Who is the Lord of the Rings in the Zeeman-spin-orbit Saga:

Majorana, Dirac or Lifshitz?

Carlos A. R. Sa de Melo
Georgia Institute of Technology

College Station, Texas : December 15th, 2011

Acknowledgements

I would like to thank all the organizers, and in particular Jairo Sinova, for putting together such a nice conference, for their hospitality and for the opportunity to speak here today.

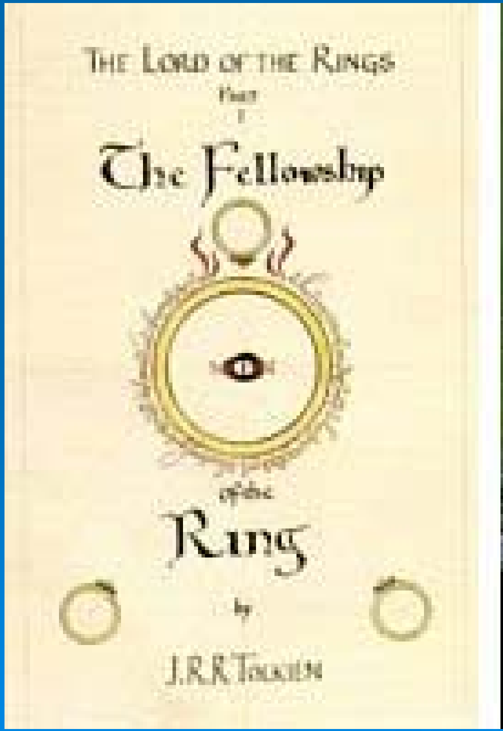
Also I would like to thank Ian Spielman for many discussions over the last several years.

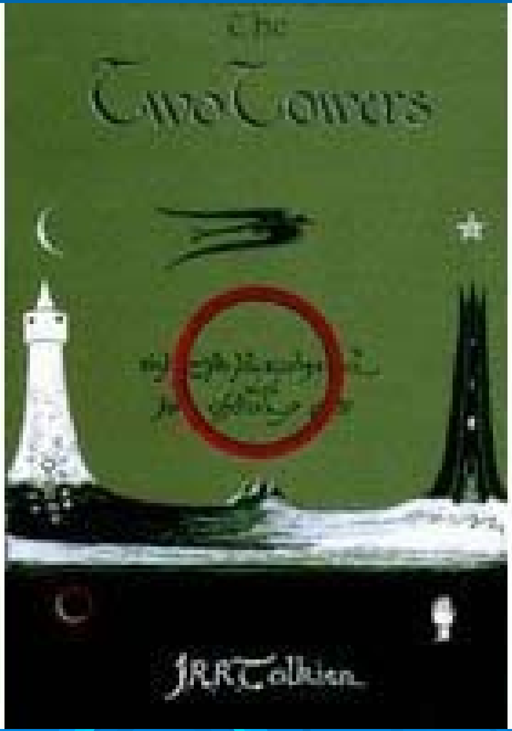
Acknowledgements

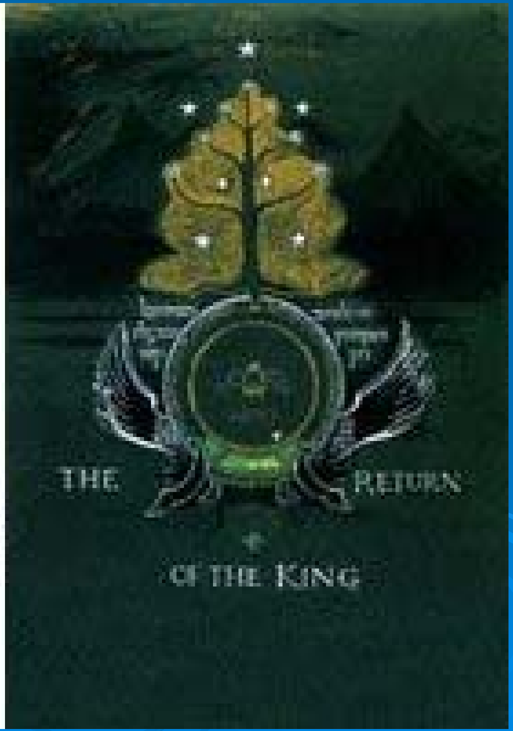
Kangjun Seo

Li Han

Who is the Lord of the Rings…



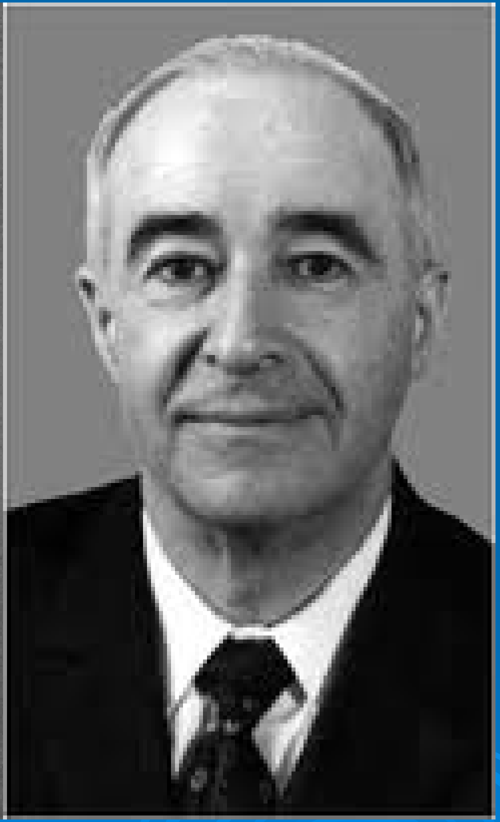




in the Zeeman-spin-orbit saga: Majorana, Dirac or Lifshitz?

Ettore Majorana

Paul Dirac

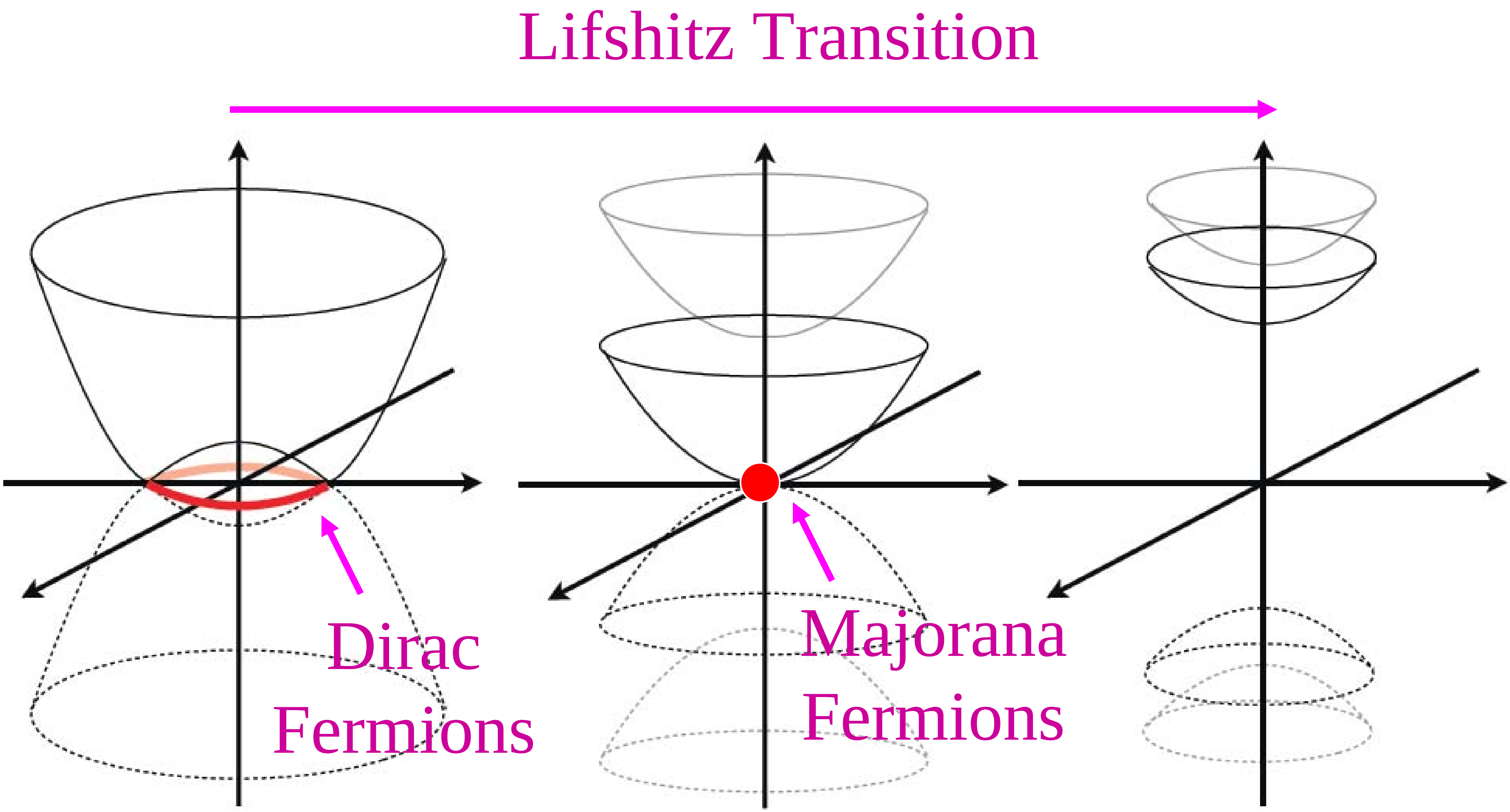


Ilya Lifshitz

Outline

- Introduction: BCS-to-BEC evolution
- What are topological phase transitions?
- Zeeman and spin-orbit effects in ultra-cold Fermi superfluids
- Conclusions.

Who is the Lord of the Rings?



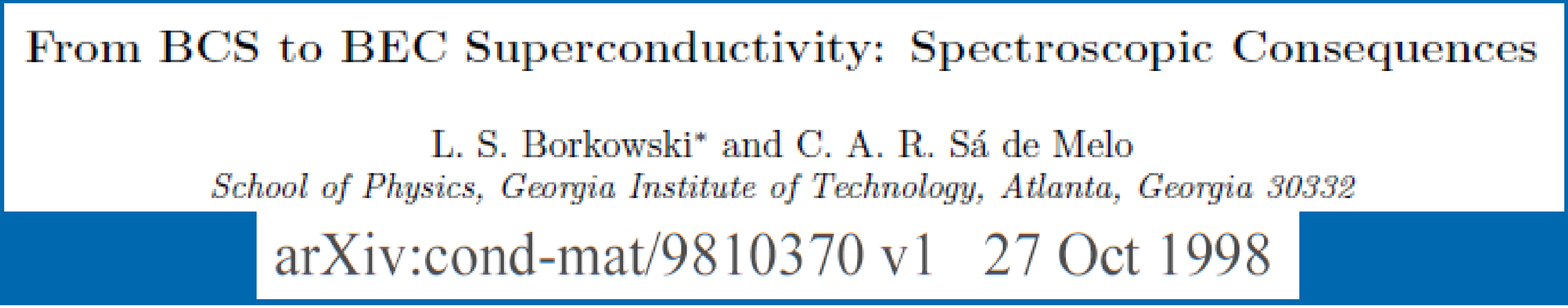
Conclusions

- The evolution from BCS to BEC superfluids in the presence of Zeeman, spin-orbit fields and interactions exhibit several topological phase transitions, where the symmetry of the superfluid order parameter does not change but the topology of excitations and of ground state properties change across phase boundaries.

- Such topological transitions belong to the Lifshitz class, where Dirac fermions disappear at phase boundaries creating gaps in the excitation spectrum in momentum space. In some of the phase transitions Dirac Fermions annihilate leading to the emergence of bulk Majorana Fermions as quasi-particles becomes massive.

- These momentum space topological phase transitions can be probed via spectroscopic properties such as the excitation spectrum, momentum distribution, spectral function , density of states, or via thermodynamic properties such as the compressibility matrix or the spin-susceptibility tensor.

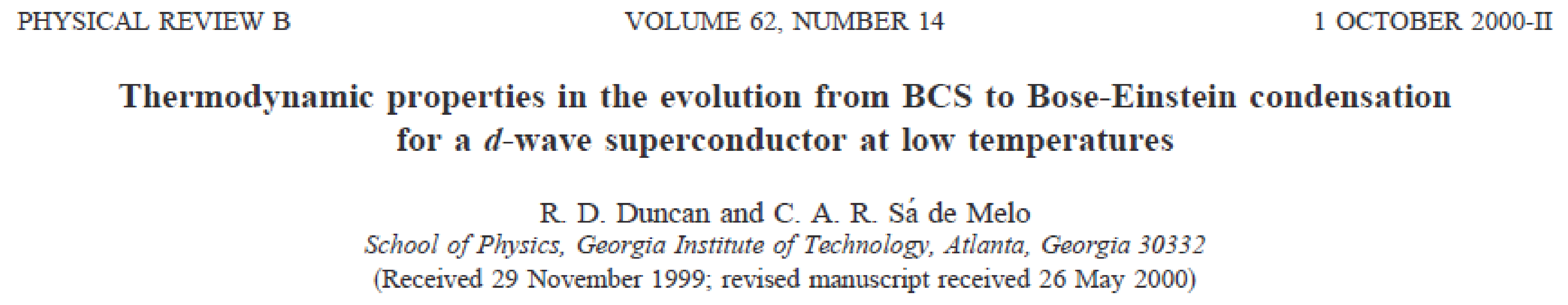
Some of our earlier work on topological superfluids



From BCS to BEC Superconductivity: Spectroscopic Consequences

L. S. Borkowski* and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332

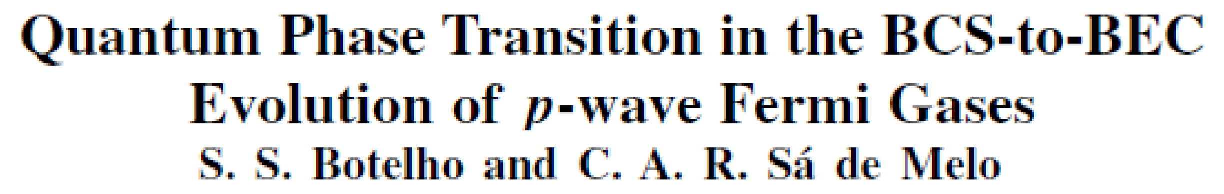
arXiv:cond-mat/9810370 v1 27 Oct 1998



PHYSICAL REVIEW B VOLUME 62, NUMBER 14 1 OCTOBER 2000-II

Thermodynamic properties in the evolution from BCS to Bose-Einstein condensation for a *d*-wave superconductor at low temperatures

