

Itinerant Magnetism in Cold Atoms



Inti Sodemann Dima Pesin Rembert Duine



**"I WILL CUT OFF THE HORNS OF
ALL THE WICKED..."**

PSALMS 75:10

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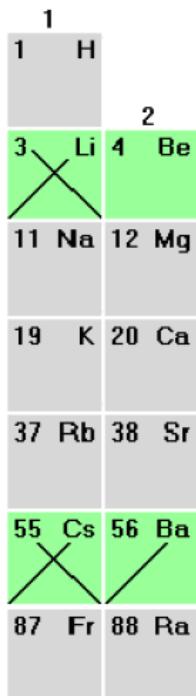
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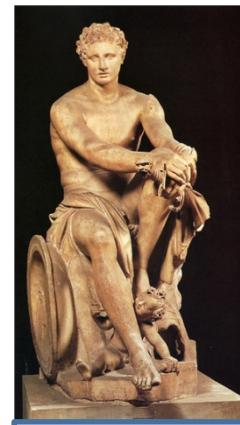
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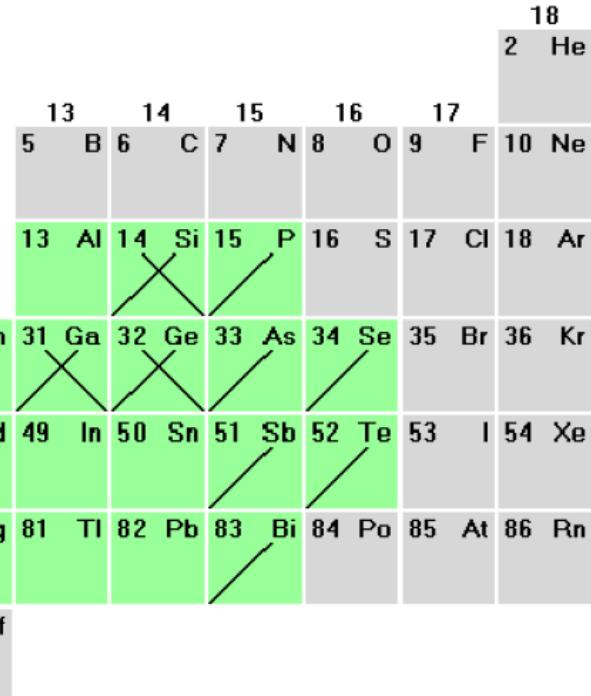
Condensed Matter Physics



