Light-induced gauge potentials and optical flux lattices for ultracold atoms

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Newspin 2, *17 December 2011*

Collaboration

(in the area of the artificial magnetic fields for ultracold atoms)

- **P. Öhberg & group,** Heriot-Watt University, *Edinburgh*
- ***M. Fleischhauer & group,*** *TU Kaiserslautern,*
- ***L. Santos & group,*** *Universität Hannover*
- ***J. Dalibard, F. Gerbier & group, ENS, Paris***
- ***I. Spielman, D. Campbel, C. Clark and J. Vaishnav, B. Anderson, NIST, USA***
- ***V. Galitski, UMD, USA***
- ***M. Lewenstein, ICFO, ICREA, Barcelona***

Quantum Optics Group @ ITPA, Vilnius University

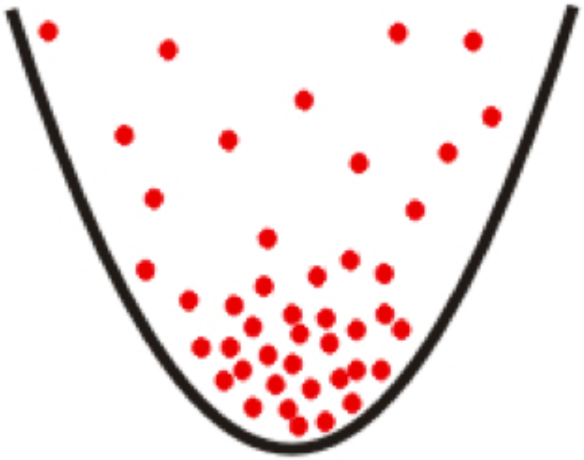
V. Kudriasov, J. Ruseckas, G. J., A. Mekys, T. Andrijauskas
Not in the picture: V. Pyragas and S. Grubinskas

OUTLINE

- Background
- Gauge potentials using optical lattices
- Light-induced (Abelian) geometric gauge potentials (for ultracold atoms)
- Optical flux lattices (OFL)
  - Non-staggered artificial magnetic flux
  - Ways of producing of OFL
- Non-Abelian gauge potentials
- Conclusions

Cold atoms are trapped using:

1. (Parabolic) trapping potential *produced by magnetic or optical means:*



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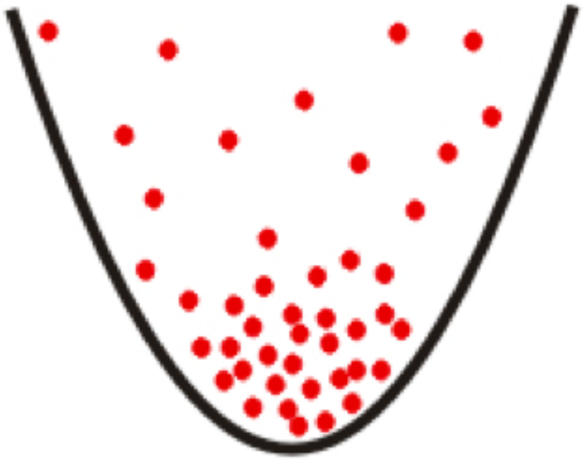
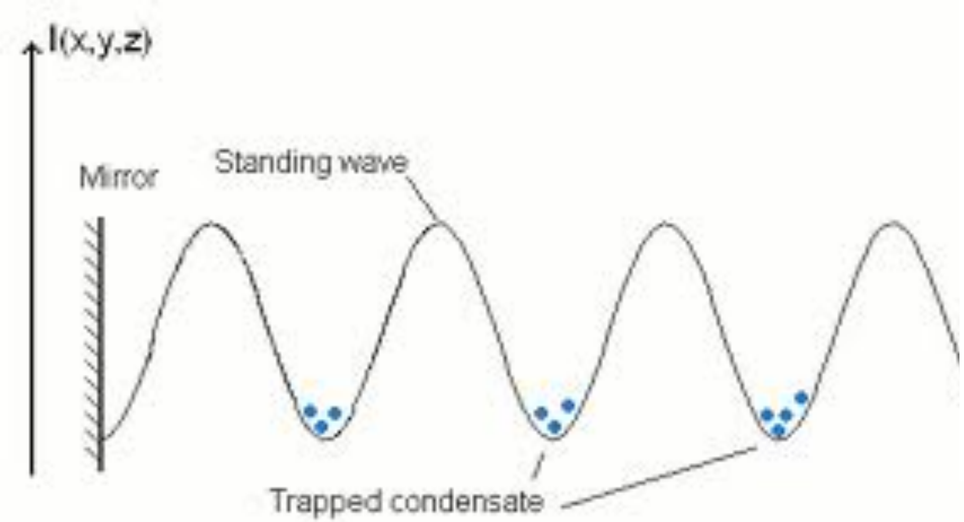