R. D. Duncan and C. A. R. Sá de Melo
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332
(Received 29 November 1999; revised manuscript received 26 May 2000)



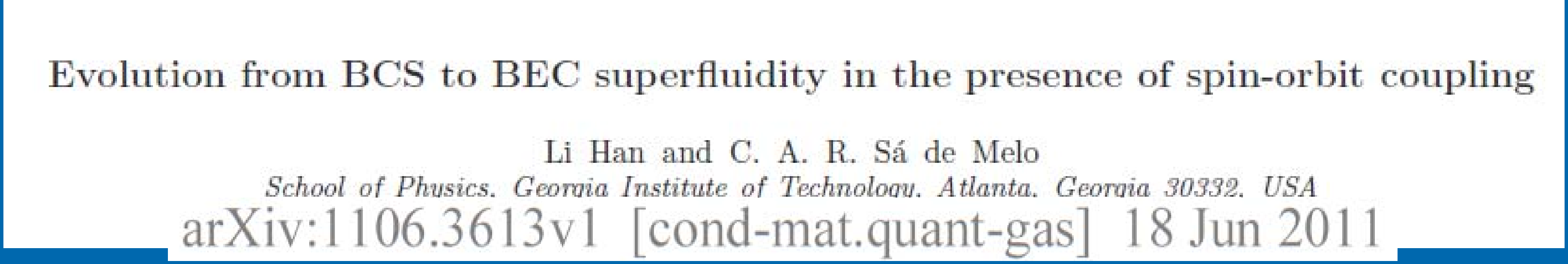
Quantum Phase Transition in the BCS-to-BEC Evolution of *p*-wave Fermi Gases

S. S. Botelho and C. A. R. Sá de Melo

JLTP 140, 409 (2005)

G. E. Volovik’s book “Exotic Properties of Superfluid ^{3}He” (1992).

Main References for Talk



Evolution from BCS to BEC superfluidity in the presence of spin-orbit coupling

Li Han and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

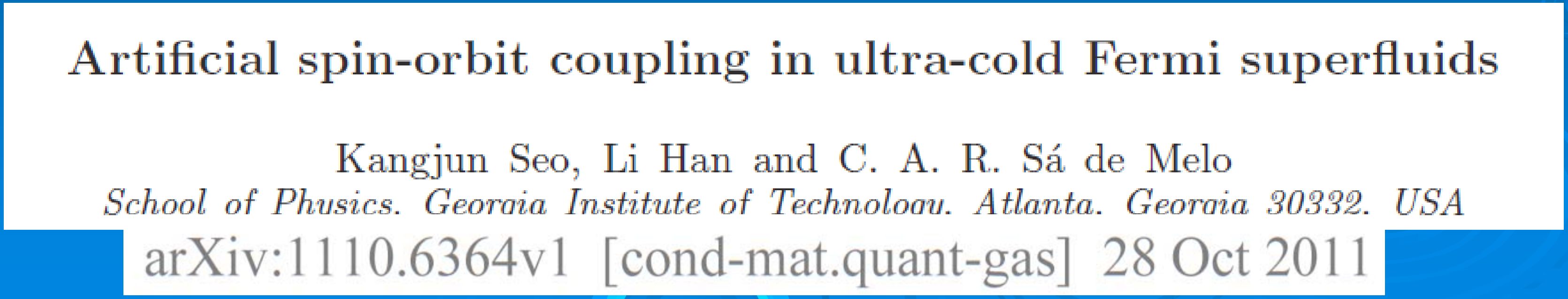
arXiv:1106.3613v1 [cond-mat.quant-gas] 18 Jun 2011

Topological phase transitions in ultra-cold Fermi superfluids: the evolution from BCS to BEC under artificial spin-orbit fields

Kangjun Seo, Li Han and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

arXiv:1108.4068v2 [cond-mat.quant-gas] 23 Aug 2011



Artificial spin-orbit coupling in ultra-cold Fermi superfluids

Kangjun Seo, Li Han and C. A. R. Sá de Melo

School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

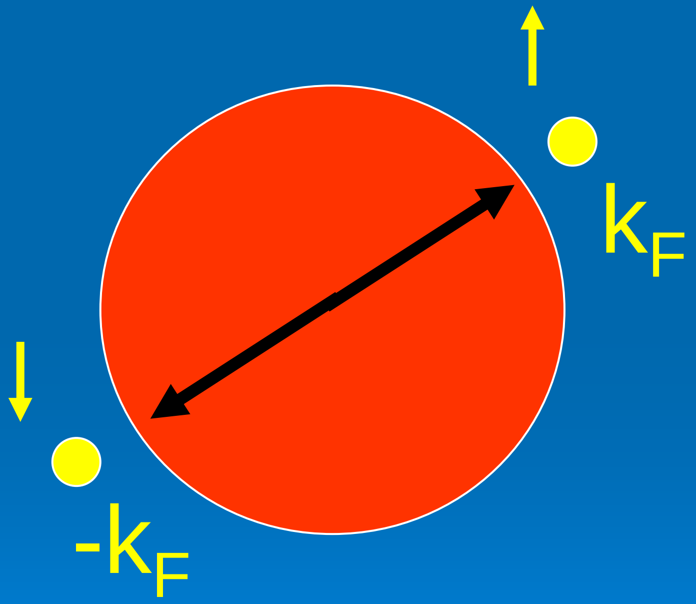
arXiv:1110.6364v1 [cond-mat.quant-gas] 28 Oct 2011

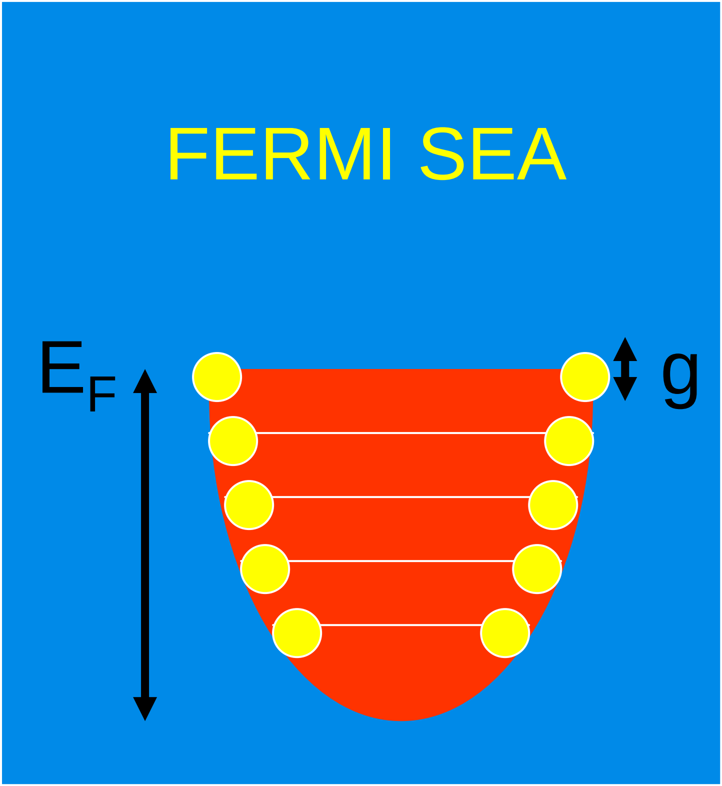
Outline

- Introduction: BCS-to-BEC evolution
- What are topological phase transitions?
- Zeeman and spin-orbit effects in ultra-cold Fermi superfluids
- Conclusions

BCS Pairing ($g \ll E_F$ or $k_F a_s \rightarrow 0^-$)

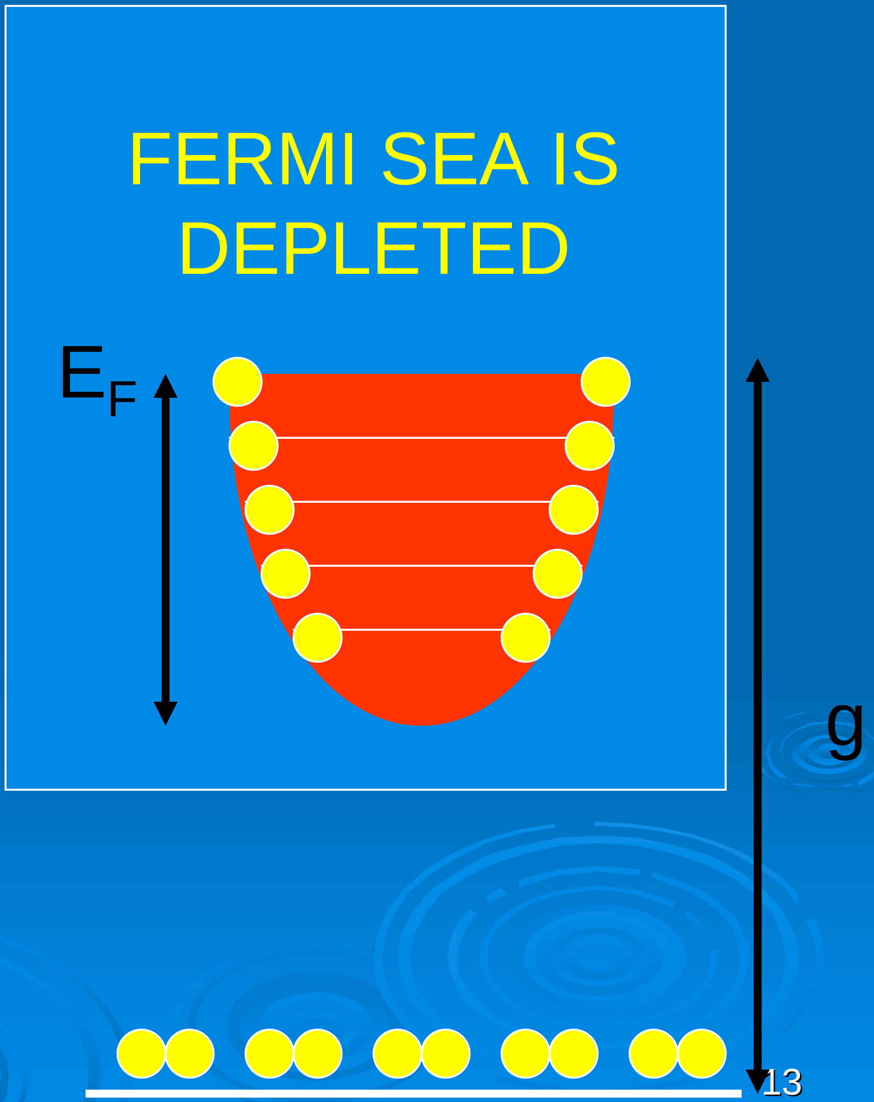
$\mu = E_F > 0$





BEC Pairing (g >> E_F or $k_F a_s \rightarrow 0^+$)

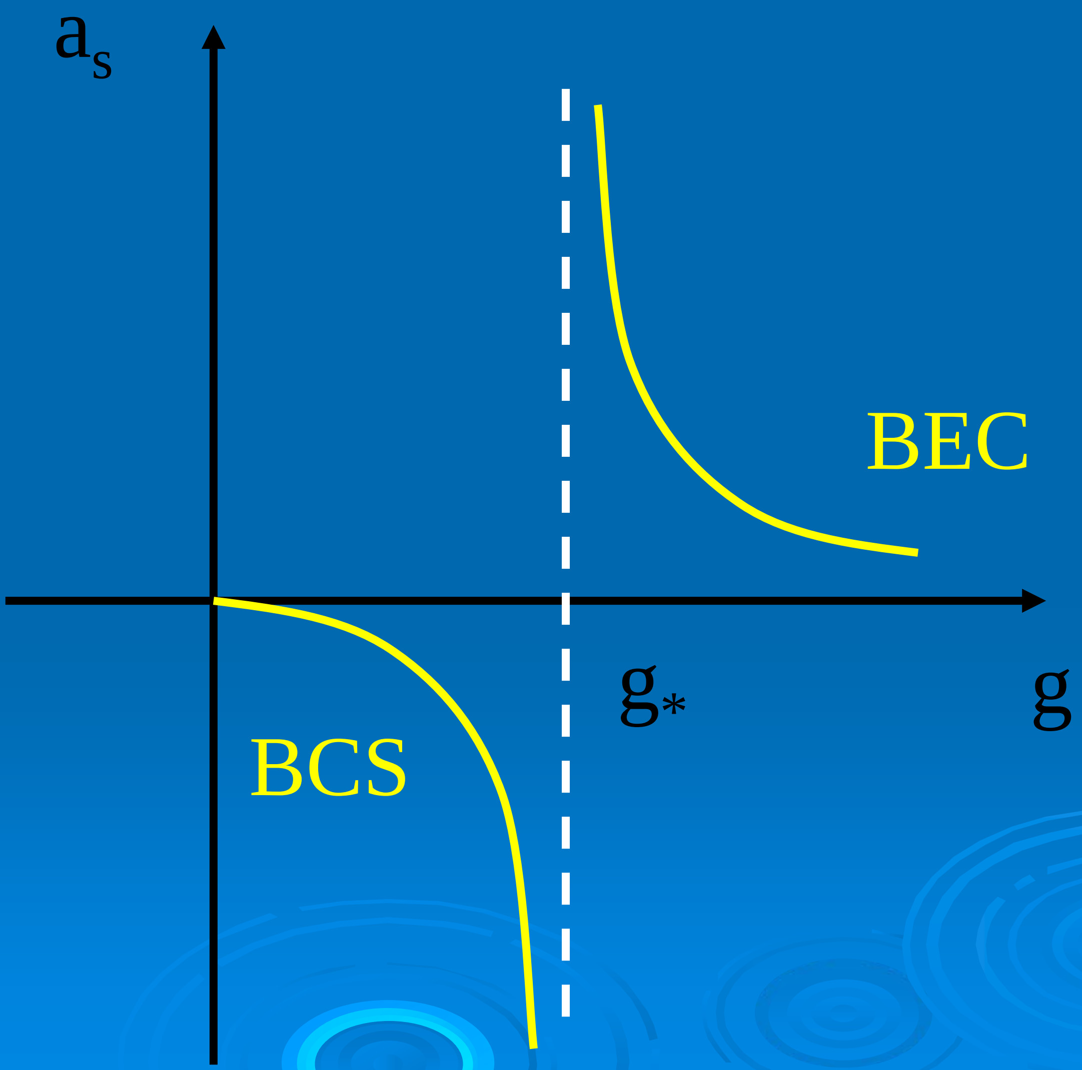
Weakly interacting gas of tightly bound “Cooper” molecules