VENUS

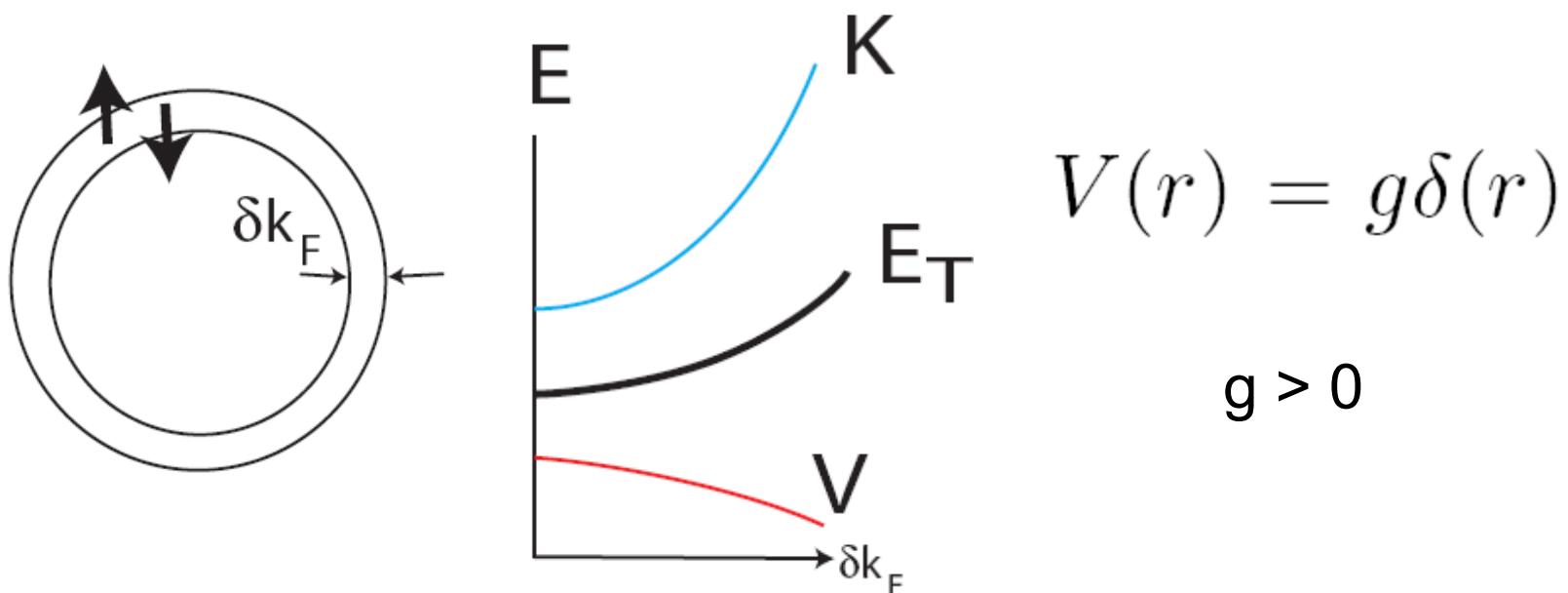


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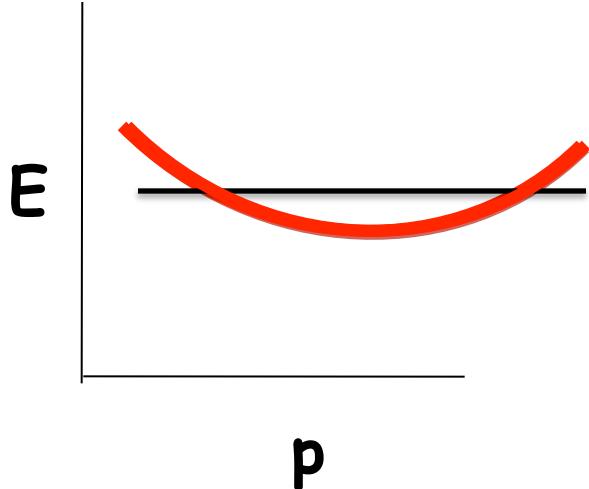


Itinerant ferromagnetism

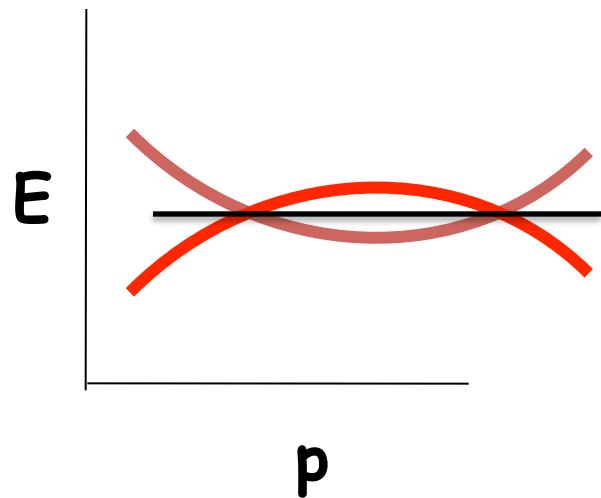
- Ferromagnetism results from cooperation between Pauli principle and Coulomb repulsion. The “Stoner” instability is a strong interaction instability determined by competition between kinetic and interaction energy:



Two Stoner Criteria

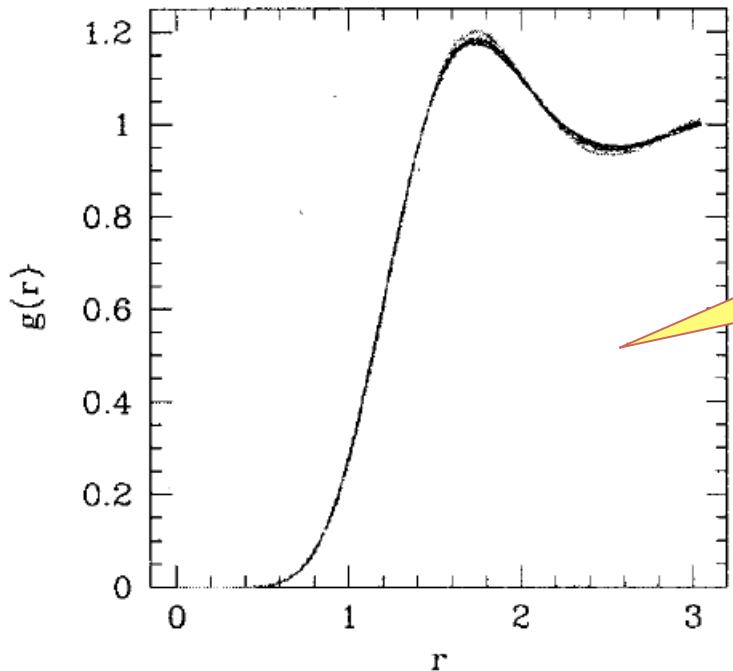


$$m_x = D(\epsilon_F) U m_x$$

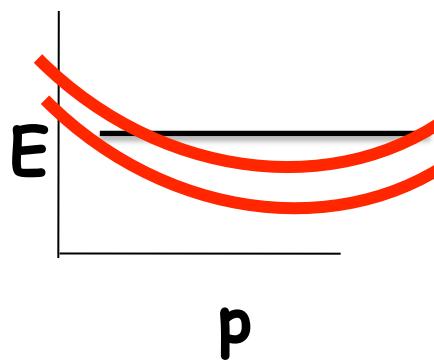


$$m_x = \frac{UD(\epsilon_F)}{2} \int dE \frac{m_x}{(E^2 + (U * m_x/2)^2)^{1/2}}$$