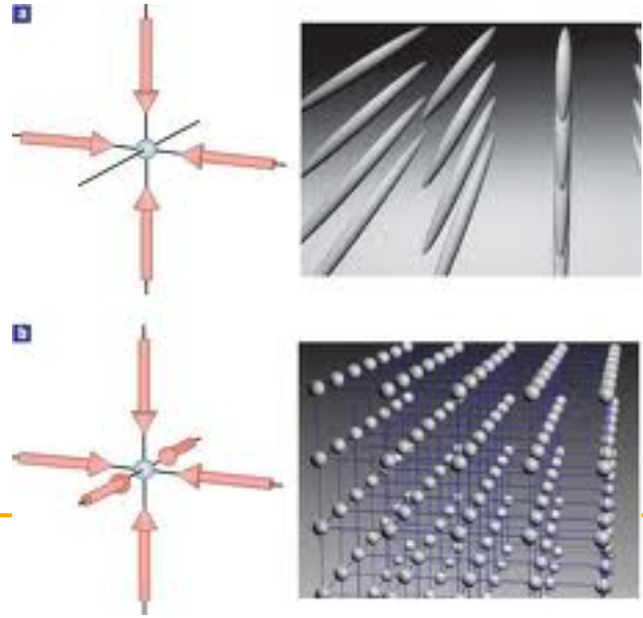
2. Optical lattice (periodic potential):



Optical lattices (ordinary): [Last 10 years]

- A set of **counter-propagating** light beams (*off resonance to the atomic transitions*)

I. Bloch, Nature Phys. 1, 23 (2005)



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- → Atoms are trapped at **intensity minima** (or **intensity maxima**) of the interference pattern

  (depending on the sign of atomic polarisability)

$$V_{dip}(\mathbf{r}) = -\mathbf{d}\cdot\mathbf{E}(\mathbf{r}) = -\alpha(\omega_L)|\mathbf{E}(\mathbf{r})|^2$$

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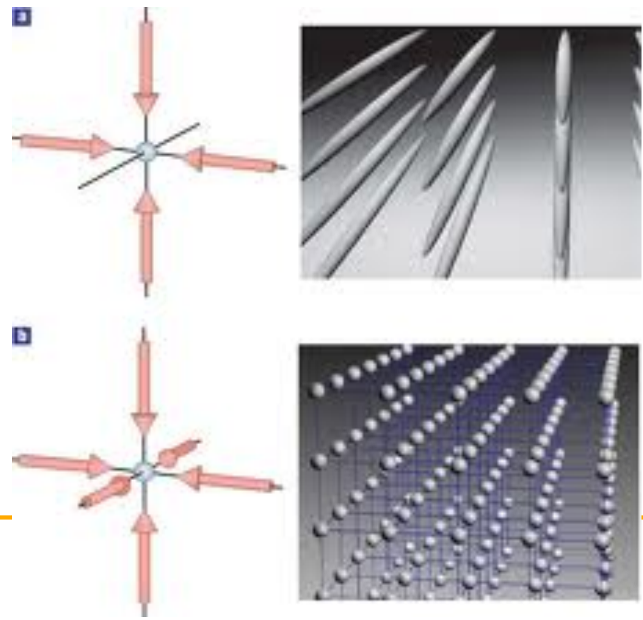