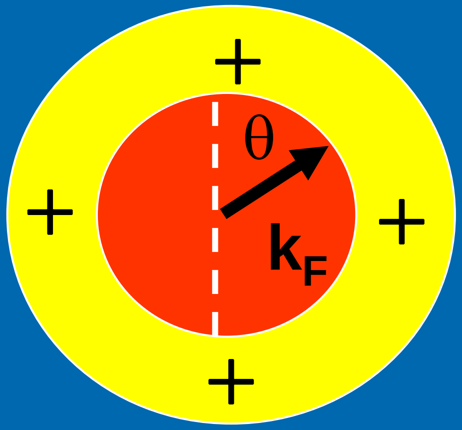
FERMI SEA IS DEPLETED

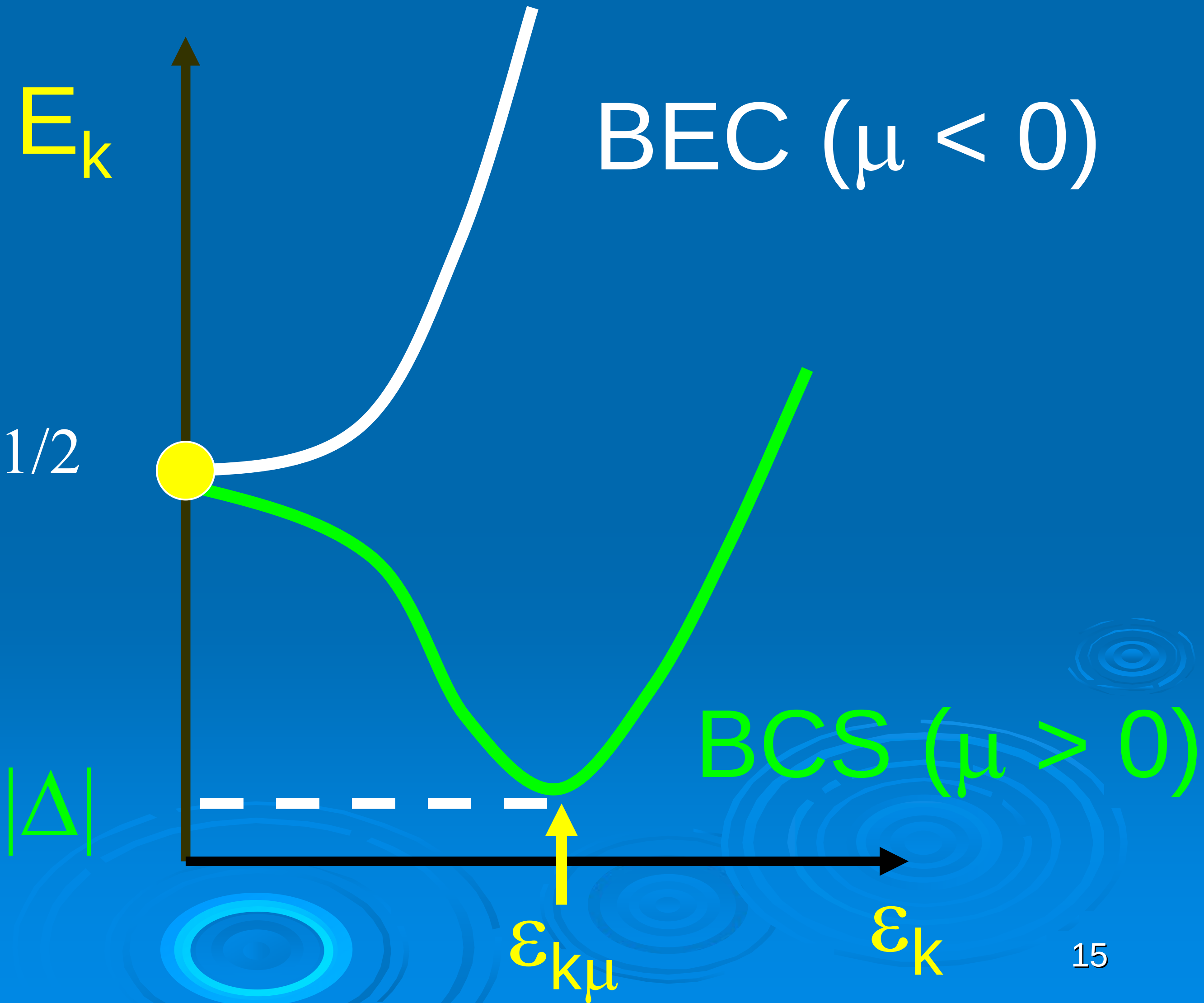
$2\mu = -E_b < 0$

Scattering Length

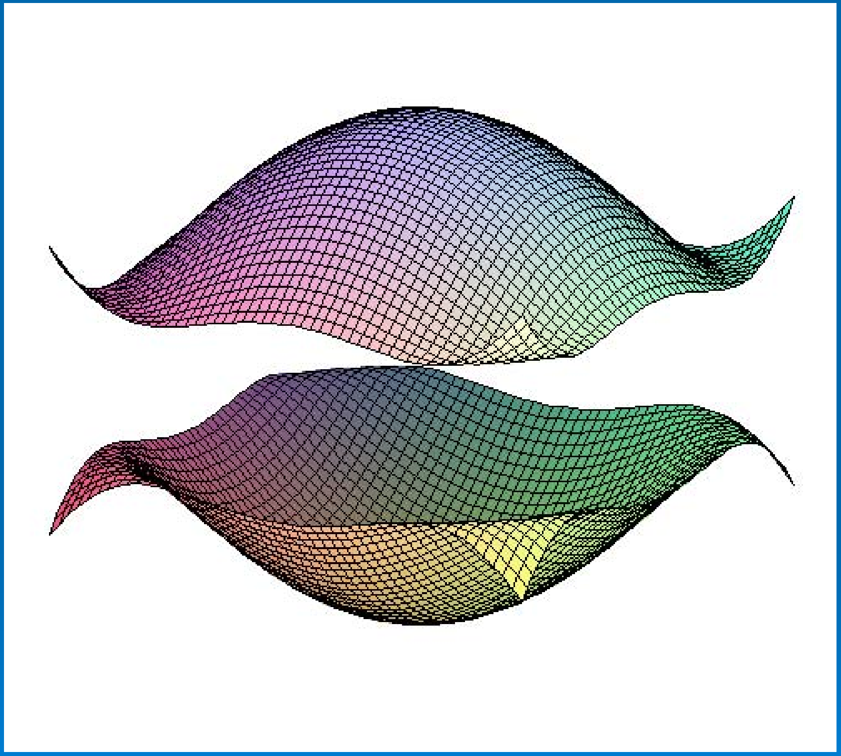


$$E(k) = [(\varepsilon_k - \mu)^2 + \Delta^2]^{1/2}$$

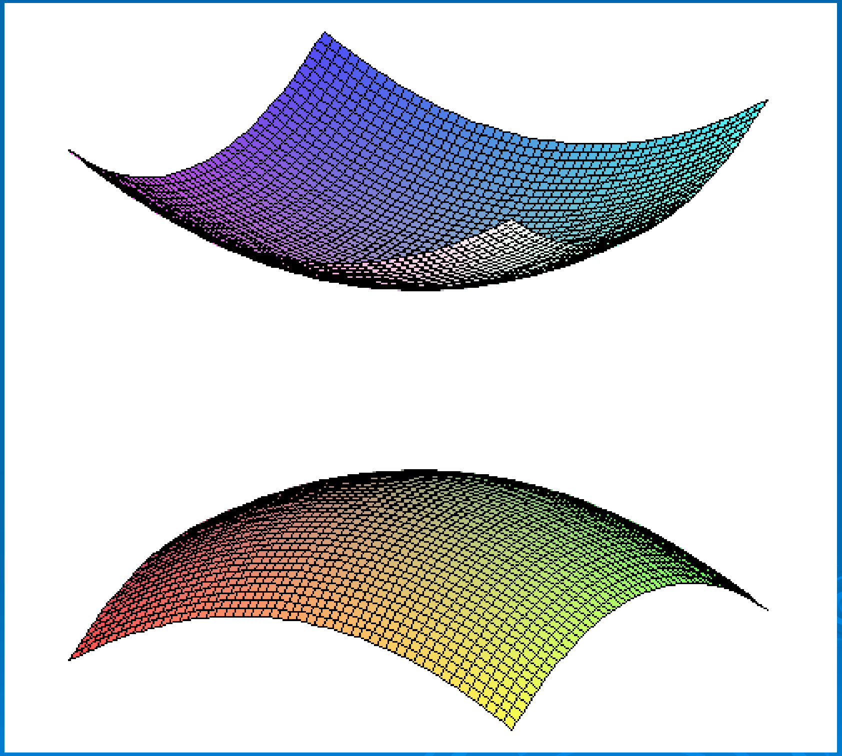




E(k) at T = 0 and k_x = 0 (S-wave)



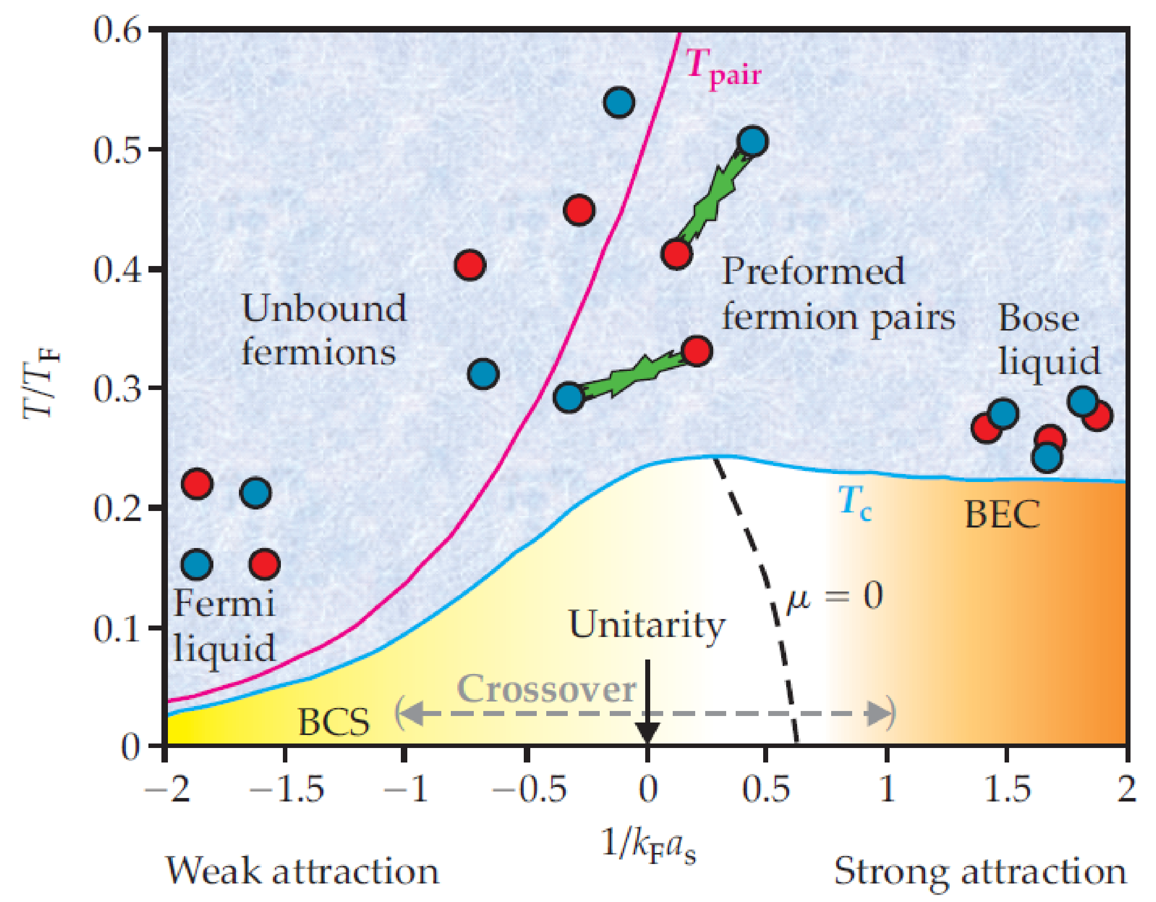
$\mu > 0$



$\mu < 0$

Same Topology

BCS-BEC evolution in 3D for s-wave is just a crossover.