Magnetic Stoner Criteria

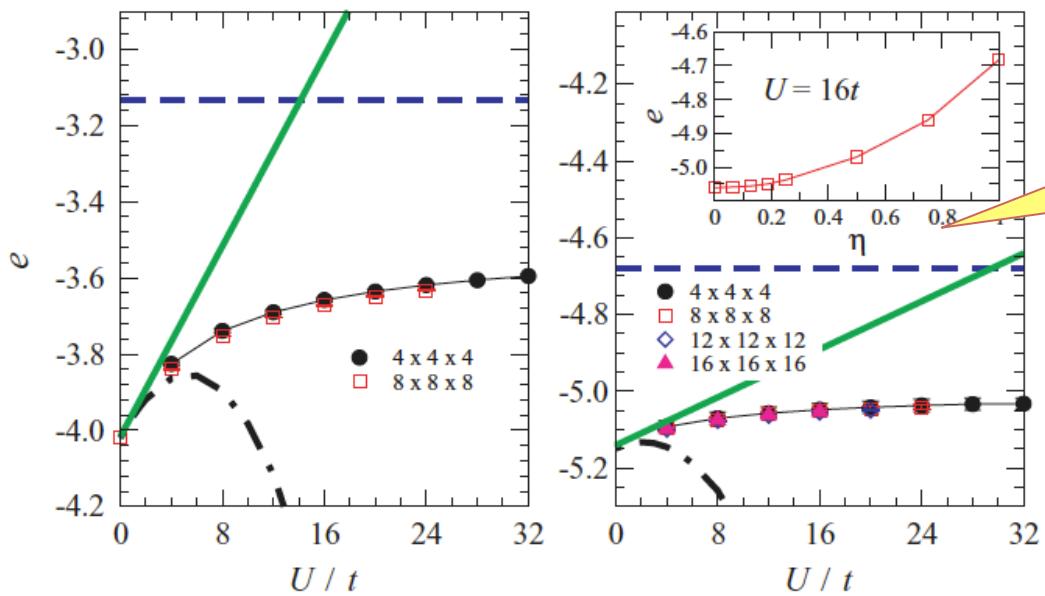


3D Electron Gas
Zong et al. PRL (2002)
 $(DU)_c \approx 10$

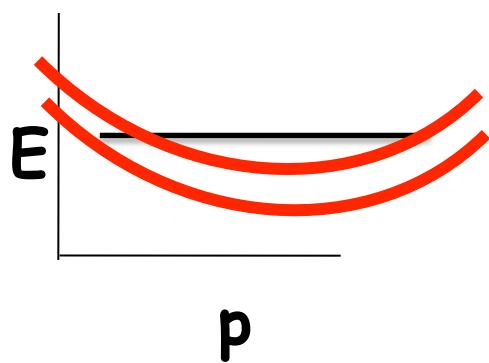


$$m_x = D(\epsilon_F) U m_x$$

Dilute Hubbard Model



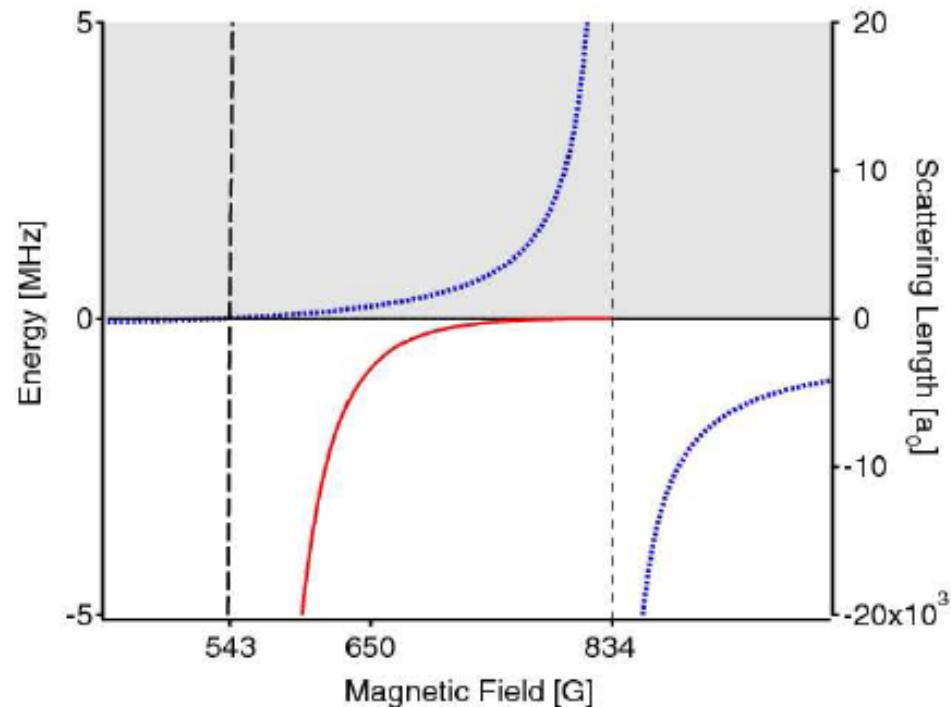
Chang *et al.*
PRA (2010)



$$m_x = D(\epsilon_F) U m_x$$

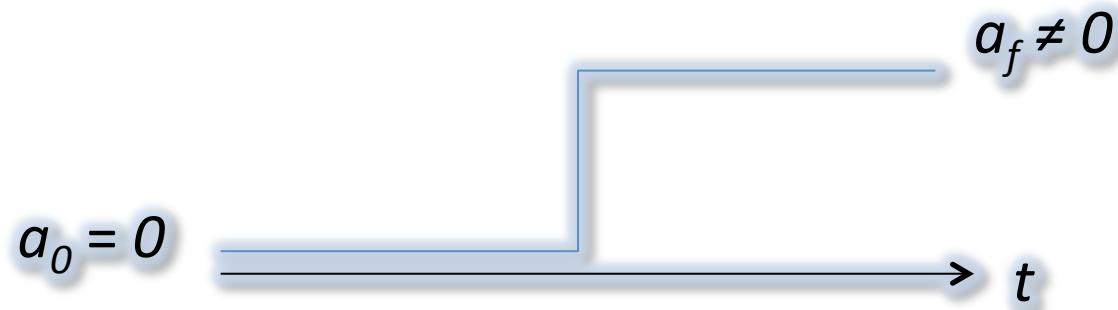
The Difference Between Boys and Girls

Even when the potential is attractive, the interaction appears to be repulsive when a bound state is just below the continuum.



