The two-component Mott insulator as an emulator of quantum magnetism

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Goal

Bose-Hubbard Hamiltonian for a two-component system:

$$H = -\sum_{\langle ij\rangle\sigma}\left(J_{\mu\sigma}a_{i\sigma}{}^{\dagger}a_{j\sigma} + H.c.\right) + \frac{1}{2}\sum_{i,\sigma}U_\sigma n_{i\sigma}(n_{i\sigma}-1) + U_{\uparrow\downarrow}\sum_i n_{i\uparrow}n_{i\downarrow}$$

2CMI $\longrightarrow$ **anisotropic-XXZ Heisenberg Hamiltonian**:

$$H = \sum_{\langle i,j\rangle}\left[\lambda_{\mu z}S_i^z S_j^z \pm \lambda_{\mu\perp}(S_i^x S_j^x + S_i^y S_j^y)\right]$$

Tunable experimental parameters:

$$\lambda_{\mu z} = \frac{J_{\mu\uparrow}^2 + J_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}} - \frac{J_{\mu\uparrow}^2}{U_\uparrow} - \frac{J_{\mu\downarrow}^2}{U_\downarrow}$$

$$\lambda_{\mu\perp} = \frac{J_{\mu\uparrow}J_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

Outline

Why Quantum Magnetism?

Simulation of spin Hamiltonians

Ferromagnetic phases

- Spin dynamics
- Proof-of-principle

Antiferromagnetic phases

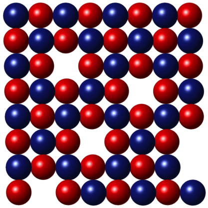
- Frustration
- Spin liquids
- RVB states
- High-T_c superconductivity $\leftrightarrow$ frustrated AF

Can these be modeled with atoms in optical lattices?

Condensed matter and dilute ultracold gases

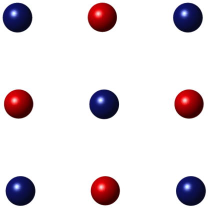
Normal matter (electrons in ionic lattice)

- Tightly packed atoms
- Complicated interactions
- Fixed parameters



"Artificial" matter (atoms in optical lattices)

- 10^8 times lower density
- Tunable interactions
- Controllable parameters



Condensed matter and dilute ultracold gases

Relative Scales

	YBCO	Rb in YAG lattice
Lattice Constant	10^{-9} m	5×10^{-7} m
Site Density	10^{21} cm^{-3}	10^{13} cm^{-3}
Interaction U	600 THz	700 Hz
Tunneling J	100 THz	20 Hz
Exchange J^2/U	1500 K	5×10^{-11} K
Néel temperature	300 K	2×10^{-10} K

B. Capogrosso-Sansone, S. Soyler, N. Prokof'ev, and B. Svistunov, Phys. Rev. A **81**, 053622 (2010)

Paths towards quantum magnetism

Anisotropic-XXZ Heisenberg Hamiltonian:

$$H = \sum_{\langle i,j \rangle} \left[\lambda_{\mu z} S_i^z S_j^z \pm \lambda_{\mu \perp} (S_i^x S_j^x + S_i^y S_j^y) \right] - B_z \sum_i S_i^z$$

- Fermions in harmonic traps
- Ultracold polar molecules in optical lattices
- Trapped ions
- Bosons in optical lattices (number or spin sector)

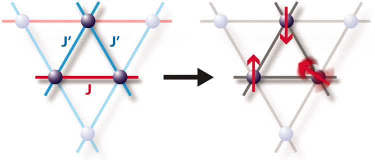
G. Jo *et al.*, Science, **325**, 1521 (2009)
A. V. Gorshkov *et al.*, Phys. Rev. Lett., **107**, 115301 (2011)
K. Kim *et al.*, Nature, **465**, 590 (2010)
J. Simon *et al.*, Nature, **472**, 307 (2011)
L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

Quantum emulation of classical magnetism

Classical vector spin:

$$S_i = [\cos(\theta_i), \sin(\theta_i)]$$

$$E(\{\theta_i\}) = -\sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j) = -\sum_{\langle i,j \rangle} J_{ij} S_i \cdot S_j$$



- Matrix elements J_{ij} assume the role of the spin-spin coupling
- $J_{ij} > 0$: ferromagnetic, $J_{ij} < 0$: antiferromagnetic
- No need for superexchange interaction permits higher temperatures

J. Struck *et al.*, Science, **333**, 996 (2011)

Gross-Pitaevskii equation

Time-dependent

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\left(r\right) + g|\Psi\left(\mathbf{r},t\right)|^2\right)\Psi\left(\mathbf{r},t\right)$$

Time-independent

$$\mu\Psi(\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\left(r\right) + g|\Psi\left(\mathbf{r}\right)|^2\right)\Psi\left(\mathbf{r}\right)$$

Chemical potential can be determined when we now the number of atoms $N = \int |\Psi(\mathbf{r})|^2 d^3 r$

$$g = \frac{4\pi\hbar^2\alpha_s}{m}$$

$\alpha_{Rb} = +5.2nm$(repulsive) and $\alpha_{Li} = -1.45nm$(attractive)*.

