean field phase diagram with SO coupling

MFT: minimize classical energy

$$E_{\mathrm{MFT}} = \frac{1}{2}\int d^3r \left[\left(c_0 + \frac{c_2}{2}\right)\left(\rho_{\downarrow'} + \rho_{\uparrow'}\right)^2 + \frac{c_2}{2}\left(\rho_{\downarrow'}^2 - \rho_{\uparrow'}^2\right) + \left(c_2 + c'_{\uparrow,\downarrow}\right)\rho_{\downarrow'}\rho_{\uparrow'} + \delta\left(\rho_{\uparrow'} - \rho_{\downarrow'}\right)\right]$$



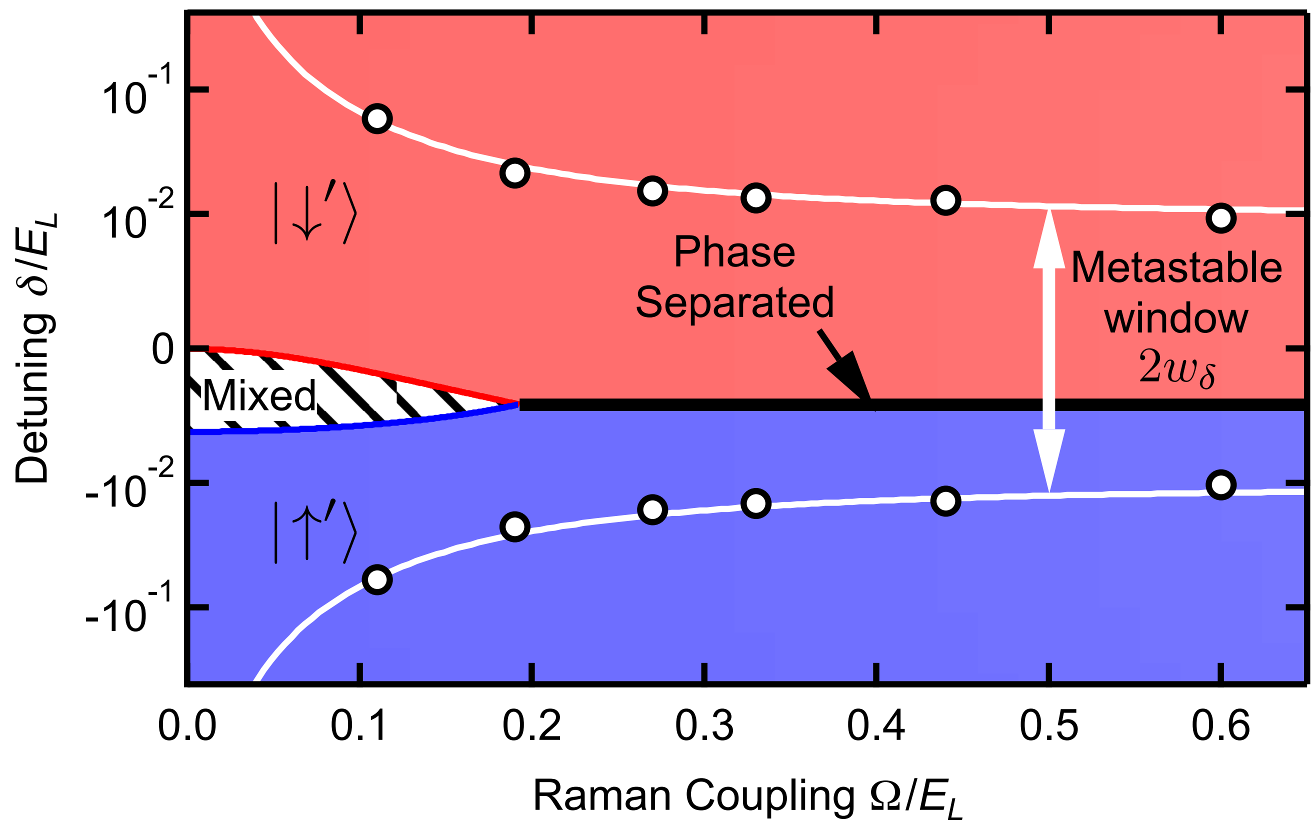

Fixed total spin or magnetization


Refs.