Can we study ferromagnetism?

- How does a system behave after an interaction quench?



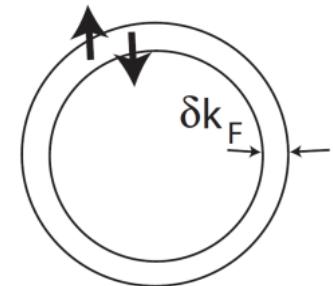
- Unpolarized gas:
 - D. Pekker et al., PRL 106, 050402 (2011).
 - M. Babadi et al., arXiv:0908.3483 (2009).

Can we study ferromagnetism?

Extra “knob” available in cold atoms:

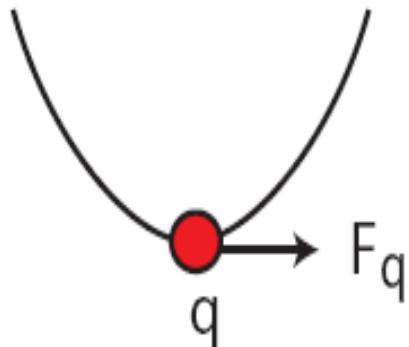
- Total spin is conserved on experimental time scales

$$\delta = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}} = \frac{k_{F\uparrow}^3 - k_{F\downarrow}^3}{k_{F\uparrow}^3 + k_{F\downarrow}^3}$$



Linear instability analysis

- *System is in the ideal polarized Fermi gas initially.*
- *Response to small perturbations:*

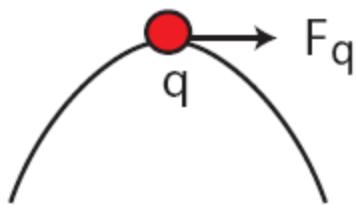


$$q(\omega) = \chi_q(\omega) F_q(\omega)$$

$$q(t) \rightarrow q_0 \exp(i\omega_0 t), \quad \chi_q(\omega_0) = \infty$$

Linear instability analysis

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$$q(t) \rightarrow q_0 \exp(i\omega_0 t), \quad \chi_q(\omega_0) = \infty$$

- Instabilities appear as poles of the susceptibility with positive imaginary part: growth rate of the instability.

Observables

- Pairing at zero and finite momentum:

$$\Delta(r) = \langle \psi_{\uparrow}^{\dagger}(r) \psi_{\downarrow}^{\dagger}(r) \rangle \quad \longleftrightarrow \quad \Delta_P = \sum_k \langle c_{\frac{1}{2}P+k\uparrow}^{\dagger} c_{\frac{1}{2}P-k\downarrow}^{\dagger} \rangle$$

- Spin density and particle density:

$$n(r) = \langle n_{\uparrow}(r) + n_{\downarrow}(r) \rangle \quad s_z(r) = \frac{\hbar}{2} \langle n_{\uparrow}(r) - n_{\downarrow}(r) \rangle$$

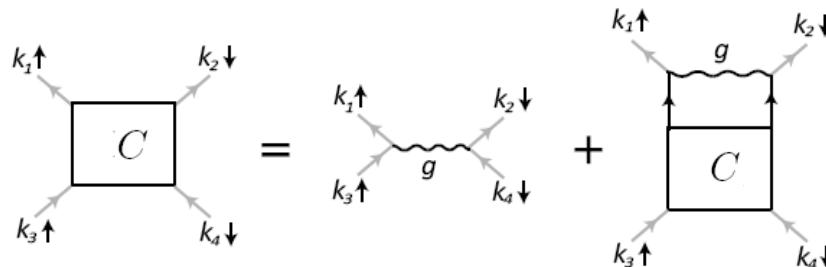
- Transverse spin density:

$$s_x = \frac{\hbar}{2} \langle \psi_{\uparrow}^{\dagger}(r) \psi_{\downarrow}(r) \rangle + h.c.$$

Model

- Interactions near Feshbach resonance are controlled by scattering length.
- Eliminate explicit appearance of interaction potential in favor of scattering Fermi sea amplitude: Cooperon.

$$C_{\uparrow\downarrow}^{-1}(E, P) = \frac{m}{4\pi a} + i\frac{\pi}{2}\sqrt{E - \frac{P^2}{4m}} - \int_q \frac{n_{\uparrow\frac{1}{2}P-q} + n_{\downarrow\frac{1}{2}P+q}}{E - \xi_{\uparrow\frac{1}{2}P-q} - \xi_{\downarrow\frac{1}{2}P+q}}$$



$$\delta = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$$

Pairing instability

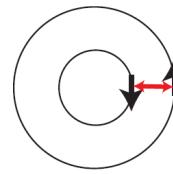
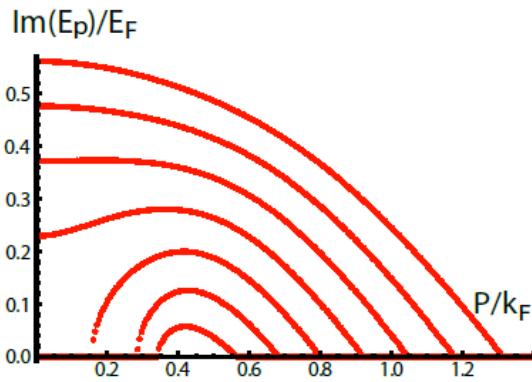
- Pairing instability diagram