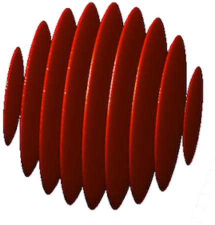
*Dalfovo *et al.*. Rev. Mod. Phys. **71**, 463 (1999)

Dipole traps for optical lattices

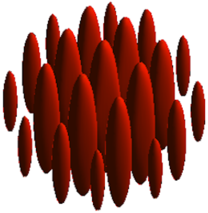
$$V\left(x,y,z\right)=\sum_{i=\{x,y,z\}}\left[V_i cos^2\left(\frac{\pi}{L_i}i\right)+\frac{m}{2}\omega_i^2 i^2\right]$$

For a far detuned optical lattice potential:

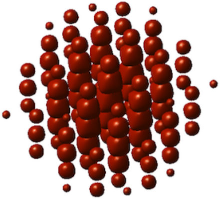
$$\frac{V_{lat}}{E_r}=\frac{2m}{\hbar^2 k^2}\frac{3\pi c^2}{2\omega_0^3}\frac{\Gamma}{\Delta}\frac{2P}{\pi w_0^2}$$



1-D



2-D



3-D

The Bose-Hubbard Hamiltonian

$$H = \int \psi^\dagger(r) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{latt}(r) + V_{trap}(r) \right) \psi(x) d^3r +$$
$$+ \frac{2\pi\alpha_s\hbar^2}{m} \int \psi^\dagger(r)\psi^\dagger(r)\psi(r)\psi(r) d^3r$$

The **Bose-Hubbard Hamiltonian**:

$$H = -J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \sum_i \epsilon_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right)$$

$\hat{a}_j$ and $\hat{a}_j^\dagger$:annihilation and creation operators.
$\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ is the number operator
J is the tunneling energy
U is the interaction energy due to $s-$wave scattering

M. P. A. Fisher *et al.*, Phys. Rev. B, **40**, 546 (1989)
D. Jaksch *et al.*, Phys. Rev. Lett., **81**, 3108 (1998)

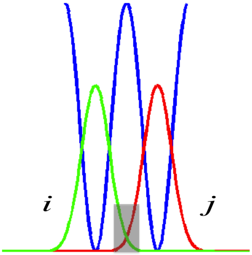
The tunneling energy J

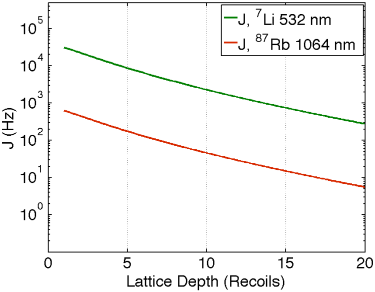
Hopping term:

$$H = -\frac{J}{2} \sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + a_i a_j^\dagger \right)$$

Tunneling matrix element

$$J = \int w^*(x - x_i) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{latt} \right] w(x - x_j) d^3x$$





The interaction energy U

Self-interaction term:

$$H = \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1\right)$$

This term deviates from G-P treatment for low $\langle n \rangle$
On-site interaction:

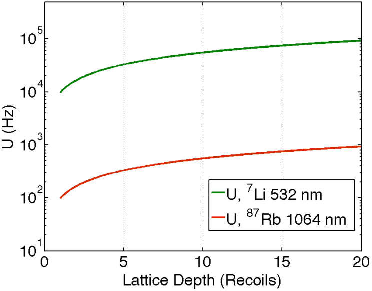
$$U = \frac{4\pi\alpha_s\hbar^2}{m} \int |w(x)|^4 d^3x$$

For ^{87}Rb in a $1064nm$ lattice:

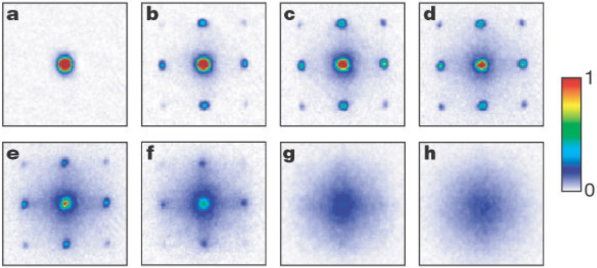
$$U_{\uparrow\uparrow} \approx U_{\downarrow\downarrow} \approx U_{\uparrow\downarrow}$$

and

$$\Lambda = \frac{\lambda_{latt}}{2} >> \alpha_s$$



Superfluid to Mott insulator transition



- Pure condensate for $0E_r$
- For low lattice energies a coherent periodic wavefunction
- For higher energies discrete isolated atoms

M. Greiner *et al.*, Nature, **415**, 39 (2002)

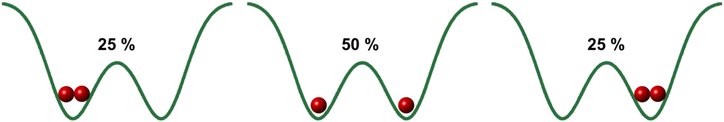
SF to MI in a double well

The symmetric: $|\phi_S\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle + |\phi_R\rangle)$

The anti-symmetric: $|\phi_A\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle - |\phi_R\rangle)$

Superfluid state:

$$\frac{1}{\sqrt{2}}(|\phi_L\rangle + |\phi_R\rangle) \otimes \frac{1}{\sqrt{2}}(|\phi_L\rangle + |\phi_R\rangle)$$



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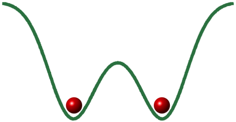
The anti-symmetric: $|\phi_A\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle - |\phi_R\rangle)$

Mott insulator state:

$$\frac{1}{\sqrt{2}}(|\phi_L\rangle \otimes |\phi_R\rangle) + \frac{1}{\sqrt{2}}(|\phi_R\rangle \otimes |\phi_L\rangle)$$