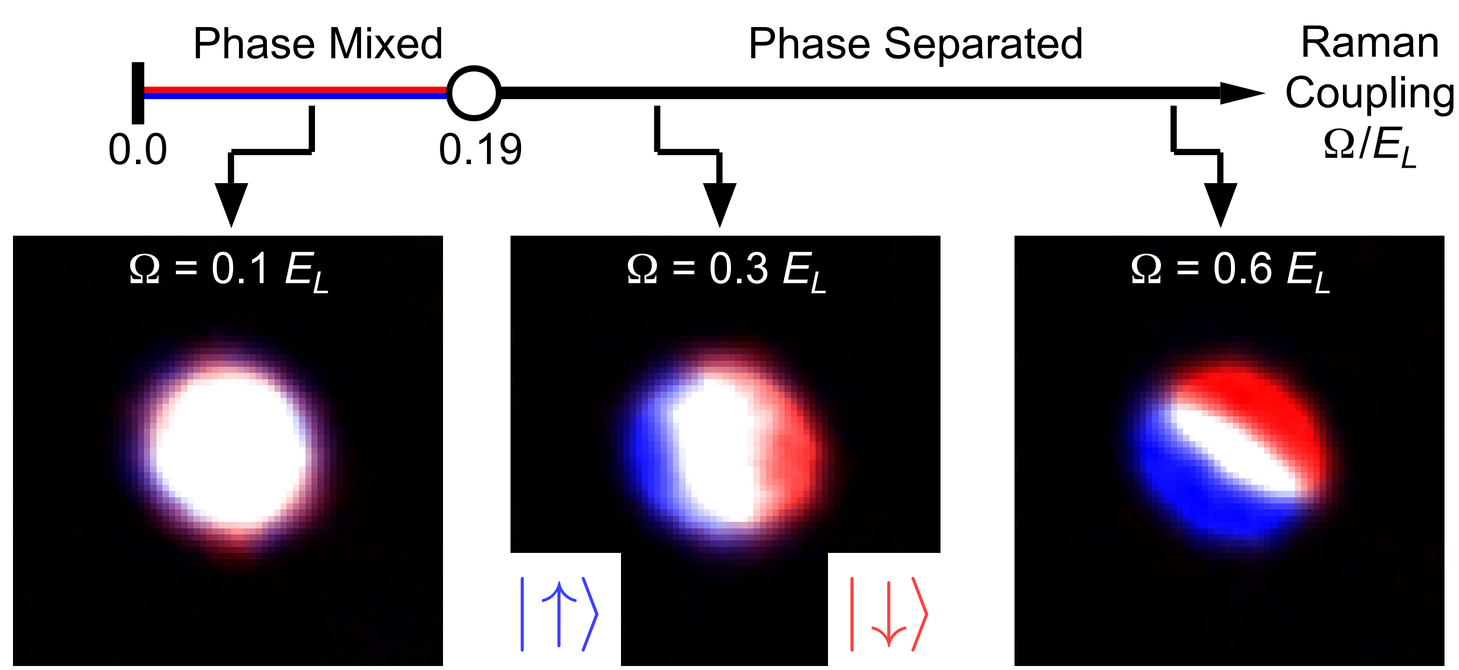

Y.-J. Lin et al, Nature (2011), C. Wang et al (arXiv:1006.5148), T.-L. Ho and S. Zhan (arXiv:1007.0650)