- Finite momentum pairing:

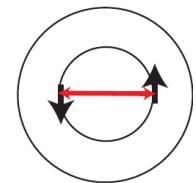
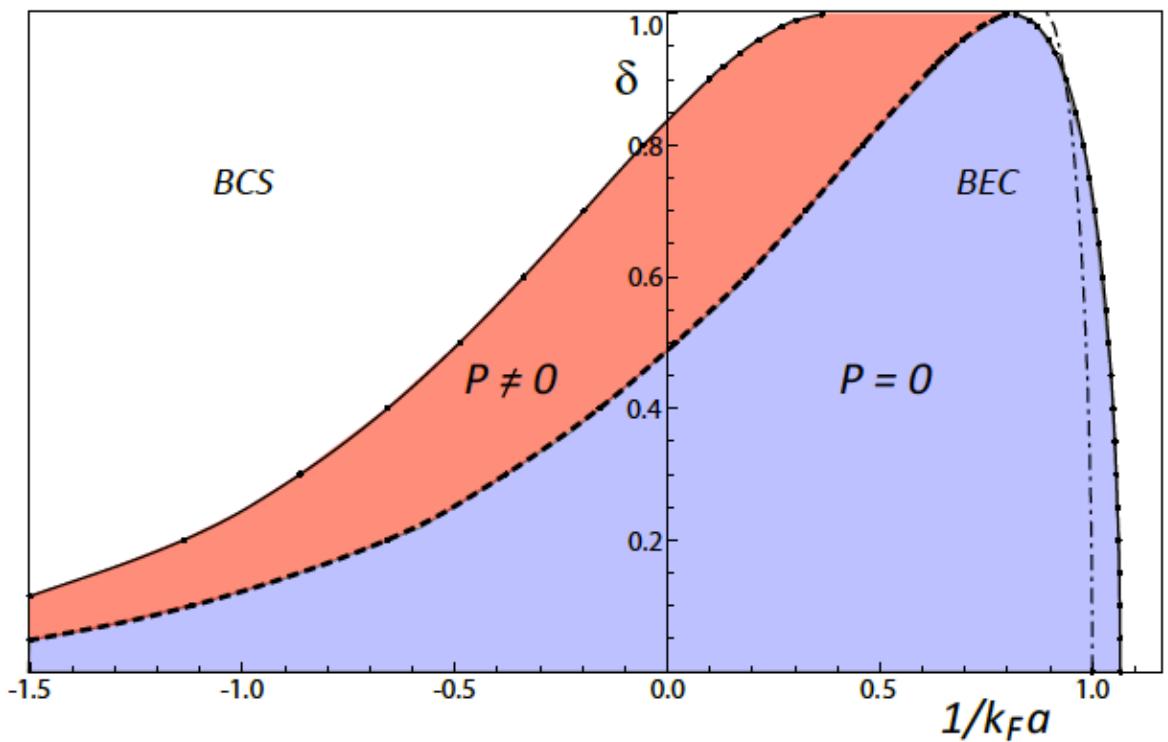
$$P \neq 0$$

- Zero momentum pairing:

$$P = 0$$



$$\frac{k_\uparrow^2}{2m} + \frac{k_\downarrow^2}{2m} = \frac{1}{ma^2}$$

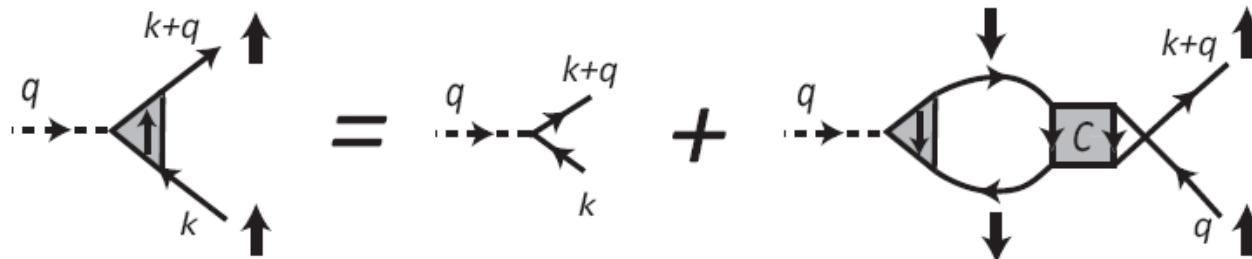


Density and longitudinal spin instability

- Density and spin responses are coupled in polarized gas,

$$\chi_{nn}(r, t), \chi_{zz}(r, t) \quad \chi_{nz}(r, t)$$

RPA with interaction potential replaced by cooperon,



$$\Gamma_q^\uparrow(k) = 1 + i \int \frac{d^4 k'}{(2\pi)^4} G_\downarrow(q + k') G_\downarrow(k') C(k + k' + q) \Gamma_q^\downarrow(k')$$