Superfluid and phase coherence

For $J >> U$ the many-body ground state for a homogeneous system is then given by:

$$\left|\Psi_{SF}\right\rangle_{U=0} \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger\right)^N \left|0\right\rangle$$

- All atoms occupy the identical extended Bloch state.
- A macroscopic wavefunction with long-range phase coherence throughout the lattice.
- Poissonian density fluctuations: $Var\left(n_i\right) = \left\langle \hat{n}_i \right\rangle$

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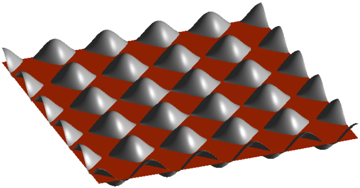

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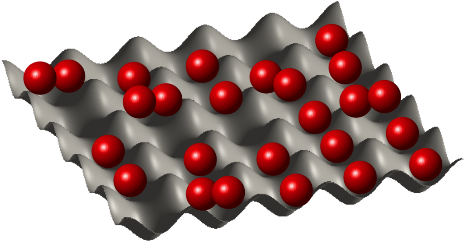

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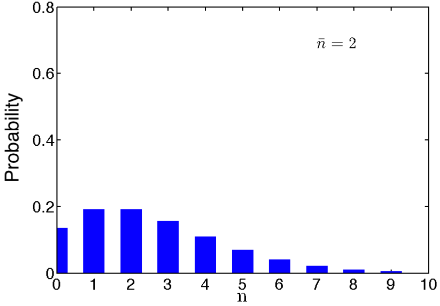
Density fluctuations and Poissonian statistics

$$|\Phi_i\rangle = \sum_{n=0}^{\infty} f_n^{(i)} |n\rangle \text{ and } \langle n\rangle = \sum_n n|f_n|^2$$

$$f_n = g^{\frac{n(n-1)}{2}} \frac{\bar{n}^{n/2}}{\sqrt{n!}}$$

Where g is a squeezing parameter depending on U/J.



Mott-insulator and number states

For $U >> J$ localized atomic wavefunctions with a fixed number of atoms per site. The ground state of the many-body system is given by:

$$|\Psi_{MI}\rangle_{J=0} \propto \prod_{i=1}^{M} \left(\hat{a}_i^\dagger\right)^n |0\rangle$$

- Gross-Pitaevskii equation or Bogoliubov's theory do not apply
- Perfect correlations in the atom number exist between sites
- Phase coherence is lost (Coherent $\Longrightarrow$ Fock states)

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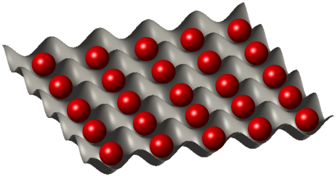

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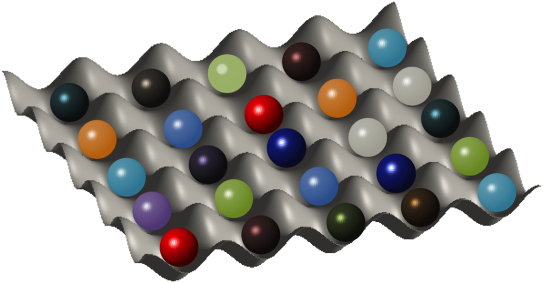

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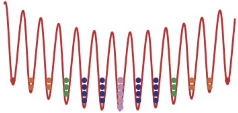
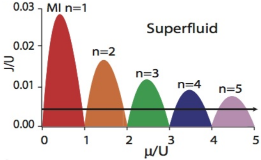
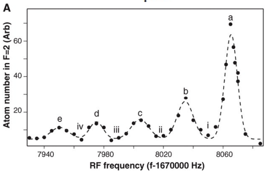
Phase diagram and the wedding cake

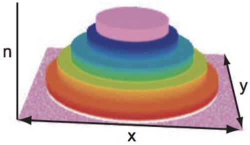


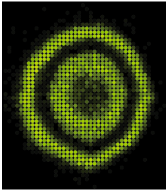
Phase diagram





Mott shells





G. K. Campbell *et al.*, Science, **313**, 649 (2006)
W . S. Bakr *et al.*, Science, **329**, 547 (2010)

The effect of temperature on the Mott insulator

On-site interaction energy: $E(n) = Un(n-1)/2$
Creating a particle costs energy: $E_{particle} = Un_0$
Creating a hole gives energy: $E_{hole} = -U(n_0 - 1)$
Partition function:

$$Z_0 = \sum_n exp(-\frac{E(n) - \mu n}{k_B T})$$

Local chemical potential μ:

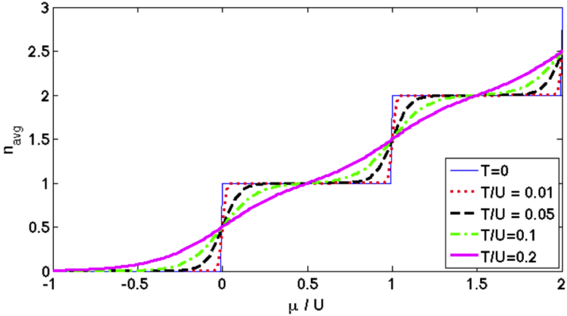
$$\mu(r) = \mu_0 - \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

Gerbier, F. ,Phys. Rev. Lett. **99**, 120405 (2007)
T.-L. Ho, & Q. Zhou, Phys. Rev. Lett., **99**, 120404 (2007)