# Transition from miscible to immiscible

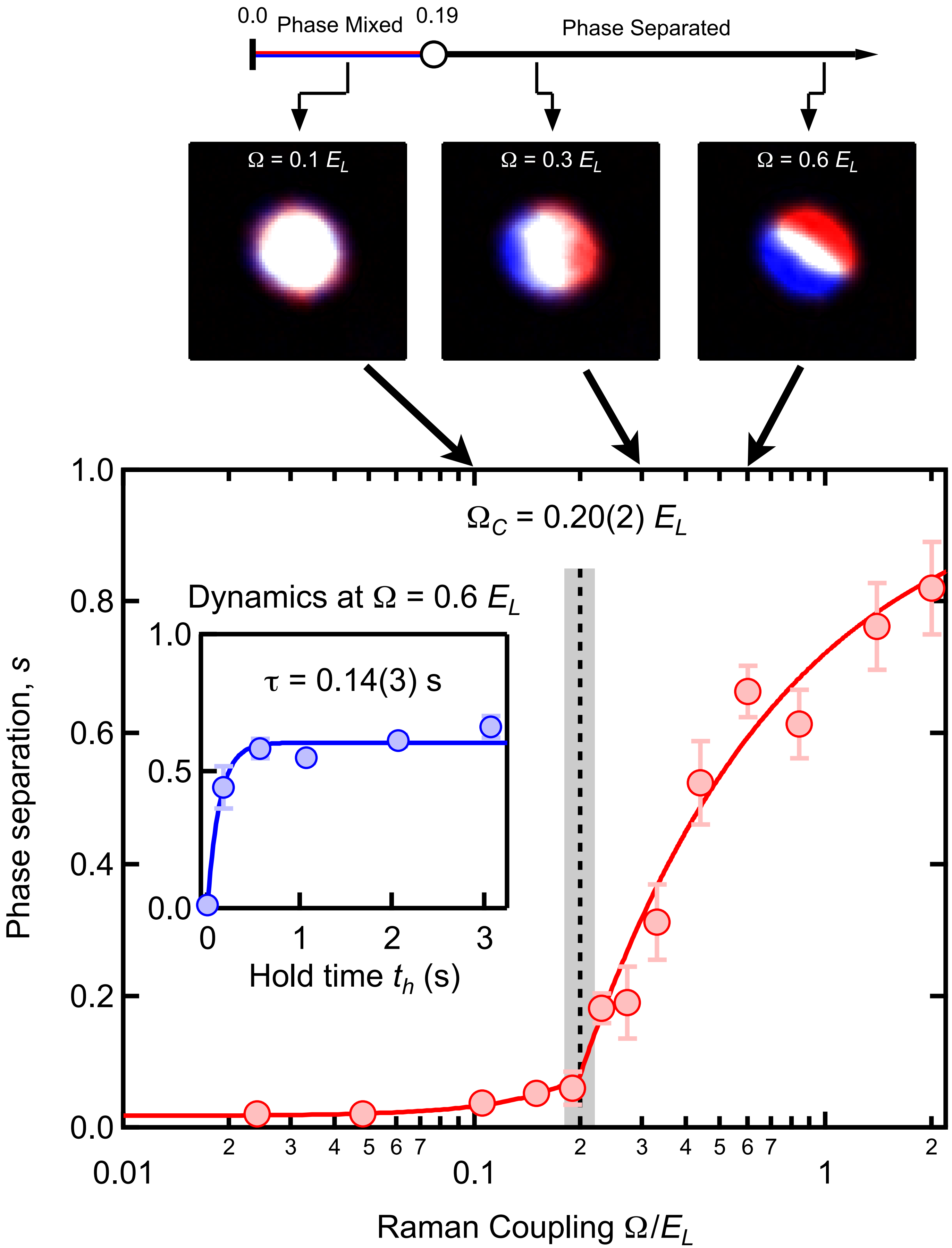
**b** Phase diagram, inset




**c** Miscible to immiscible transition



# Transition from miscible to immiscible




<u>A quantum phase transition</u>

Previously unexpected

<u>Our MFT prediction</u>

Phase separation at $\Omega = 0.19\, E_L$

<u>Refs.</u>

Y.-J. Lin et al, Nature (2011)
C. Wang et al (arXiv:1006.5148),
T.-L. Ho and S. Zhan (arXiv:1007.0650)