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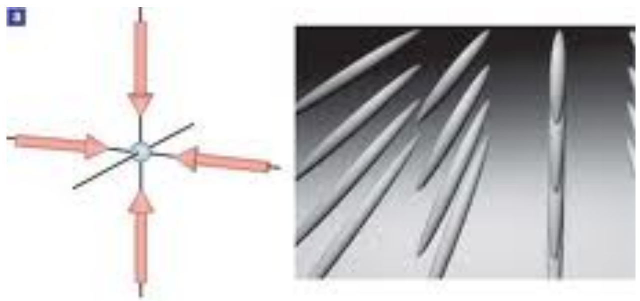
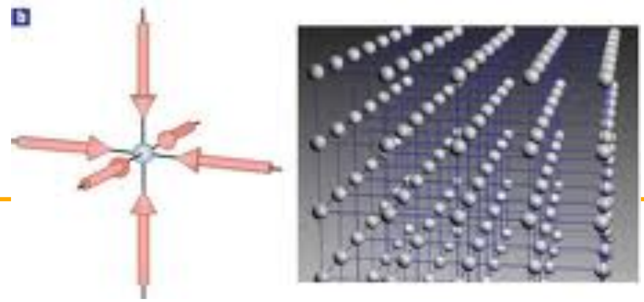
2D square optical lattice:



3D cubic optical lattice:



Optical lattices (more sophisticated)

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Experiment:

New Journal of Physics
The open-access journal for physics

Ultracold quantum gases in triangular optical lattices

New Journal of Physics **12** (2010) 065025 (17pp)

C Becker[1,3], P Soltan-Panahi[1], J Kronjäger[2], S Dörscher[1], K Bongs[2] and K Sengstock[1]

nature physics
ARTICLES
PUBLISHED ONLINE: 13 FEBRUARY 2011 | DOI: 10.1038/NPHYS1916

Multi-component quantum gases in spin-dependent hexagonal lattices

P. Soltan-Panahi[1], J. Struck[1], P. Hauke[2], A. Bick[1], W. Plenkers[1], G. Meineke[1], C. Becker[1], P. Windpassinger[1], M. Lewenstein[2,3] and K. Sengstock[1]*

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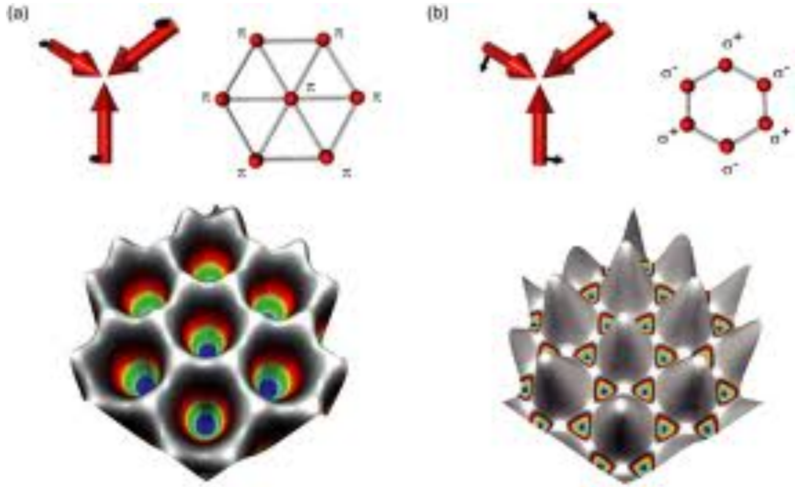