Physics Today (2008)

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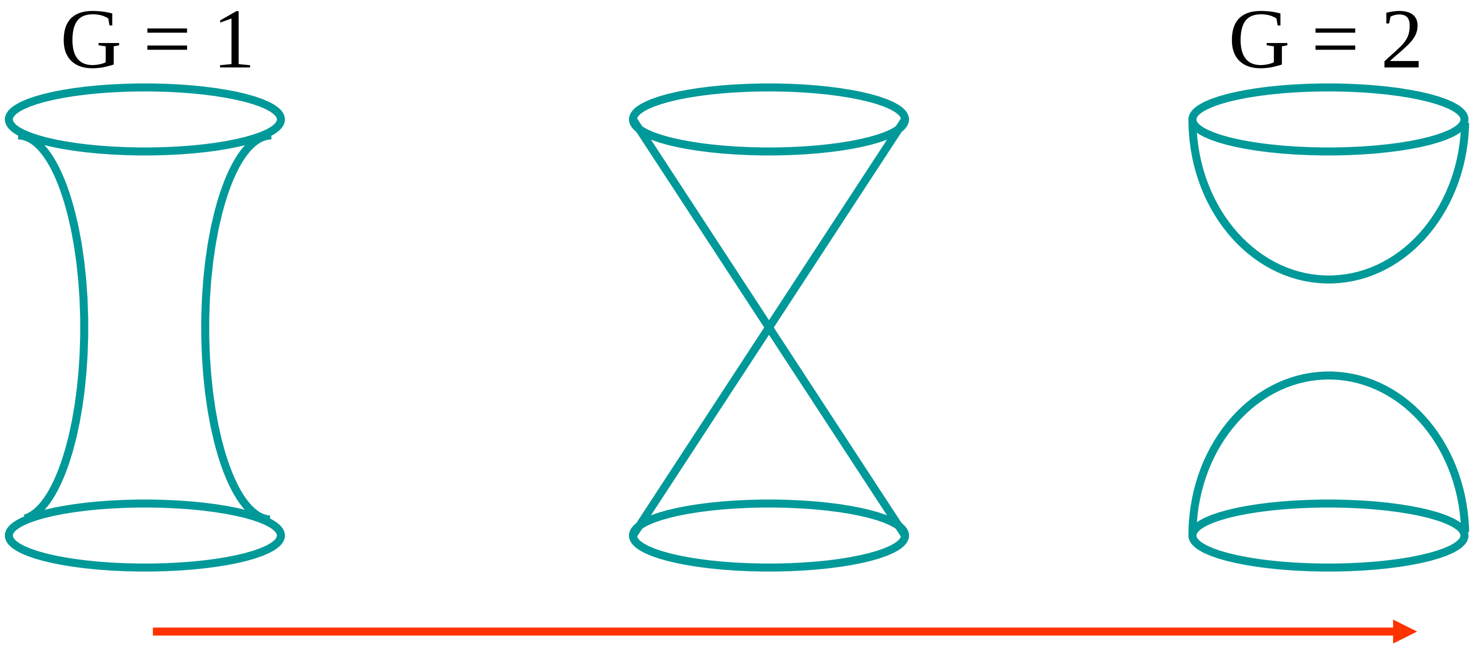
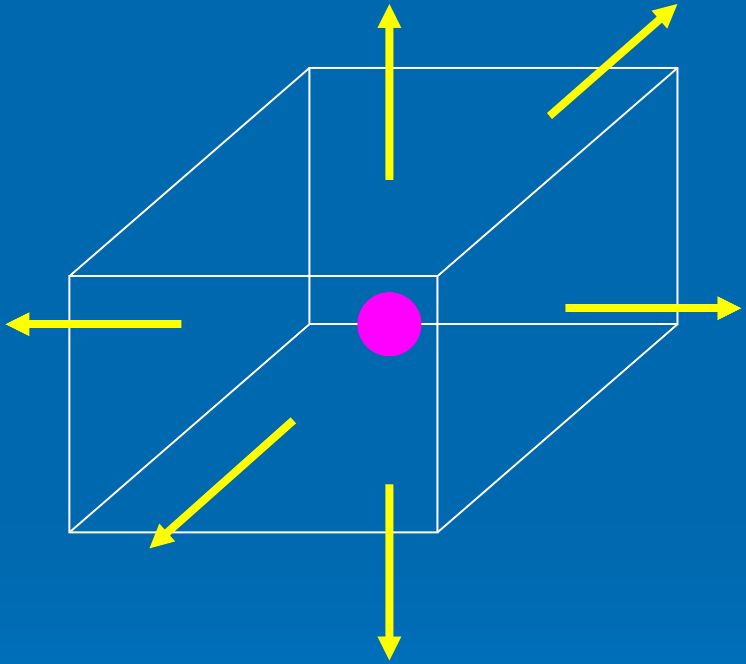
Connection to the Lifshitz Transition (Weakly or non-interacting Fermi Systems)



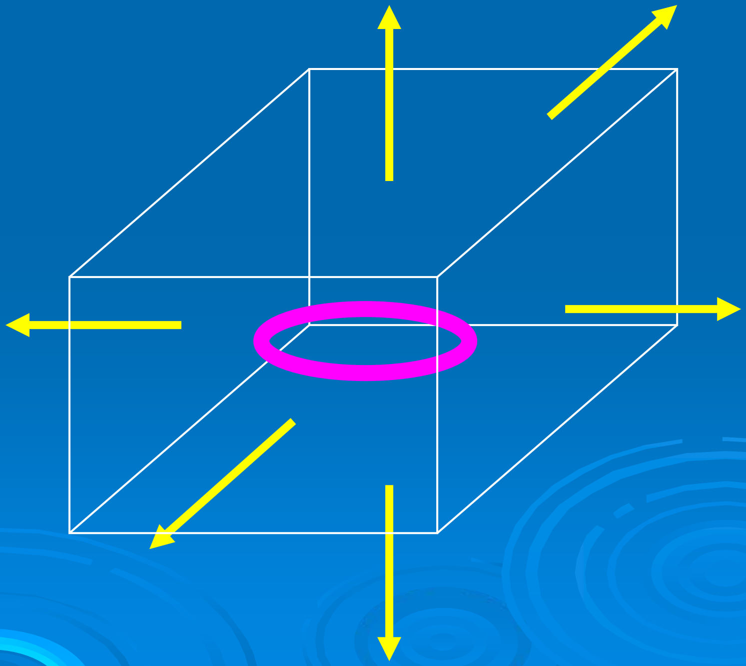
Increasing Pressure

Change in topology of the Fermi surface changes density of states and thus alters thermodynamic properties.

Topological Invariants:
Gauss' Law and Electric Charge



$$\oint d\vec{S}.(\varepsilon_0 \vec{E}) = q$$



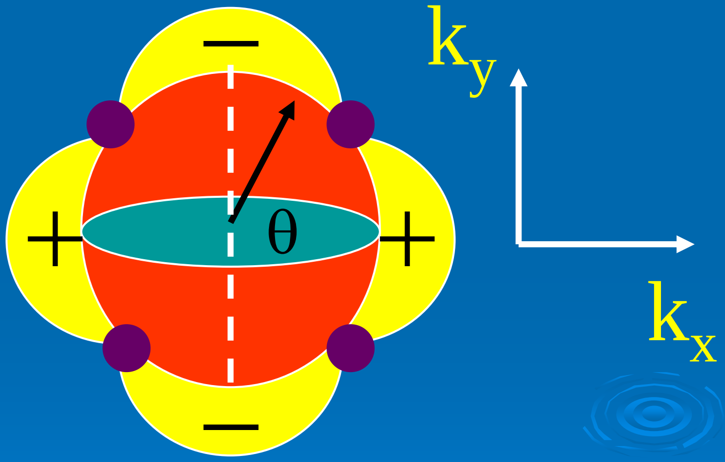
$$\nabla.(\varepsilon_0 \vec{E}) = \rho$$

d-wave Superconductors

Order Parameter $\Delta(k) = \Delta_0 (k_x{}^2 - k_y{}^2)$

$k_x = |k|\cos\theta$

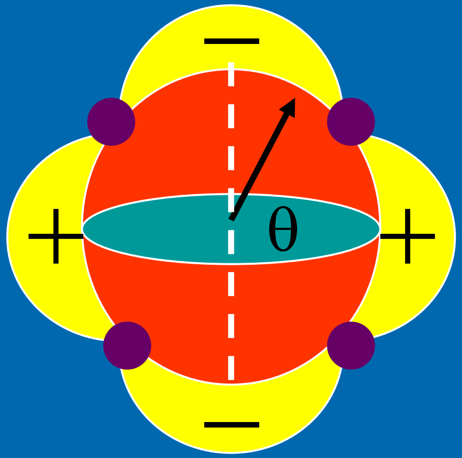
$k_y = |k|\sin\theta$

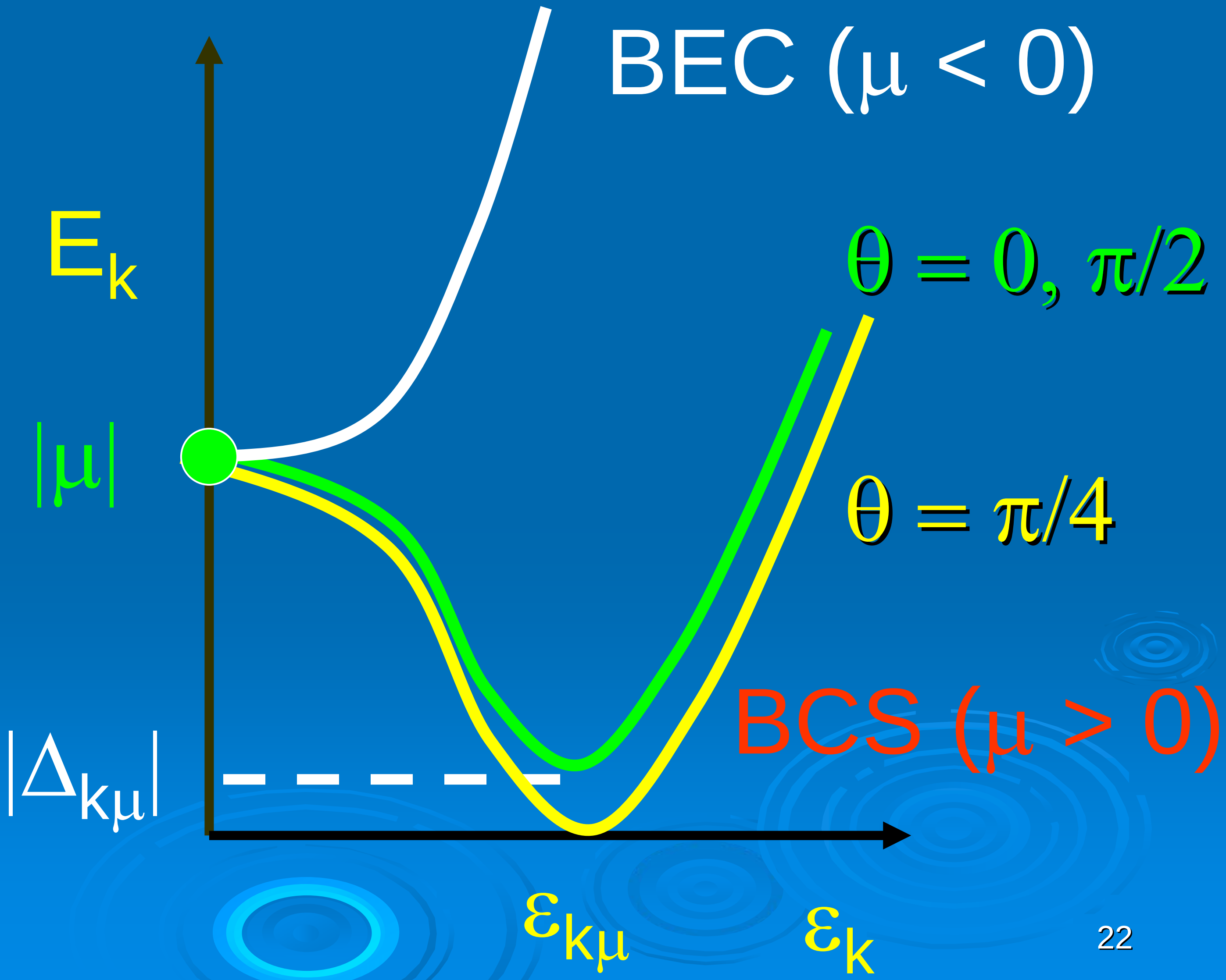


Excitation Spectrum

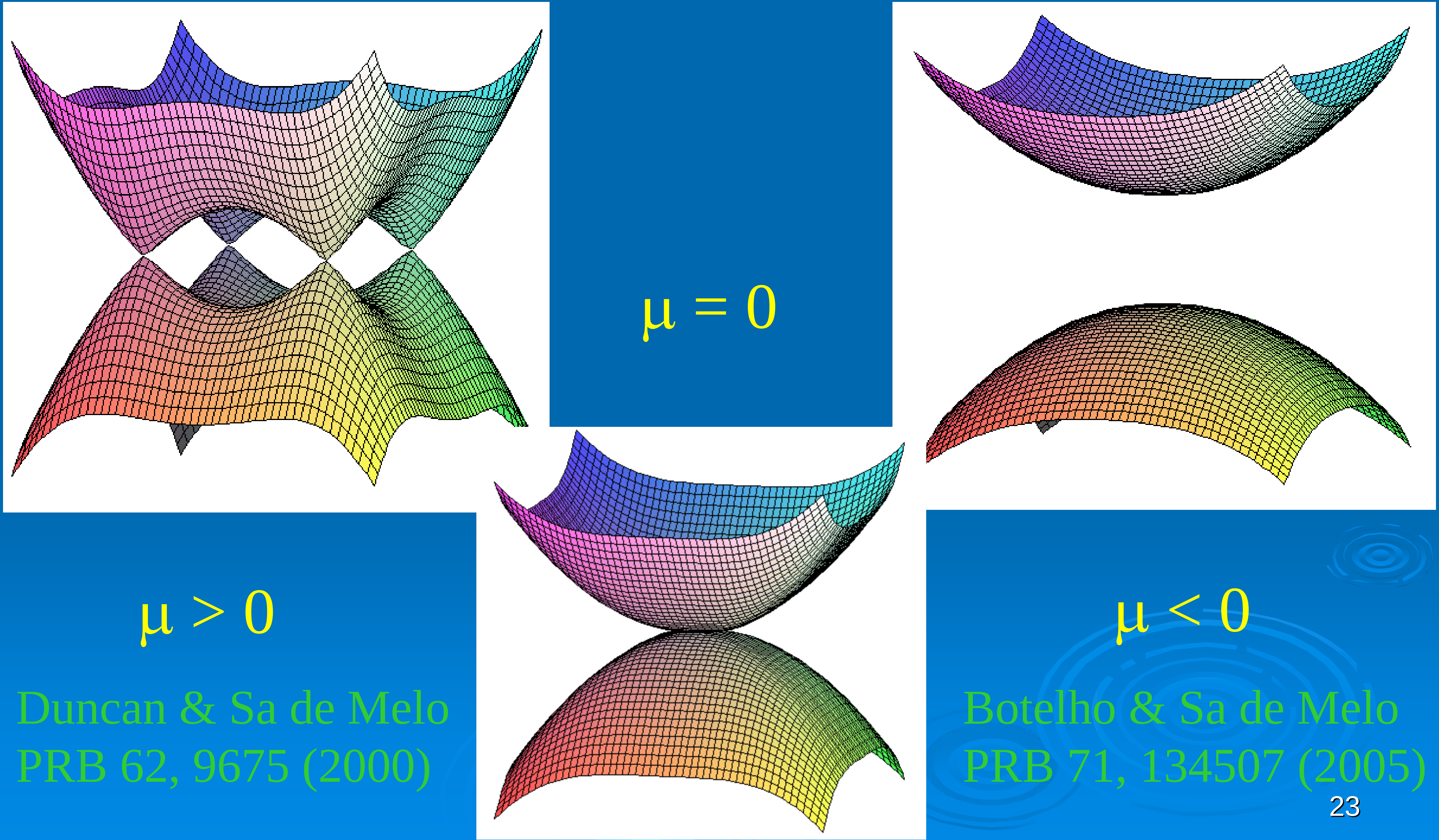
$$E(k) = [(\varepsilon_k - \mu)^2 + |\Delta(k)|^2]^{1/2}$$

$$E(k) = [(\varepsilon_k - \mu)^2 + \Delta_0{}^2|k|^2 (\cos^2\theta - \sin^2\theta)^2]^{1/2}$$