The effect of temperature on the Mott insulator

Particle-hole approximation:

$$\bar{n} \approx n_0 + \left(e^{-\frac{E_{particle-\mu}}{k_B T}} - e^{-\frac{E_{hole+\mu}}{k_B T}}\right)/Z_0$$



- For $T = 0$: $n_0 = floor(\mu/U) + 1$
- Melting temperature $T_m \approx 0.2U/k_B$
- Similar with binary system of spin-gradient thermometry

Two component Mott insulator

Bose-Hubbard Hamiltonian for a two-component system:

$$H = -\sum_{\langle ij \rangle \sigma} \left(J_{\mu\sigma} {a_{i\sigma}}^{\dagger} a_{j\sigma} + H.c. \right) + \frac{1}{2} \sum_{i,\sigma} U_{\sigma} n_{i\sigma} (n_{i\sigma} - 1) + U_{\uparrow\downarrow} \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Tunneling energy:

$$J_{\mu\sigma} \approx \left(\frac{4}{\sqrt{\pi}}\right) E_R^{1/4} V_{\mu\sigma}^{3/4} exp \left[-2\sqrt{\frac{V_{\mu\sigma}}{E_R}} \right]$$

Strong dependence on lattice depth

L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

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Intraspecies interaction energies:

$$U_\sigma \approx \sqrt{8\pi} k \alpha_{s\sigma} (E_R \bar{V}_{x\sigma} \bar{V}_{y\sigma} \bar{V}_{z\sigma})^{1/4}$$

Interspecies interaction energy:

$$U_{\uparrow\downarrow} \approx \sqrt{8\pi} k \alpha_{s\uparrow\downarrow} (E_R \bar{V}_{x\uparrow\downarrow} \bar{V}_{y\uparrow\downarrow} \bar{V}_{z\uparrow\downarrow})^{1/4}$$

Spin average potential along each direction $\mu = x, y, z$:

$$\bar{V}_{\mu\uparrow\downarrow} = \frac{4 V_{\mu\uparrow} V_{\mu\downarrow}}{(V_{\mu\uparrow}^{1/2} + V_{\mu\downarrow}^{1/2})^2}$$

L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

Two component Mott insulator and quantum magnetism

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2CMI $\longrightarrow$ **anisotropic-XXZ Heisenberg Hamiltonian**:

$$H = \sum_{\langle i,j \rangle} \left[\lambda_{\mu z} S_i^z S_j^z \pm \lambda_{\mu\perp} (S_i^x S_j^x + S_i^y S_j^y) \right]$$

Tunable experimental parameters:

$$\lambda_{\mu z} = \frac{J_{\mu\uparrow}^2 + J_{\mu\downarrow}^2}{2 U_{\uparrow\downarrow}} - \frac{J_{\mu\uparrow}^2}{U_{\uparrow}} - \frac{J_{\mu\downarrow}^2}{U_{\downarrow}}$$

$$\lambda_{\mu\perp} = \frac{J_{\mu\uparrow} J_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

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Derivation of the Heisenberg Hamiltonian

Basic assumptions:

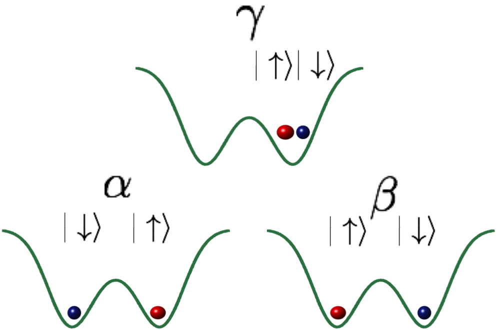
- Second order perturbation theory in the parameter $t/U \ll 1$
- Half-filling
- Our lowest-order effective Hamiltonians will involve only on-site and nearest-neighbor interactions
- Only the lowest one-particle state

$$H_{12} = H_{interaction} + H_{tunneling} = H_{12}^{(0)} + V_{12} =$$
$$= \frac{1}{2} \sum_{i,\sigma,\sigma'} U_{\sigma\sigma'} n_{i\sigma} n_{i\sigma'} - \sum_{\sigma} \left(t_\sigma a^\dagger_{1\sigma} a_{2\sigma+H.c} \right)$$

A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

Derivation of the Heisenberg Hamiltonian

$$(V'_{12})_{\alpha\beta} = -\sum_{\gamma} \frac{V_{\alpha\gamma} V_{\gamma\beta}}{E_\gamma - (E_\alpha + E_\beta)/2}$$



First order correction vanishes because we prohibit number changing tunneling:

$$\langle g|V|g\rangle = 0$$

A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

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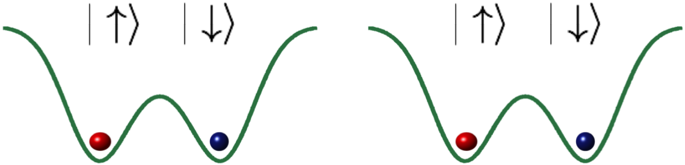
$$\left(V'_{12}\right)_{\alpha\beta} = -\sum_{\gamma} \frac{V_{\alpha\gamma} V_{\gamma\beta}}{E_\gamma - (E_\alpha + E_\beta)/2}$$

Matrix elements in terms of creation and annihilation operators:

$$\langle \uparrow\downarrow | V' | \uparrow\downarrow \rangle = -\frac{t_\uparrow^2}{U_{\uparrow\downarrow}} a^\dagger_{1\uparrow} a_{2\uparrow} a^\dagger_{2\uparrow} a^\dagger_{1\uparrow} - \frac{t_\downarrow^2}{U_{\uparrow\downarrow}} a^\dagger_{2\downarrow} a_{1\downarrow} a^\dagger_{1\downarrow} a_{2\downarrow}$$