Density and longitudinal spin instability

- Density and longitudinal spin susceptibilities:

$$\left. \begin{array}{c} \chi_{pp}(k) \\ \chi_{zz}(k) \end{array} \right\} = \begin{array}{c} \text{Diagram for } \chi_{pp}(k) \\ \text{Diagram for } \chi_{zz}(k) \end{array} \pm \begin{array}{c} \text{Diagram for } \chi_{pp}(k) \\ \text{Diagram for } \chi_{zz}(k) \end{array}$$

The diagram consists of two parts. The left part shows a loop with two horizontal arrows pointing right, labeled k above and below. A shaded region on the left contains an upward arrow labeled $q \uparrow$ and an upward arrow labeled $k+q \uparrow$ above the loop. The right part is identical but with downward arrows labeled $q \downarrow$ and $k+q \downarrow$.

- Fermi liquid form at low-frequency long-wavelength limit:

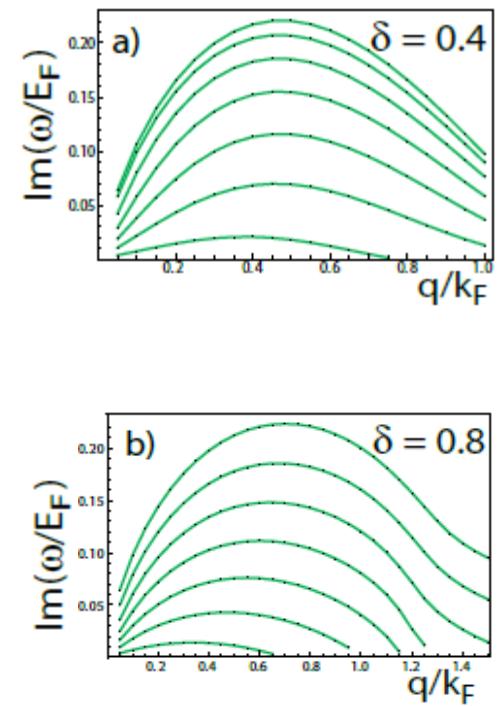
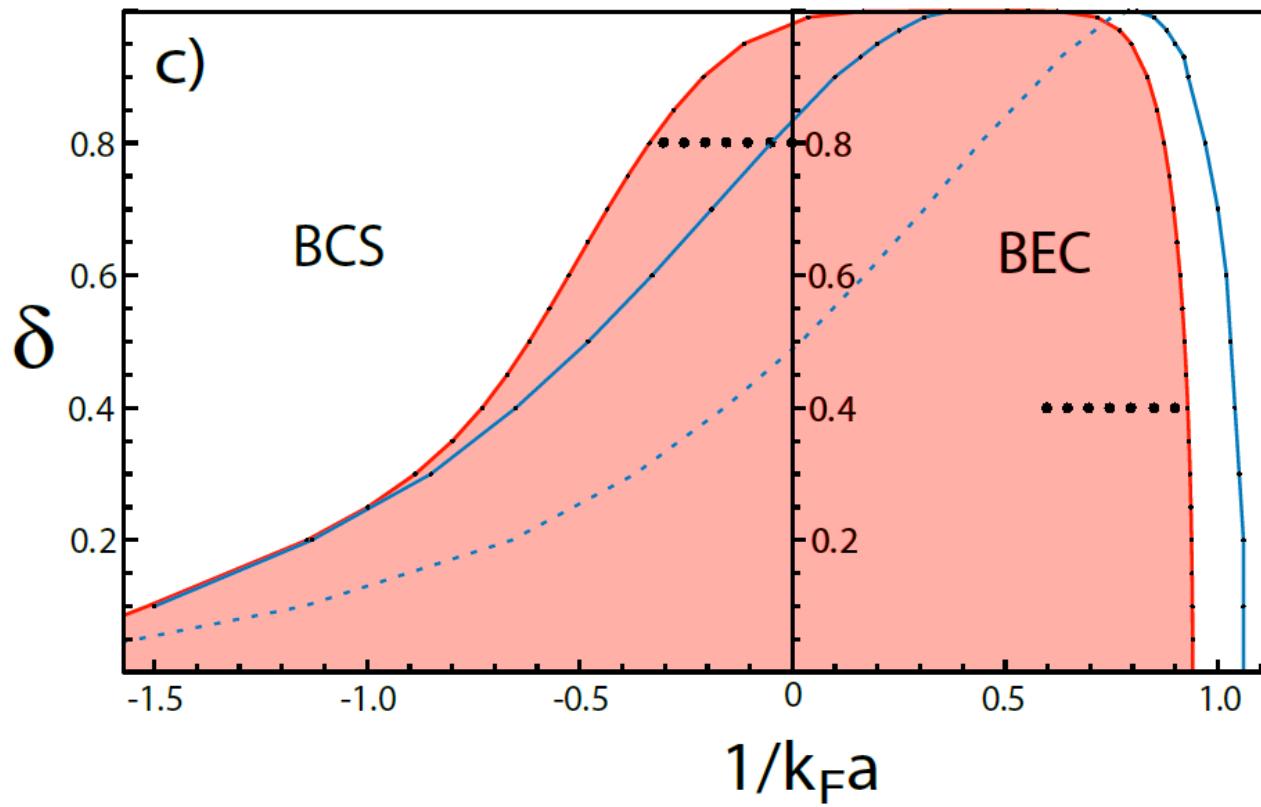
$$G_\sigma(k)G_\sigma(k+q) \rightarrow \frac{4\pi^3 i}{k_{F\sigma}^2} \zeta_\sigma(q, \hat{k}) \delta(k_0 - \epsilon_{F\sigma}) \delta(k - k_{F\sigma})$$