Lifshitz transition
in d-wave superfluids



$\mu > 0$

Duncan & Sa de Melo
PRB 62, 9675 (2000)

$\mu = 0$

$\mu < 0$

Botelho & Sa de Melo
PRB 71, 134507 (2005)

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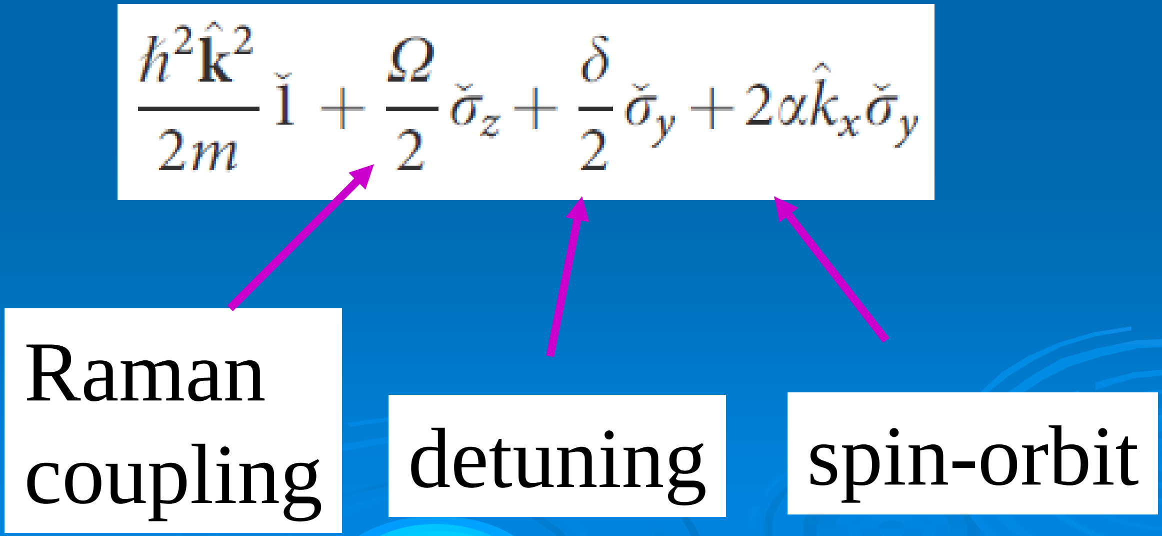
LETTER

doi:10.1038/nature09887

Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]



$$\frac{\hbar^2\hat{\mathbf{k}}^2}{2m}\breve{1}+\frac{\Omega}{2}\breve{\sigma}_z+\frac{\delta}{2}\breve{\sigma}_y+2\alpha\hat{k}_x\breve{\sigma}_y$$

Raman coupling

detuning

spin-orbit

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ATOMIC PHYSICS

NEWS & VIEWS RESEARCH

Atoms playing dress-up

The idea of using ultracold atoms to simulate the behaviour of electrons in new kinds of quantum systems — from topological insulators to exotic superfluids and superconductors — is a step closer to becoming a reality. SEE LETTER P.83

MICHAEL CHAPMAN & CARLOS SÁ DE MELO

Zeeman and Spin-Orbit Hamiltonian

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_I(\mathbf{r})$$

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}) \left[\hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r}) \sigma_{i,\alpha\beta} \right] \psi_\beta(\mathbf{r})$$

Kinetic Energy

Spin-orbit and Zeeman

$$\mathcal{H}_I(\mathbf{r}) \;\; = \;\; -g \psi^\dagger_\uparrow(\mathbf{r}) \psi^\dagger_\downarrow(\mathbf{r}) \psi_\downarrow(\mathbf{r}) \psi_\uparrow(\mathbf{r})$$

Contact Interaction

Ignore interactions first and go to momentum space

Experimental

Zeeman

Spin-Orbit

$$H_{ZSO}(\mathbf{k}) = -h_z\sigma_z - \cancel{h_y\sigma_y} - h_{ERD}(\mathbf{k})\sigma_y$$

Theoretical

$$\mathbf{H}_0(\mathbf{k}) = K_+(\mathbf{k})\mathbf{1} + \cancel{K_-}\sigma_z - h_z\sigma_z - h_y(\mathbf{k})\sigma_y - \cancel{h_x(\mathbf{k})}\sigma_x$$

Two-level system in momentum space

$$\mathcal{H}_0(\mathbf{r}) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}) \left[\hat{K}_\alpha \delta_{\alpha\beta} - h_i(\mathbf{r})\sigma_{i,\alpha\beta}\right] \psi_\beta(\mathbf{r})$$

Kinetic Energy

Spin-orbit and Zeeman

$$\mathbf{H}_0(\mathbf{k}) = K_+(\mathbf{k})\mathbf{1} + \cancel{K_-}\sigma_z - h_z\sigma_z - h_y(\mathbf{k})\sigma_y - \cancel{h_x(\mathbf{k})}\sigma_x$$

$$K_+(\mathbf{k}) = (K_\uparrow + K_\downarrow)/2 = \epsilon_\mathbf{k} - \mu_+$$

$$\mu_+ = (\mu_\uparrow + \mu_\downarrow)/2$$

$$K_- = (K_\uparrow \cancel{-} K_\downarrow)/2 = -\mu_-$$

$$\mu_- = (\mu_\uparrow \cancel{-} \mu_\downarrow)/2$$

Energy bands in helicity basis

$$\xi_{\Uparrow}(\mathbf{k}) = K_+(\mathbf{k}) - |\mathbf{h}_{\rm eff}(\mathbf{k})|$$

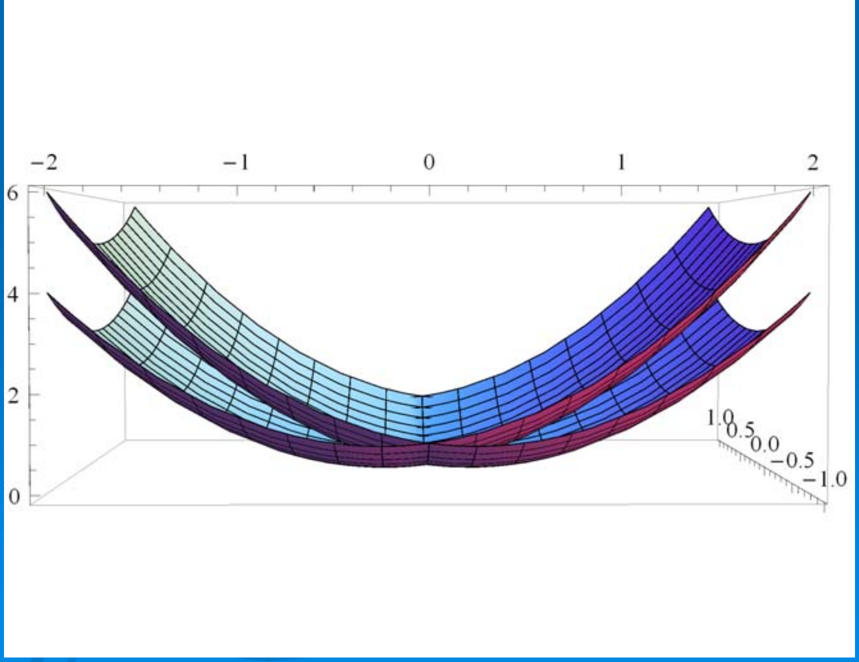
$$\xi_{\Downarrow}(\mathbf{k}) = K_+(\mathbf{k}) + |\mathbf{h}_{\rm eff}(\mathbf{k})|$$

$$h_{\rm eff}(\mathbf{k}) = (\cancel{h_x(\mathbf{k})}, h_y(\mathbf{k}), \cancel{\mu_z + h_z})$$

$$h_{\rm eff}(\mathbf{k}) = (0, vk_x, 0)$$

ERD $h_y = vk_x$ $\longrightarrow$

$$v = 0.5v_F$$



Can have intra- and inter-helicity pairing.

Energy bands in helicity basis

$$\xi_{\Uparrow}(\mathbf{k}) = \bar{K}_{+}(\mathbf{k}) - |\bar{\mathbf{h}}_{\text{eff}}(\mathbf{k})|$$

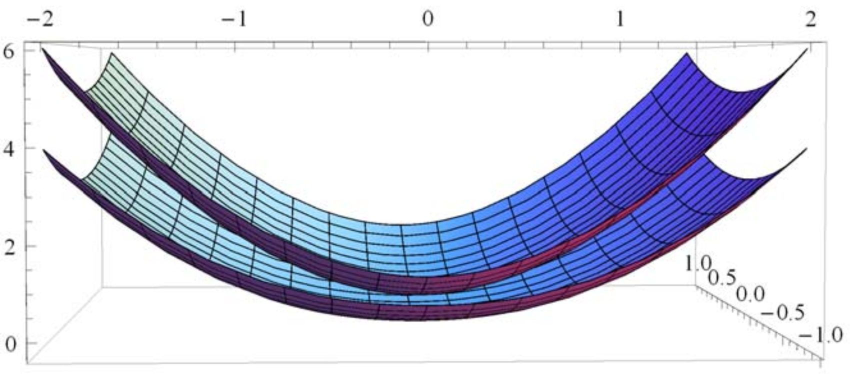
$$\xi_{\Downarrow}(\mathbf{k}) = K_{+}(\mathbf{k}) + |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$\mathrm{h}_{\text{eff}}\,(\mathbf{k}) = [\xcancel{\mathrm{h}_x(\mathbf{k})}, \mathrm{h}_y(\mathbf{k}), \xcancel{\mu_-} + \mathrm{h}_z)$$

$$\mathrm{h}_{\text{eff}}\,(\mathbf{k}) = (0, \mathrm{v}\ \mathrm{k}_x\,, \mathrm{h}_z)$$

ERD

$$h_y = v k_x \longrightarrow$$



$$\mathrm{v} = 0.5\mathrm{v}_F \quad \text{and} \quad \mathrm{h}_z = 0.3\varepsilon_F$$

Can have intra- and inter-helicity pairing.

Bring interactions back: Hamiltonian in initial spin basis

$$\mathbf{H}_0 = \begin{array}{c|cccc} & \psi_{k\uparrow} & \psi_{k\downarrow} & \psi^{+}_{-k\uparrow} & \psi^{+}_{-k\downarrow} \\ \hline \psi^{+}_{k\uparrow} & \tilde{K}_{\uparrow}(\mathbf{k}) & -h_{\perp}(\mathbf{k}) & 0 & -\Delta_0 \\ \psi^{+}_{k\downarrow} & -h^{*}_{\perp}(\mathbf{k}) & \tilde{K}_{\downarrow}(\mathbf{k}) & \Delta_0 & 0 \\ \psi_{-k\uparrow} & 0 & \Delta^{\dagger}_{0} & -\tilde{K}_{\uparrow}(-\mathbf{k}) & h^{*}_{\perp}(-\mathbf{k}) \\ \psi_{-k\downarrow} & -\Delta^{\dagger}_{0} & 0 & h_{\perp}(-\mathbf{k}) & -\tilde{K}_{\downarrow}(-\mathbf{k}) \end{array}$$

Bring interactions back: Hamiltonian in the helicity basis

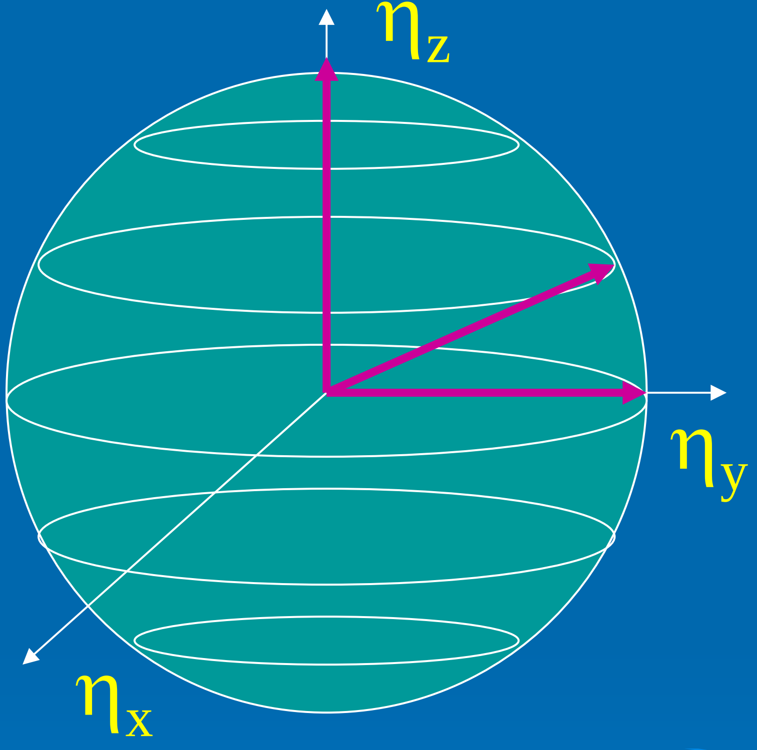
$$\tilde{\mathrm{H}}_0 = \begin{array}{c|cccc} & \Phi_{k\Uparrow} & \Phi_{k\Downarrow} & \Phi^{+}_{-k\Uparrow} & \Phi^{+}_{-k\Downarrow} \\ \hline \Phi^{+}_{k\Uparrow} & \xi_{\Uparrow}(\mathbf{k}) & 0 & \boxed{\Delta_T(\mathbf{k})e^{-i\varphi_{\mathbf{k}}}} & -\Delta_S(\mathbf{k}) \\ \Phi^{+}_{k\Downarrow} & 0 & \xi_{\Downarrow}(\mathbf{k}) & \Delta_S(\mathbf{k}) & \boxed{-\Delta_T e^{i\varphi_{\mathbf{k}}}} \\ \Phi_{-k\Uparrow} & \boxed{\Delta_T^*(\mathbf{k})e^{i\varphi_{\mathbf{k}}}} & -\Delta_S^*(\mathbf{k}) & -\xi_{\Uparrow}(\mathbf{k}) & 0 \\ \Phi_{-k\Downarrow} & \Delta_S^*(\mathbf{k}) & \boxed{-\Delta_T^*(\mathbf{k})e^{-i\varphi_{\mathbf{k}}}} & 0 & -\xi_{\Downarrow}(\mathbf{k}) \end{array}$$