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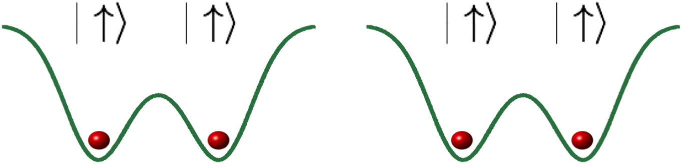


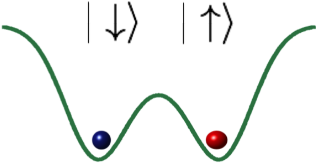
A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

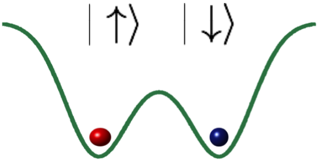
Derivation of the Heisenberg Hamiltonian

$$\left(V'_{12}\right)_{\alpha\beta} = -\sum_{\gamma} \frac{V_{\alpha\gamma} V_{\gamma\beta}}{E_\gamma - (E_\alpha + E_\beta)/2}$$

Matrix elements in terms of creation and annihilation operators:

$$\langle\downarrow\uparrow|V'|\uparrow\downarrow\rangle = -\frac{t_\uparrow t_\downarrow}{U_{\uparrow\downarrow}}\left(a^\dagger_{1\downarrow} a_{2\downarrow} a^\dagger_{2\uparrow} a_{1\uparrow} + a^\dagger_{2\uparrow} a_{1\uparrow} a^\dagger_{1\uparrow} a_{2\uparrow}\right)$$





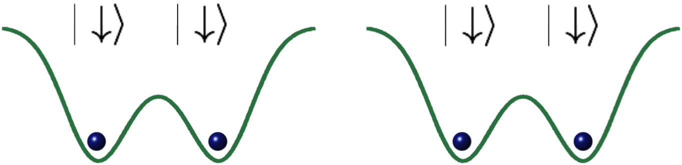
A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

Derivation of the Heisenberg Hamiltonian

$$\left(V'_{12}\right)_{\alpha\beta} = -\sum_{\gamma} \frac{V_{\alpha\gamma} V_{\gamma\beta}}{E_{\gamma} - (E_{\alpha} + E_{\beta})/2}$$

Matrix elements in terms of creation and annihilation operators:

$$\langle\downarrow\downarrow|V'|\downarrow\downarrow\rangle = -\frac{t_{\downarrow}^{2}}{U_{\downarrow\downarrow}}\left(a_{1\downarrow}^{\dagger} a_{2\downarrow} a_{2\downarrow}^{\dagger} a_{1\downarrow} + a_{2\downarrow}^{\dagger} a_{1\downarrow} a_{1\downarrow}^{\dagger} a_{2\downarrow}\right)$$



A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

Isospin operators

If we now express the previous relationships with the use of the isospin operators:

$$S^x = a_\uparrow^\dagger a_\downarrow + a_\downarrow^\dagger a_\uparrow$$
$$S^y = -i(a_\uparrow^\dagger a_\downarrow - a_\downarrow^\dagger a_\uparrow)$$
$$S^z = n_\uparrow - n_\downarrow$$

And on each site:

$$S^+ = (S^x + iS^y)/2$$
$$S^- = (S^x - iS^y)/2$$

$$B = \mu_\uparrow - \mu_\downarrow$$

A. B. Kuklov and B. V. Svistunov, Phys. Rev. Lett., **90**, 100401 (2003)

Two component Mott insulator and quantum magnetism

Bose-Hubbard Hamiltonian for a two-component system:

$$H = -\sum_{\langle ij\rangle\sigma}\left(J_{\mu\sigma} a_{i\sigma}{}^{\dagger} a_{j\sigma} + H.c.\right) + \frac{1}{2}\sum_{i,\sigma} U_{\sigma} n_{i\sigma}(n_{i\sigma}-1) + U_{\uparrow\downarrow}\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

2CMI $\longrightarrow$ **anisotropic-XXZ Heisenberg Hamiltonian**:

$$\boxed{H = \sum_{\langle i,j\rangle}\left[\lambda_{\mu z} S_i^z S_j^z - \lambda_{\mu\perp}(S_i^x S_j^x + S_i^y S_j^y)\right] - B_z \sum_i S_i^z}$$

Tunable experimental parameters:

$$\lambda_{\mu z} = \frac{J_{\mu\uparrow}^2 + J_{\mu\downarrow}^2}{2U_{\uparrow\downarrow}} - \frac{J_{\mu\uparrow}^2}{U_{\uparrow}} - \frac{J_{\mu\downarrow}^2}{U_{\downarrow}}$$

$$\lambda_{\mu\perp} = \frac{J_{\mu\uparrow} J_{\mu\downarrow}}{U_{\uparrow\downarrow}}$$

L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

Phase transitions in the XXZ model

$$H = \sum_{\langle i,j \rangle} \left[\lambda_{\mu z} S_i^z S_j^z - \lambda_{\mu \perp} (S_i^x S_j^x + S_i^y S_j^y) \right] - B_z \sum_i S_i^z$$

Trial wavefunctions:

- Antiferromagnetic Néel phase: $\langle S_i \rangle_{AF} = (-1)^i \hat{\mathbf{e}}_\mathbf{z}$
- Ferromagnetic phase: $\langle S_i \rangle_F = cos\theta \hat{\mathbf{e}}_\mathbf{x} + sin\theta \hat{\mathbf{e}}_\mathbf{z}$

The energy of each trial state will be:

$$E_{AF} = -\frac{N\lambda_z}{2}$$

$$E_F = \frac{N\lambda_z sin^2\theta}{2} - \frac{N\lambda_\perp cos^2\theta}{2} - B_z sin\theta$$

Phase transitions in the XXZ model

Energy for the *ferromagnetic* state is minimized when:

$$\frac{dE_F}{d\theta} = 0 = cos\theta \left[N(\lambda_z + \lambda_\perp) sin\theta - B_z\right]$$

- Z-ferromagnet for $\theta = \pi/2$
- Canted XY-ferromagnet for $\theta = \text{arcsin}\left[\frac{B_z}{N(\lambda_z+\lambda_\perp)}\right]$

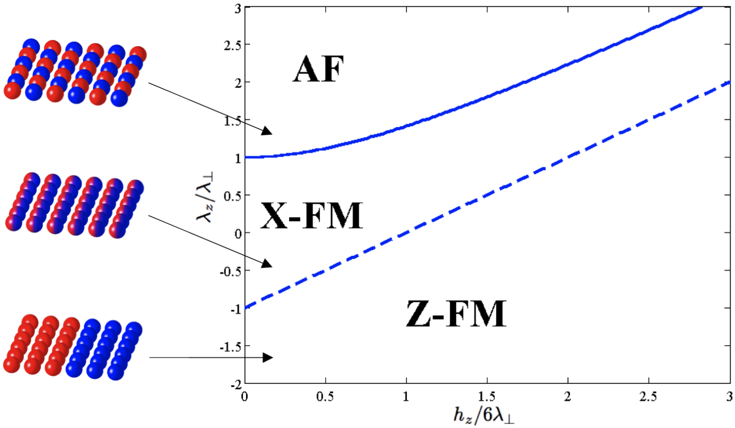
z-ferromagntet to xy-ferromagnet

We can calculate the phase diagram by equating the relevant energies:
For the z- to xy-ferromagnet transition:

$$
\begin{aligned}
E_{ZF} &= E_{XYF} \Rightarrow \\
\frac{N\lambda_z}{2} - B_z &= \frac{N\lambda_z}{2}\left(\frac{B_z^2}{N^2(\lambda_z+\lambda_\perp)^2}\right) \\
&- \frac{N\lambda_\perp}{2}\left(1 - \frac{B_z^2}{N^2(\lambda_z+\lambda_\perp)^2}\right) - \\
&- \frac{B_z^2}{N(\lambda_z+\lambda_\perp)^2}
\end{aligned}
$$

$$B_z = N(\lambda_z + \lambda_\perp)$$

Phase diagram for the XXZ model



L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

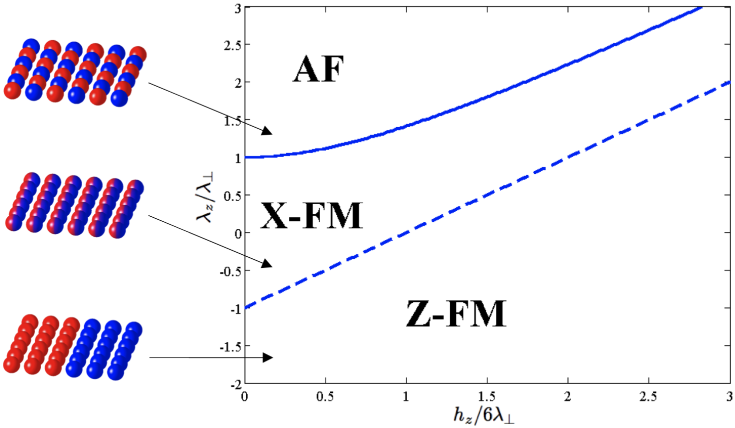
xy-ferromagntet to the antiferromagnet

Similarly for the boundary between the antiferromagnetic and xy-ferromagnetic phases:

$$
\begin{aligned}
E_{AF} &= E_{XYF} \Rightarrow \\
-\frac{N\lambda_z}{2} &= \frac{N\lambda_z}{2}\left(\frac{B_z^2}{N^2(\lambda_z+\lambda_\perp)^2}\right) \\
&\quad - \frac{N\lambda_\perp}{2}\left(1-\frac{B_z^2}{N^2(\lambda_z+\lambda_\perp)^2}\right) - \\
&\quad - \frac{B_z^2}{N(\lambda_z+\lambda_\perp)}
\end{aligned}
$$

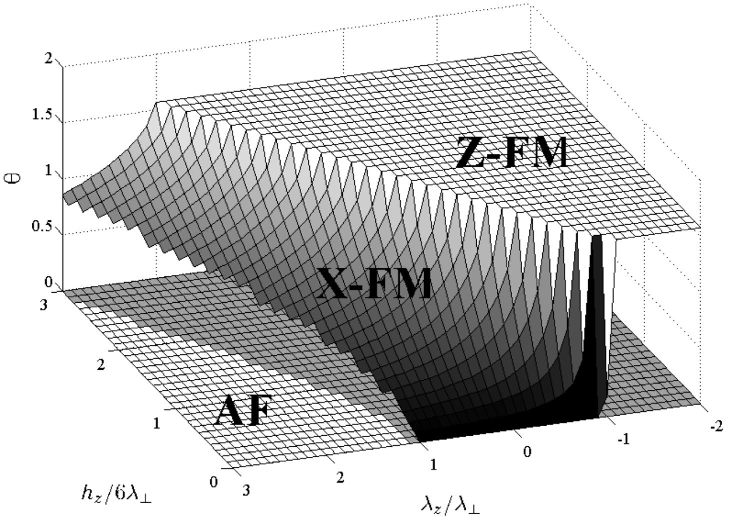
$$B_z^2 = N^2(\lambda_\perp - \lambda_z)^2$$