$$\int \frac{d\Omega_{\hat{k}}}{4\pi} \zeta_\sigma(q, \hat{k}) = \chi_\sigma^0(q) = \int_k \frac{n_{k\sigma} - n_{k+q\sigma}}{\omega + \xi_k - \xi_{k+q}}$$

$$\Gamma_\sigma^{\rho,s}(k) = \frac{1 \pm C_s(\omega) \chi_\sigma^0(k)}{1 - C_s^2(\omega) \chi_\uparrow^0(k) \chi_\downarrow^0(k)}$$

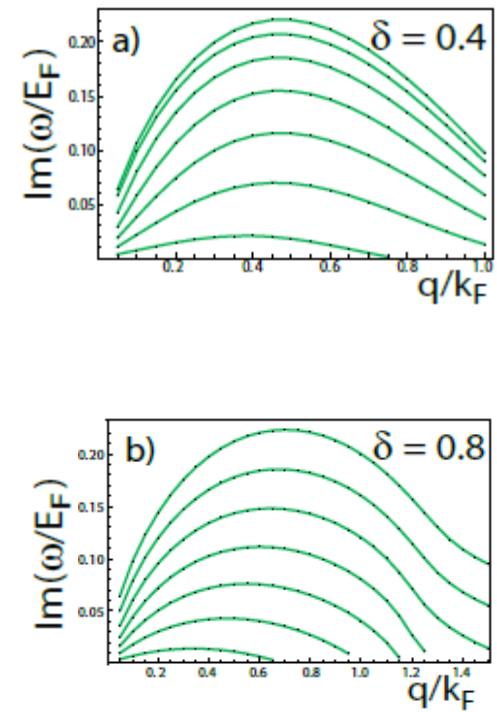
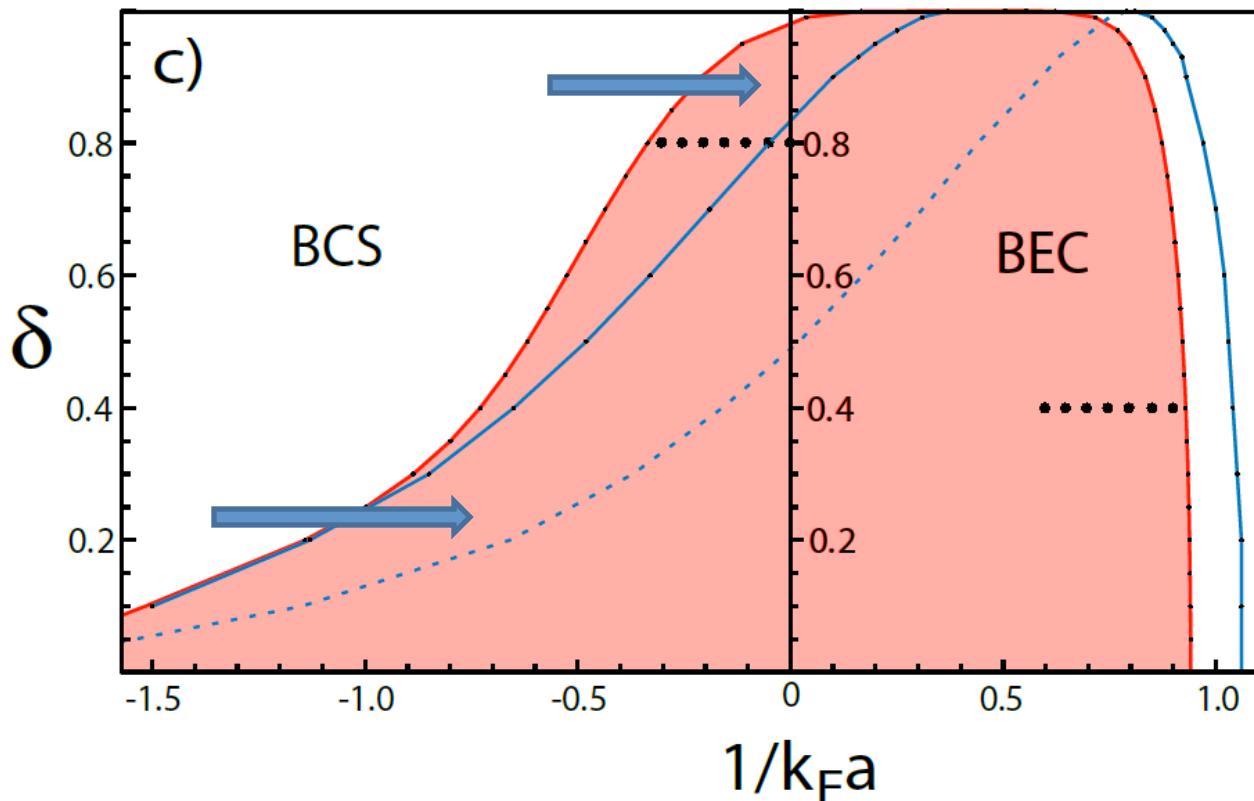
Density and longitudinal spin instabilities