Order Parameter: Singlet & Triplet

$$\Delta_S(\mathbf{k}) = \Delta_0 h_{||}(\mathbf{k})/|\mathbf{h}_{\rm eff}(\mathbf{k})|$$

$$\Delta_T(\mathbf{k}) = \Delta_0|h_\perp(\mathbf{k})|/|\mathbf{h}_{\rm eff}(\mathbf{k})|$$

$$|\Delta_T(\mathbf{k})|^2 + |\Delta_S(\mathbf{k})|^2 = |\Delta_0|^2$$



ERD

$$|h_\perp(\mathbf{k})| = v|k_x|$$

$$h_{||}(\mathbf{k}) = \cancel{\mu -} + h_z$$

$$\mathrm{h}_{\rm eff}(\mathbf{k}) = (\cancel{\mathrm{h}_x(}\mathbf{k}), \mathrm{h}_y(\mathbf{k}), \cancel{\mu_-} + \mathrm{h}_z)$$

$$\mathrm{h}_{\rm eff}(\mathbf{k}) = (0, \mathrm{v}k_x, \mathrm{h}_z)$$

Excitation Spectrum

$$E_1(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_\Uparrow - \xi_\Downarrow}{2}\right) - \sqrt{\left(\frac{\xi_\Uparrow + \xi_\Downarrow}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

★

$$E_2(\mathbf{k}) = \sqrt{\left[\left(\frac{\xi_\Uparrow - \xi_\Downarrow}{2}\right) + \sqrt{\left(\frac{\xi_\Uparrow + \xi_\Downarrow}{2}\right)^2 + |\Delta_S(\mathbf{k})|^2}\right]^2 + |\Delta_T(\mathbf{k})|^2},$$

Can be zero

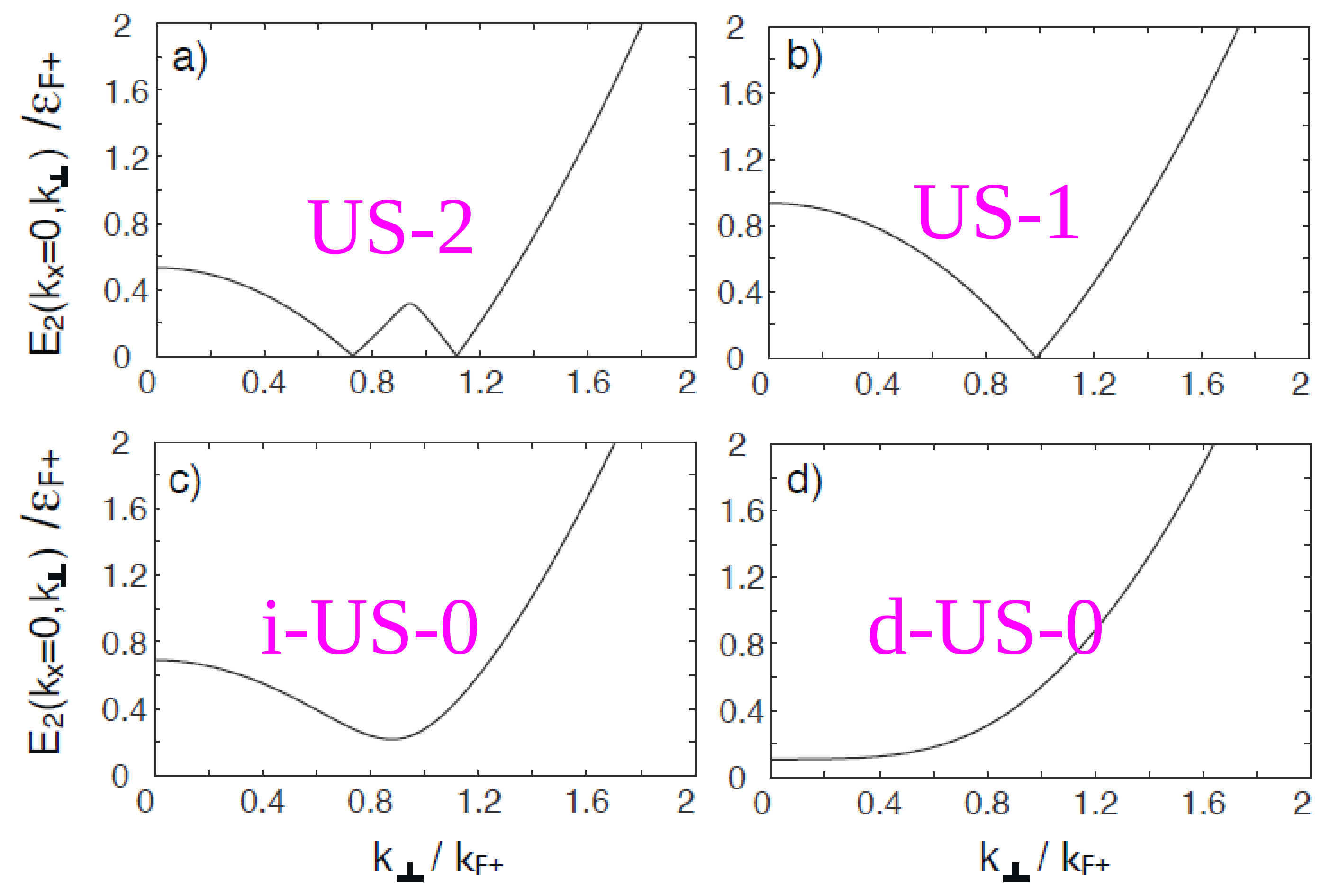
$$E_3(\mathbf{k}) = -E_2(\mathbf{k})$$

$$\xi_\Uparrow(\mathbf{k}) = K_+(\mathbf{k}) - |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

$$E_4(\mathbf{k}) = -E_1(\mathbf{k})$$

$$\xi_\Downarrow(\mathbf{k}) = K_+(\mathbf{k}) + |\mathbf{h}_{\text{eff}}(\mathbf{k})|$$

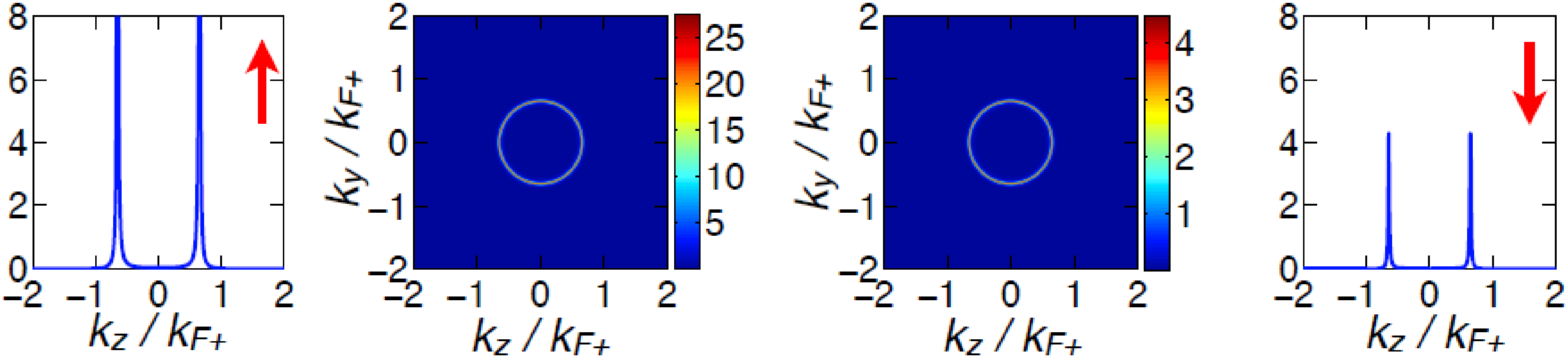
Excitation Spectrum (ERD)



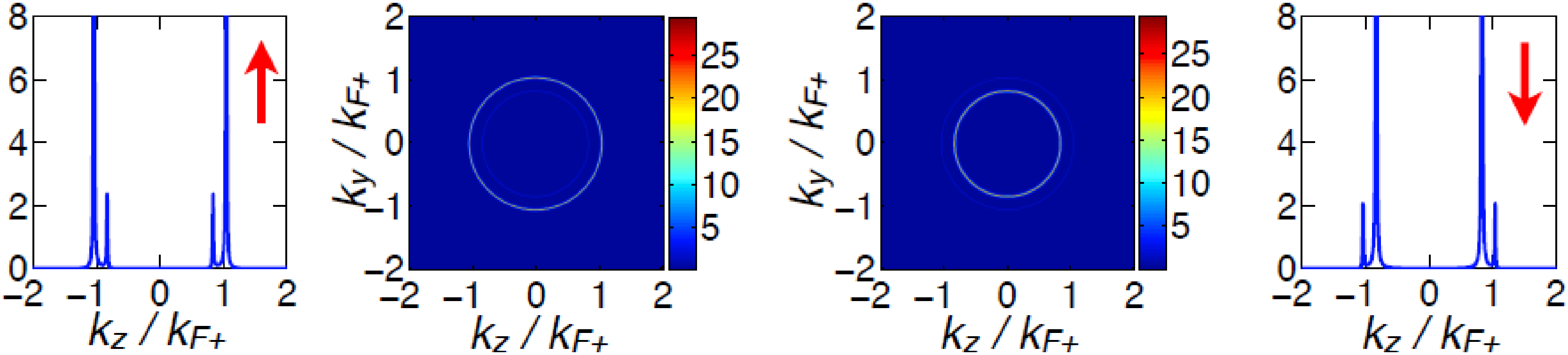
$$\Delta_T(\mathbf{k}) = \Delta_0 |h_\perp(\mathbf{k})| / |\mathbf{h}_{\text{eff}}(\mathbf{k})| \quad = 0$$

Spectral Function ($k_x = 0, \omega = 0$)