Phase diagram for the XXZ model



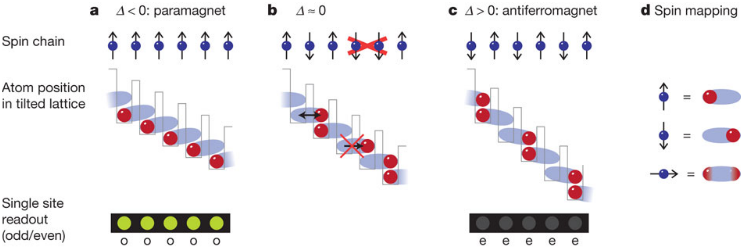
L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

Phase diagram for the XXZ model



L. Duan, E. Demler, and M. Lukin, Phys. Rev. Lett., **91**, 090402 (2003)

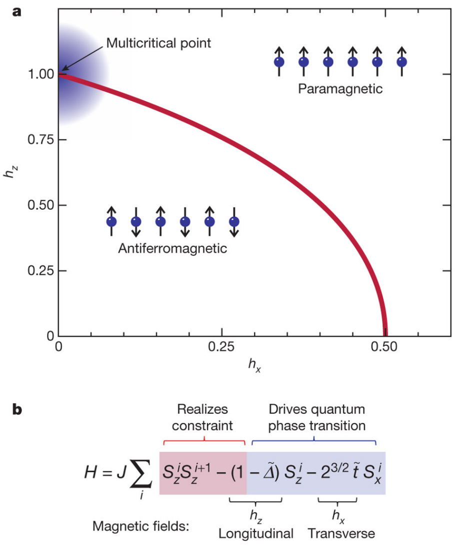
Antiferromagnetic spin chains in a tilted optical lattice



Single-component MI combined with a magnetic field gradient to achieve *resonant tunneling*.

S. Sachdev, K. Sengupta, & S. M. Girvin, Phys. Rev. B **66**, 075128 (2002)
J. Simon *et al.*, Nature, **472**, 307 (2011)

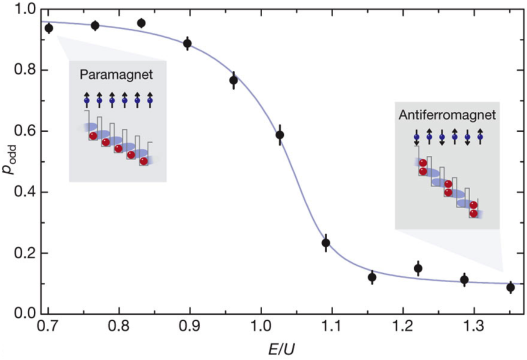
Antiferromagnetic spin chains in a tilted optical lattice



J. Simon *et al.*, Nature, **472**, 307 (2011)

Antiferromagnetic spin chains in a tilted optical lattice

AFM ordering in the number sector for a single-component MI:



J. Simon *et al.*, Nature, **472**, 307 (2011)

Superexchange interactions in an array of double wells

$$H = \sum_{\sigma=\uparrow,\downarrow} [-J(a^{\dagger}_{\sigma L} a_{\sigma R} + a^{\dagger}_{\sigma R} a_{\sigma L})] - \frac{1}{2}\Delta(n_{\sigma L} - n_{\sigma R}) + U(n_{\uparrow L} n_{\downarrow L} + n_{\uparrow R} n_{\downarrow R})$$

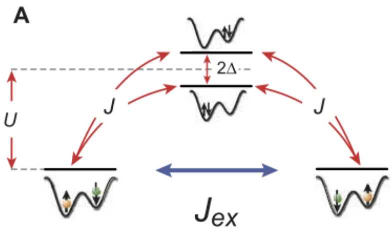
$$\boxed{H_{eff} = -J_{ex}(S_L^+ S_R^- + S_L^- S_R^+) - 2J_{ex} S_L^z S_R^z \text{ with } J_{eff} = \frac{2J^2 U}{U^2 - \Delta^2}}$$

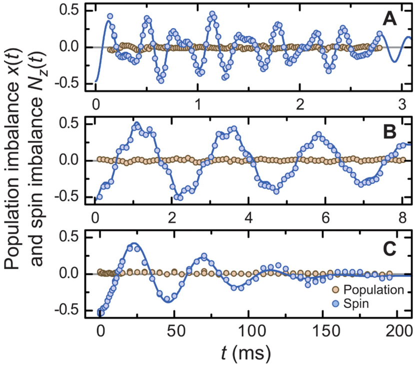
For the spin operators:

$$S^+_{L,R} = |\uparrow\rangle\langle\downarrow|_{L,R}$$
$$S^-_{L,R} = |\downarrow\rangle\langle\uparrow|_{L,R}$$
$$S^z_{L,R} = (n_{\uparrow L,R} - n_{\downarrow L,R})/2$$