- Instability diagram



Density and longitudinal spin instabilities

- Instability diagram



- Quenches in the polarized BCS side might bring new interesting unstable dynamics

Transverse spin instability

- We use simpler model with contact interaction pseudo-potential:

$$V_{\uparrow,\downarrow}(r) = g\delta(r)$$

- RPA takes a simpler form:

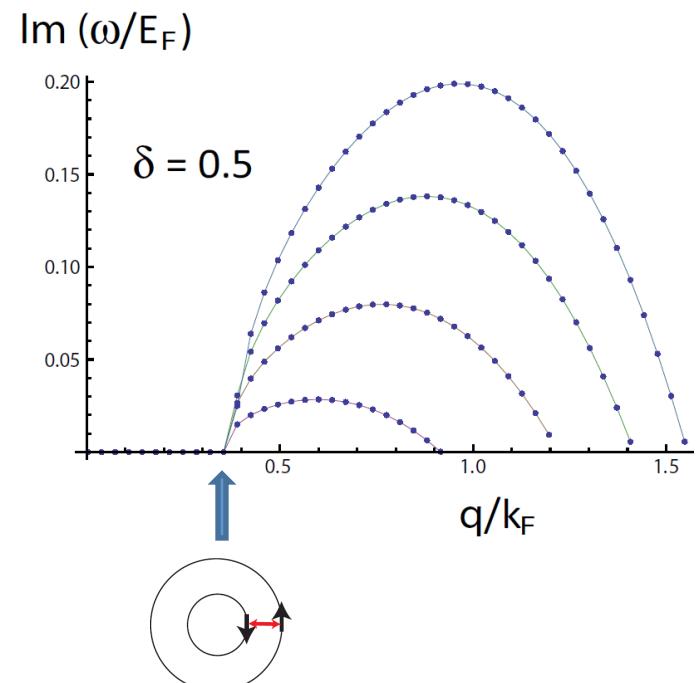
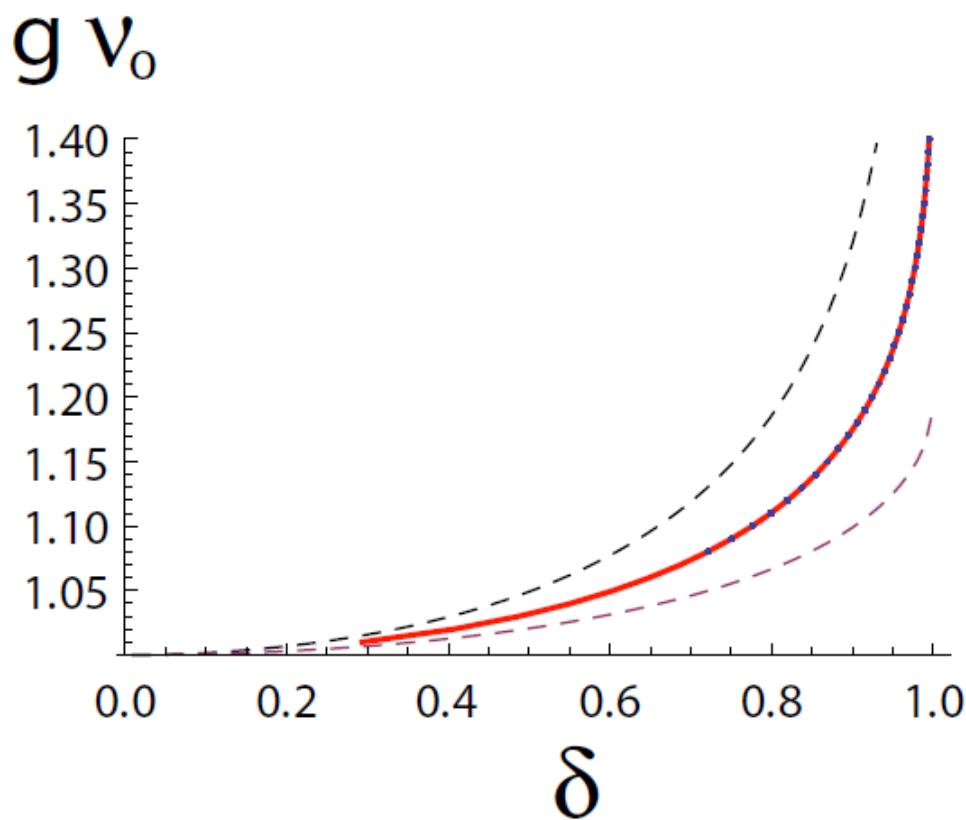
$$\chi_{\uparrow\downarrow}^{RPA}(q, \omega) = \frac{\chi_{\uparrow\downarrow}^0(q, \omega)}{1 + g\chi_{\uparrow\downarrow}^0(q, \omega)}$$

- Collective excitations stable (spin waves) and unstable (exponentially growing modes):

$$1 + g\chi_{\uparrow\downarrow}^0(q, \omega) = 0$$

Transverse spin instability

- Critical interaction



Summary

- Pairing instabilities in polarized gas are dominated by finite momentum pairing in a large region of polarization - scattering length parameters.
- Density instabilities "cover" finite momentum pairing instability and can be precursors of phase separation.
- Repulsive longitudinal spin instability is covered by pairing on BEC side.
- Transverse instabilities are at finite momentum, and occur through spontaneous creation of spin waves and particle-hole excitations.