US-1



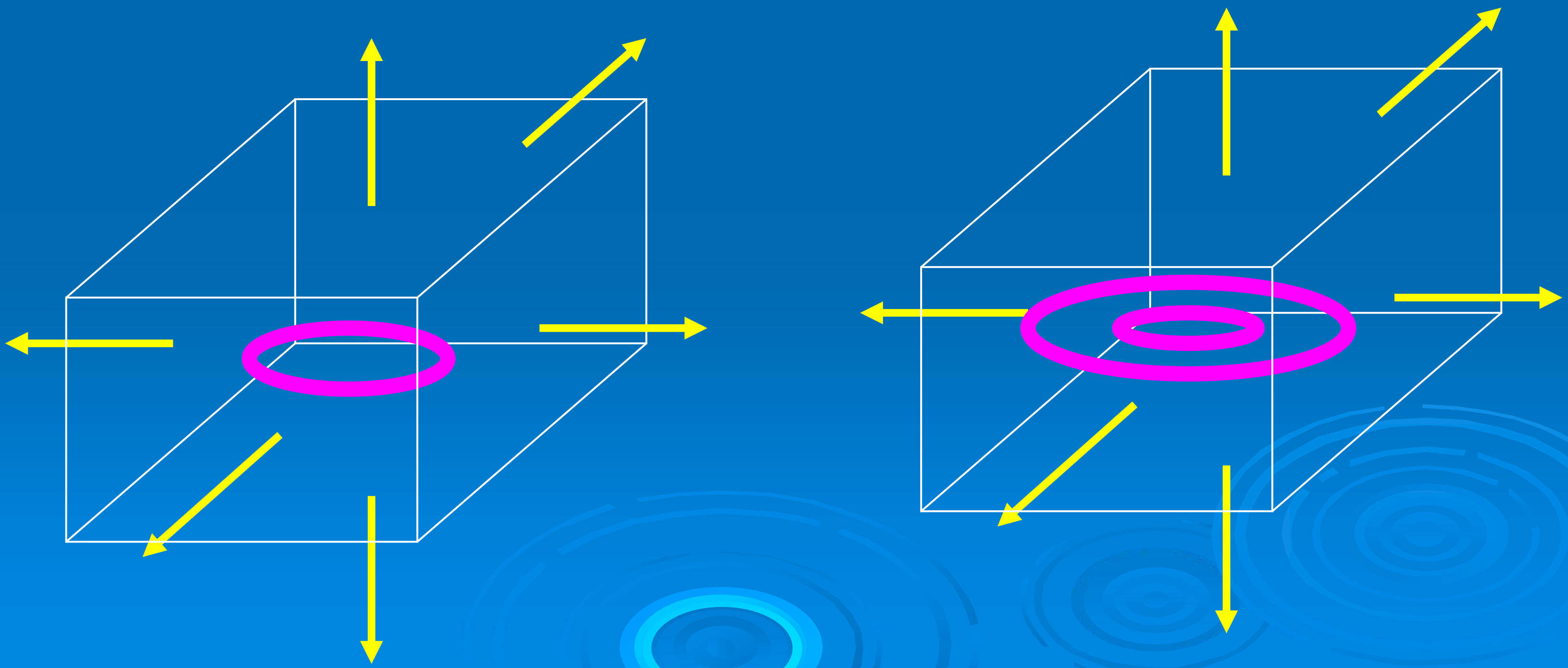
US-2



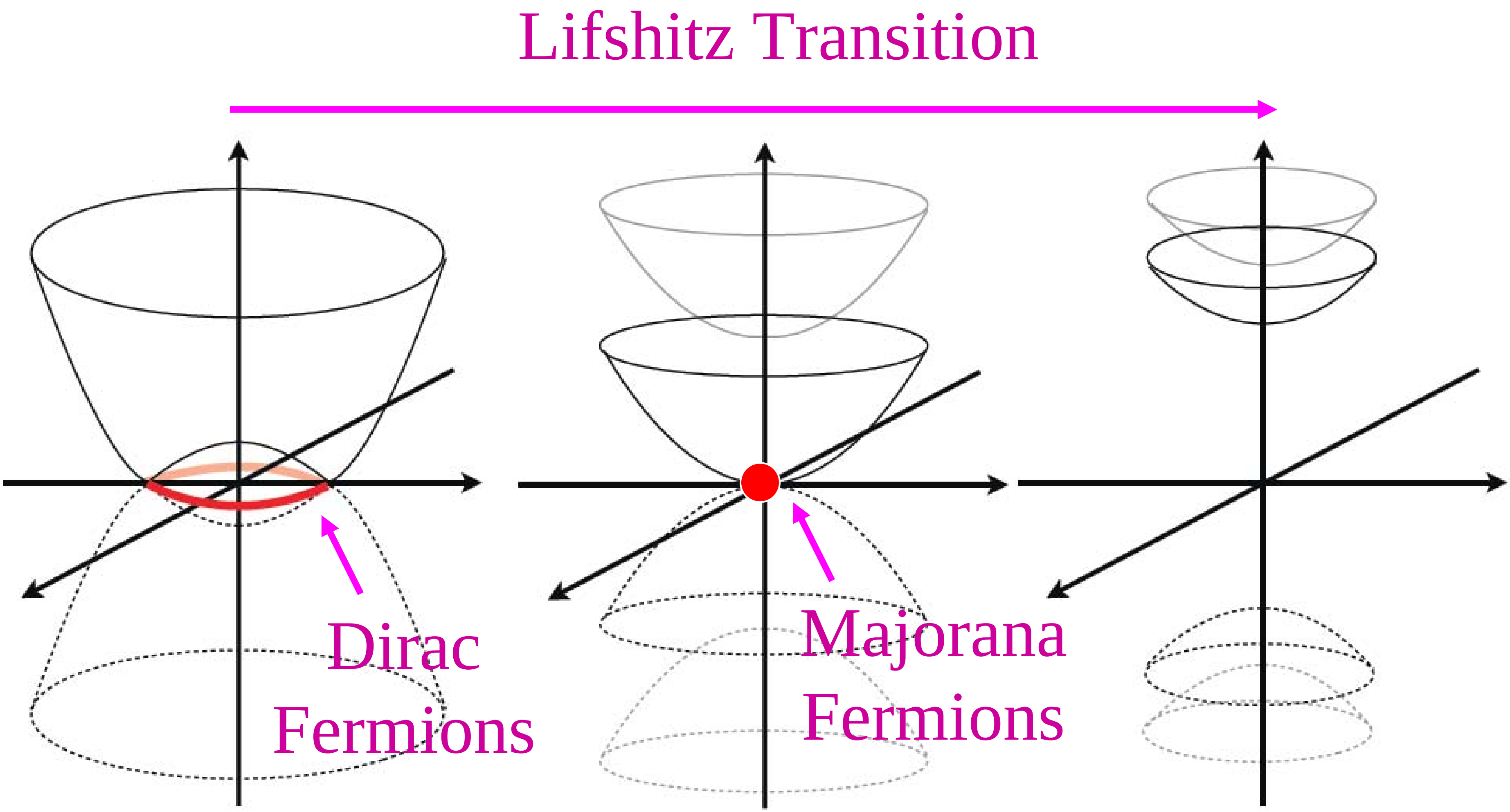
Topological invariant (charge)

$$\mathbf{M}(i\omega, \mathbf{k}) = [i\omega \mathbf{1} - \mathbf{H}_0(\mathbf{k})]^{-1} \qquad \mathbf{\Lambda}_{k_\mu} = \mathbf{M}\partial_{k_\mu}\mathbf{M}^{-1}$$

$$F^\gamma = \epsilon^{\mu\nu\lambda\gamma}\mathrm{Tr}\left[\mathbf{\Lambda}_{k_\mu}\mathbf{\Lambda}_{k_\nu}\mathbf{\Lambda}_{k_\lambda}\right]/24\pi^2 \qquad m = \int_{\mathcal{D}} dS_\gamma F^\gamma$$



Who is the Lord of the Rings?



Need to Determine Phase Diagram

Controllable variables: h_z, v, $1/(k_F a_s)$, T

Some Options at T = 0

a) μ_+ versus h_z
b) μ_+ versus v
c) h_z versus $1/(k_F a_s)$
d) P_{ind} versus $1/(k_F a_s)$

Some Options at finite T

a) T versus h_z
b) T versus v
c) T versus $1/(k_F a_s)$
d) P_{ind} versus T

For phase diagram need chemical potential and order parameter

$$\Omega_0 = V\frac{|\Delta_0|^2}{g} - \frac{T}{2}\sum_{\mathbf{k},j}\ln\{1+\exp[-E_j(\mathbf{k})/T]\} + \sum_{\mathbf{k}}\bar{K}_+,$$

$$\bar{K}_+ \quad = \quad \left[\widetilde{K}_\uparrow(-\mathbf{k}) + \widetilde{K}_\downarrow(-\mathbf{k})\right]/2$$