S. Fölling *et al.*, Nature, **448**, 1029 (2007)
S. Trotzky *et al.*, Science, **319**, 295 (2008)

Superexchange interactions in an array of double wells





S. Fölling *et al.*, Nature, **448**, 1029 (2007)
S. Trotzky *et al.*, Science, **319**, 295 (2008)

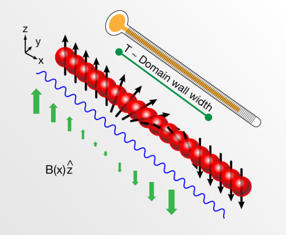
The necessary ingredients towards quantum magnetism

- Thermometry for the Mott state
- Reliable cooling
- Diagnostics of ordering
- Spin-interactions between nearest neighbors

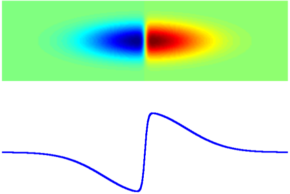
Spin-gradient thermometry in optical lattices

Mean spin $\langle s \rangle$ as a function of position x_i:

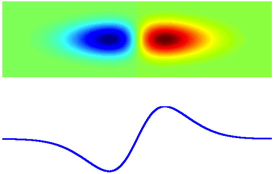
$$\langle s \rangle = tanh\left(-\frac{\Delta\mu_\sigma B\left(x_i\right)}{2k_B T}\right)$$



Low T



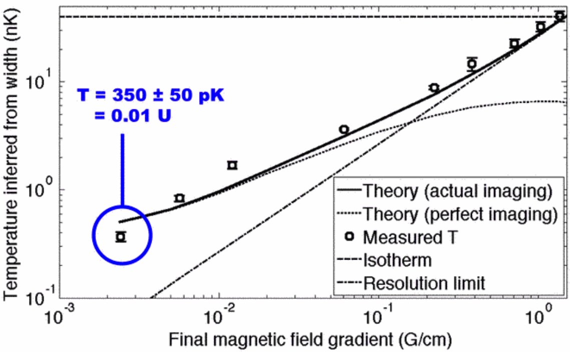
High T



D. M. Weld *et al.*, Phys. Rev. Lett., **103**, 245301 (2009)
A. M. Rey, Physics, **2**, 103 (2009)

$|\uparrow\rangle|\downarrow\rangle$ at the Newspin2

Adiabatic demagnetization cooling

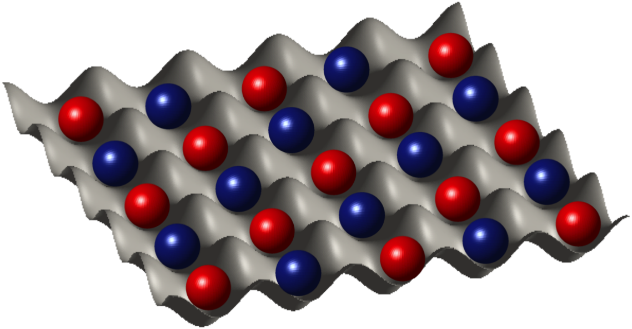


- Record low temperatures for an equilibrated gas
- Entropy redistribution between spin and kinetic degrees of freedom leads to cooling

P. Medley *et al.*, Phys. Rev. Lett., **106**, 195301 (2011)

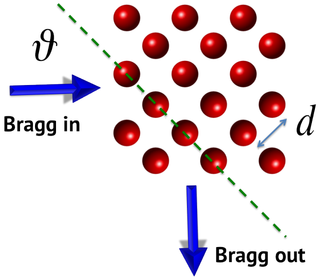
Bragg scattering from a crystal of ultracold atoms

Antiferromagnet

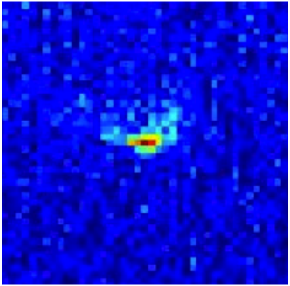


T. A. Corcovilos *et al.*, Phys. Rev. A, **81**, 013415 (2010)

Bragg scattering from a crystal of ultracold atoms



Bragg for $|F = 2, m_F = -2\rangle$



$$n\lambda = 2dsin\theta$$

$$d = 532nm, \lambda = 780nm, \theta = 47^\circ$$

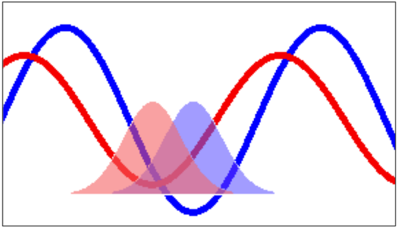
The atoms are in the lowest vibrational state

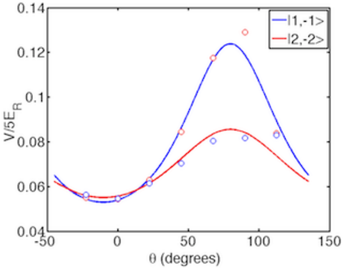
H. Miyake *et al.*, Phys. Rev. Lett., **107**, 175302 (2011)

Spin-dependent lattices

Concept: Control $U_{\uparrow\downarrow}$ by splitting σ_+ and σ_- by $\Delta x = \theta\lambda_x/(2\pi)$.

$$V_+(x,\theta) = V_0^+ cos^2(k_x x + \theta/2) \text{ and}$$
$$V_-(x,\theta) = V_0^- cos^2(k_x x - \theta/2)$$





O. Mandel *et al.*, Phys. Rev. Lett., **91**, 010407 (2003)

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