$$\frac{\delta\Omega_0}{\delta\Delta_0} = 0$$

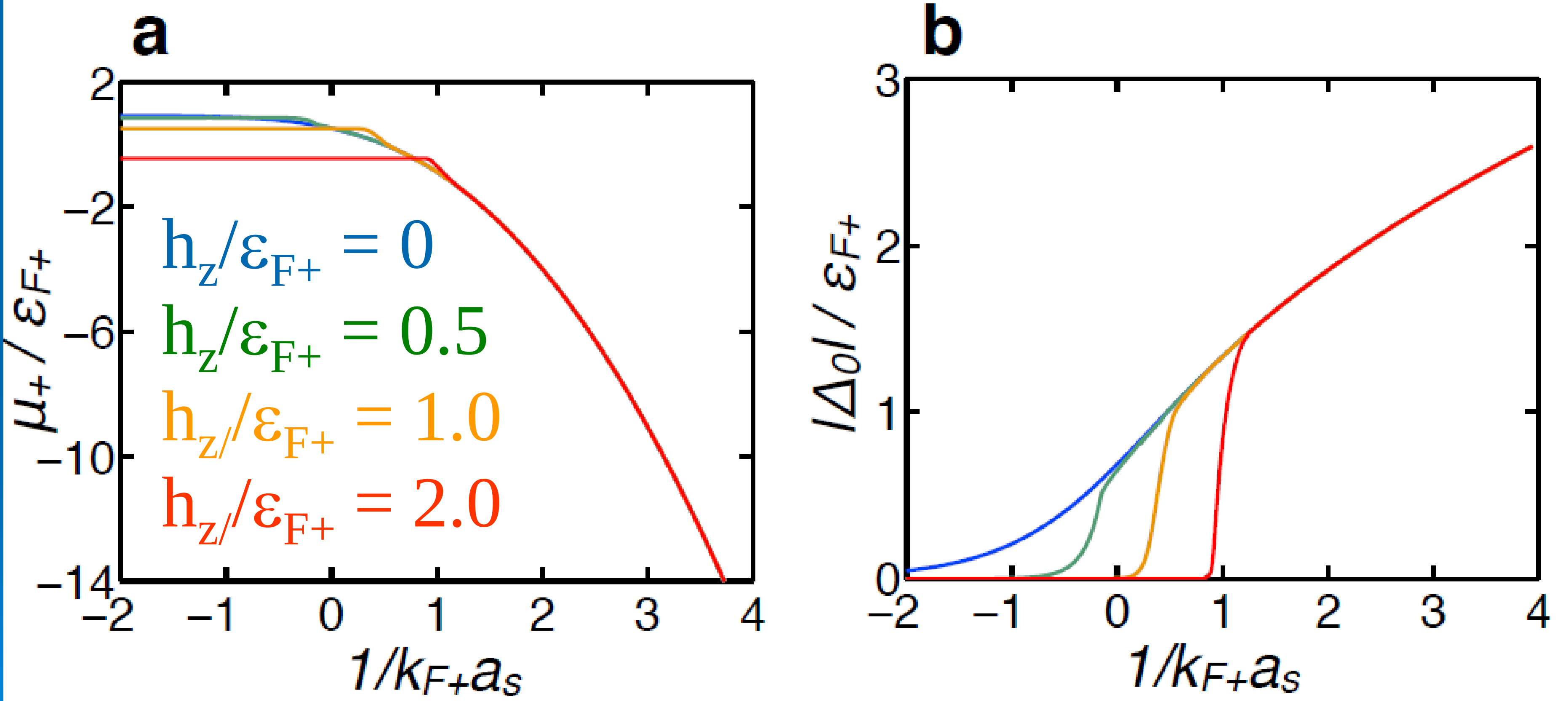
Order Parameter Equation

$$N_+ = -\frac{\partial\Omega_0}{\partial\mu_+} = 0$$

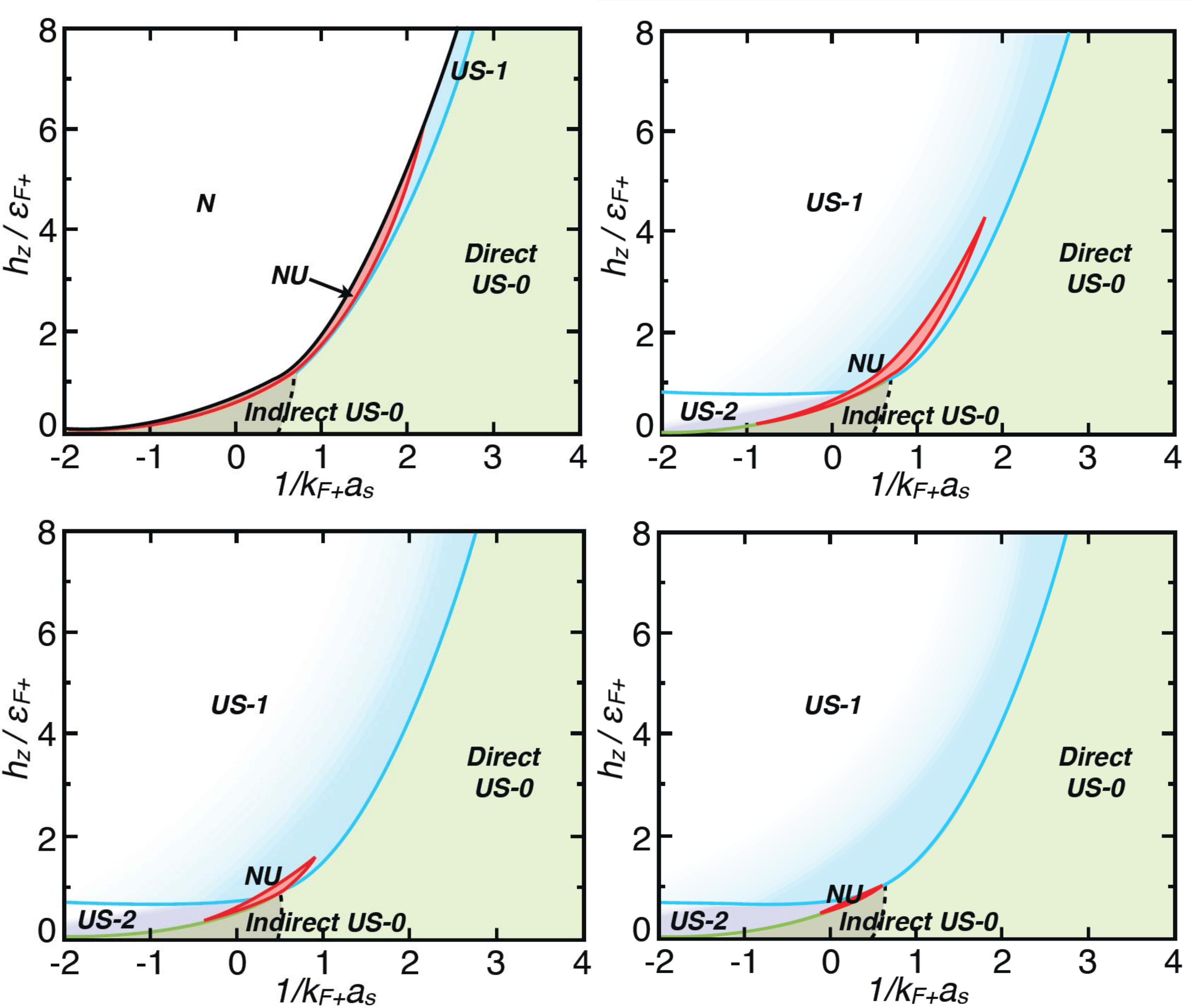
Number Equation

μ_+ and Δ_0 versus interaction

ERD SPIN-ORBIT $v/v_{F+} = 0.28$



h_z versus $1/(k_{F+}a_s)$ at T = 0



0

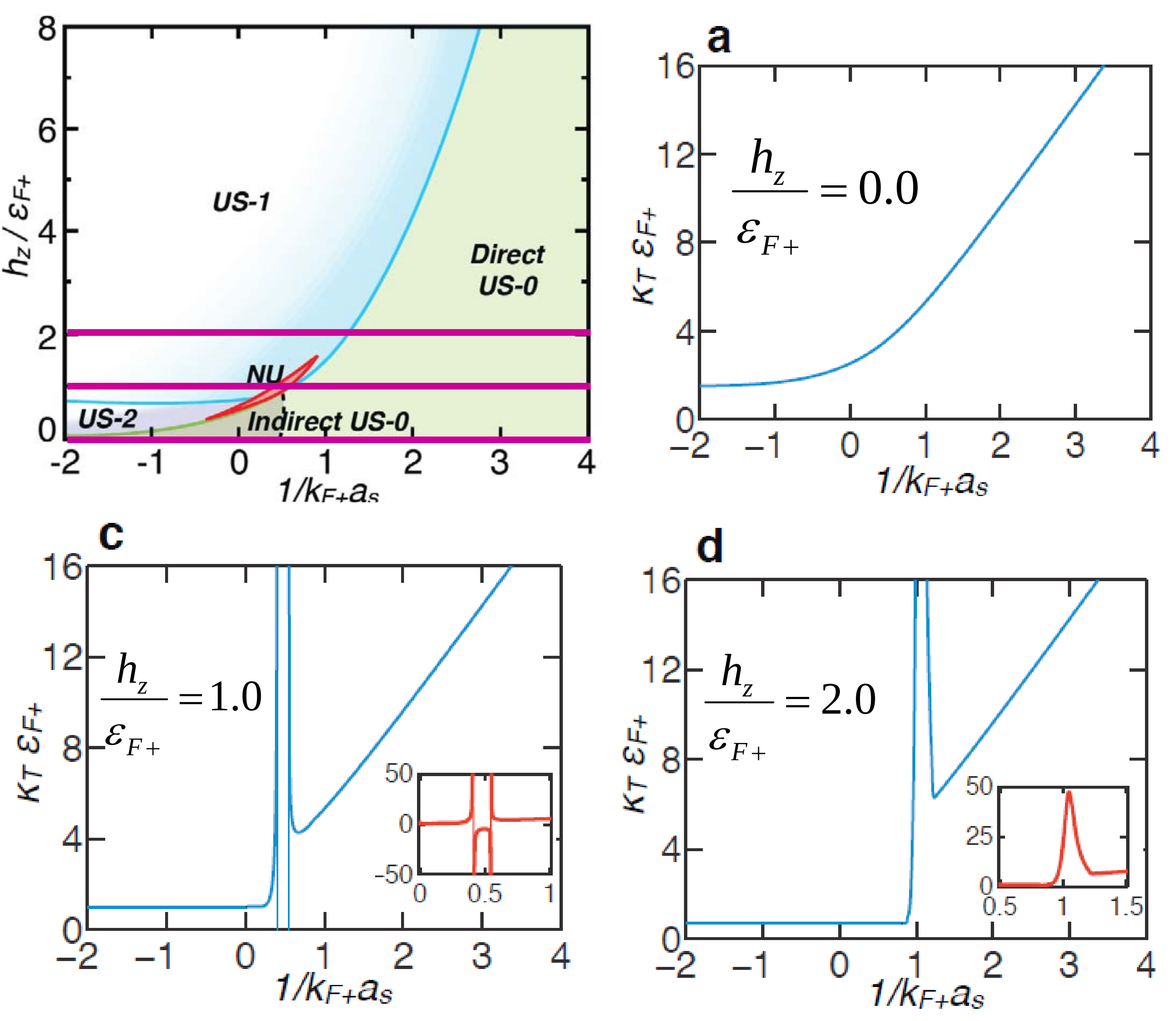
0.14

0.28

0.42

Isothermal Compressibility

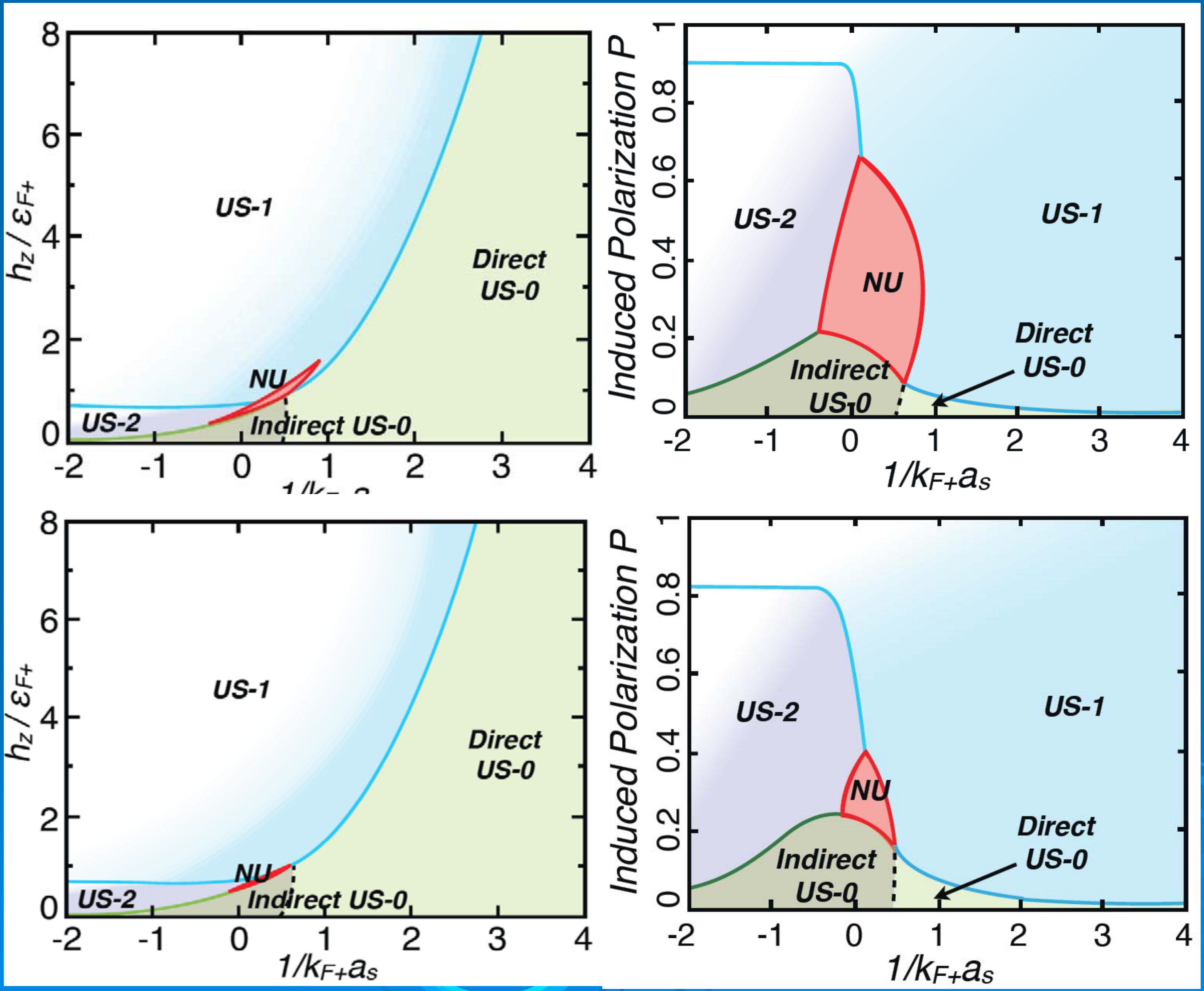
0.28



Critical Polarization

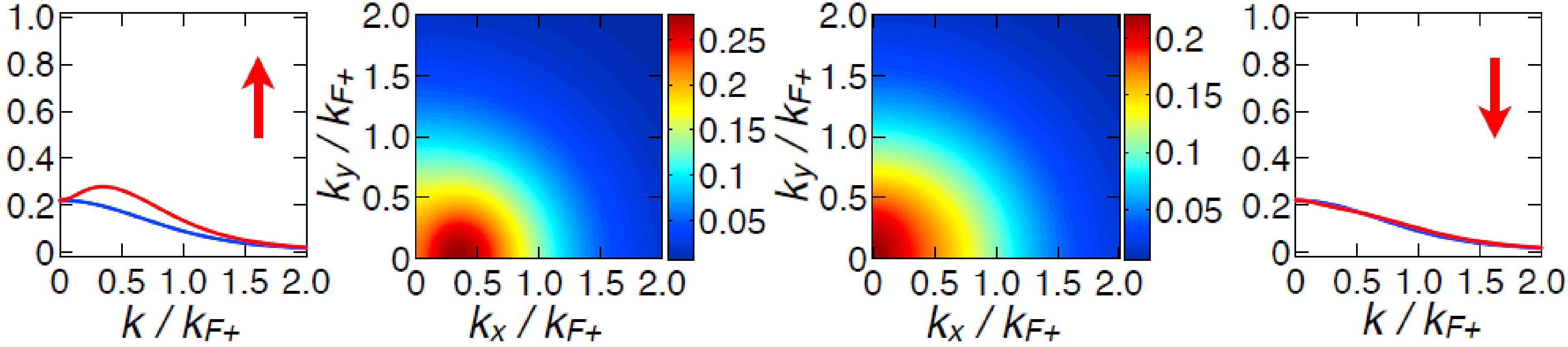
0.28

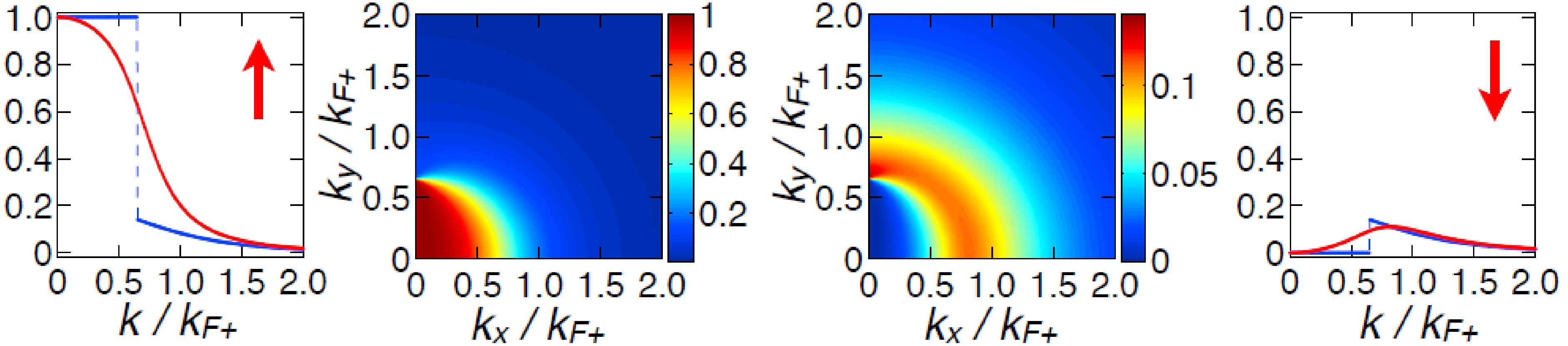
0.42



Momentum Distribution ($k_z = 0$)

$v = 0.28$ $h_z = 1.44$ $P = 0.02$ d-US-0 $1/(k_{F+}a_s) = -1$





$v = 0.28$ $h_z = 1.75$ $P = 0.18$ US-1 $1/(k_{F+}a_s) = -1$

Outline

- Introduction: BCS-to-BEC evolution
- What are topological phase transitions?
- Zeeman and spin-orbit effects in ultra-cold Fermi superfluids
- Conclusions.

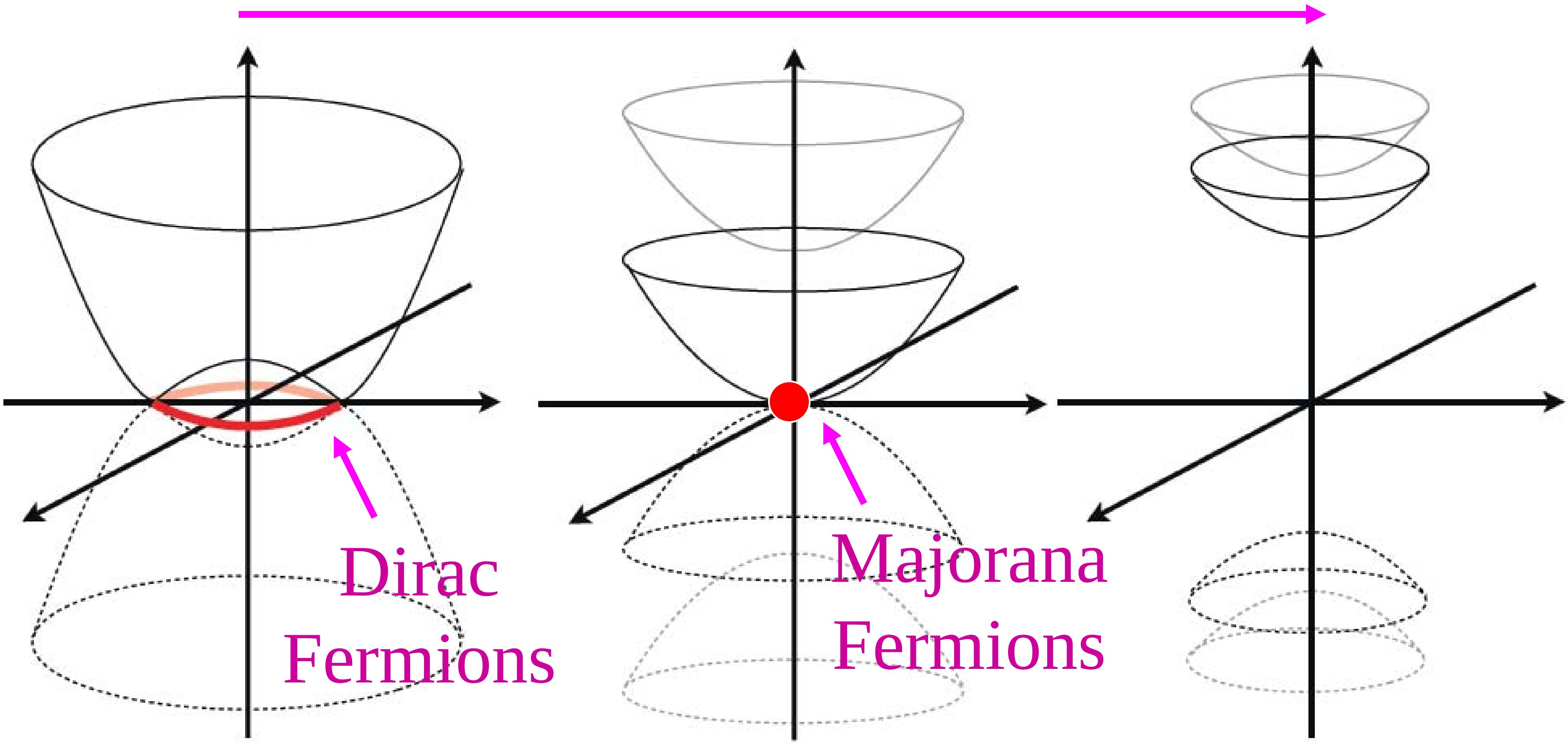
Conclusions

- The evolution from BCS to BEC superfluids in the presence of Zeeman, spin-orbit fields and interactions exhibit several topological phase transitions, where the symmetry of the superfluid order parameter does not change but the topology of excitations and of ground state properties change across phase boundaries.

- Such topological transitions belong to the Lifshitz class, where Dirac fermions disappear at phase boundaries creating gaps in the excitation spectrum in momentum space. In some of the phase transitions Dirac Fermions annihilate leading to the emergence of bulk Majorana Fermions as quasi-particles becomes massive.

- These momentum space topological phase transitions can be probed via spectroscopic properties such as the excitation spectrum, momentum distribution, spectral function , density of states, or via thermodynamic properties such as the compressibility matrix or the spin-susceptibility tensor.

Who is the Lord of the Rings?



Lifshitz Transition

Dirac Fermions

Majorana Fermions

END OF TALK