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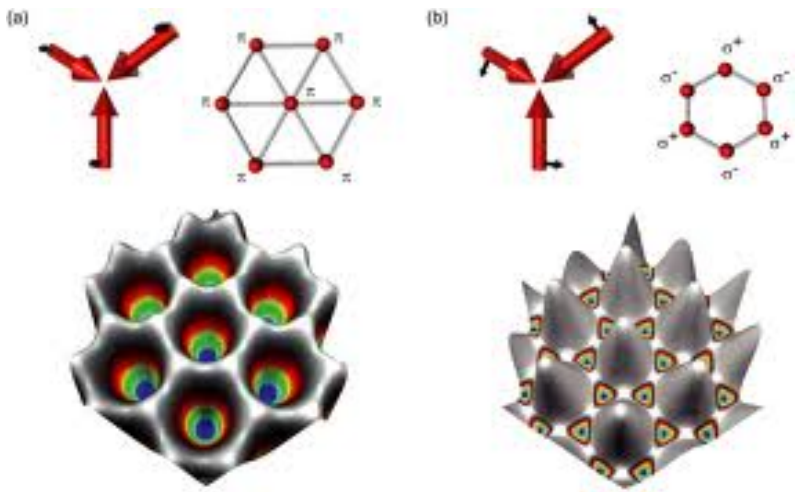

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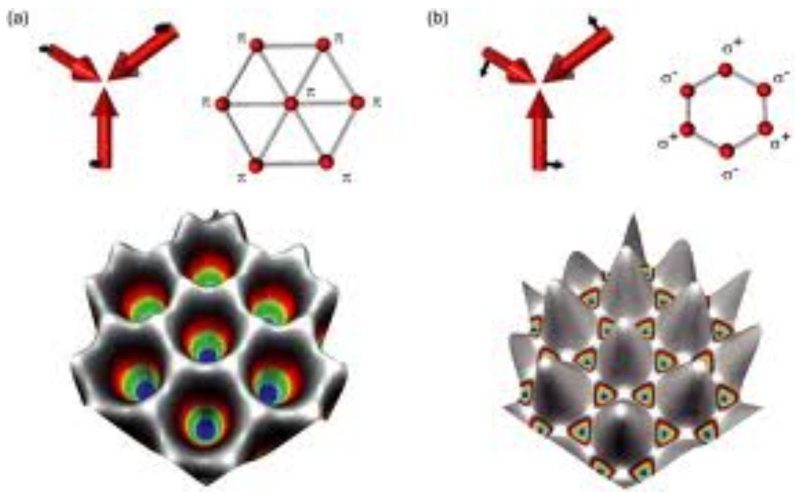

- (a) Polarisations are perpendicular to the plane → **Triangular** lattice
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→ Analogies with electrons graphene

Optical lattices (more sophisticated)

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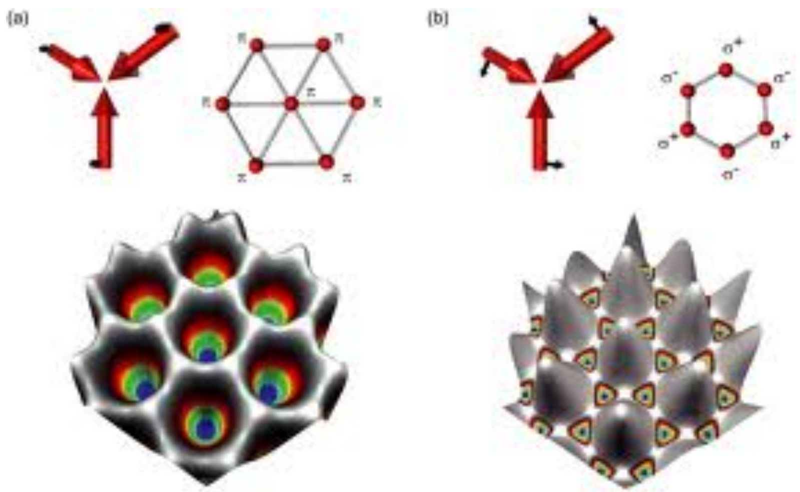

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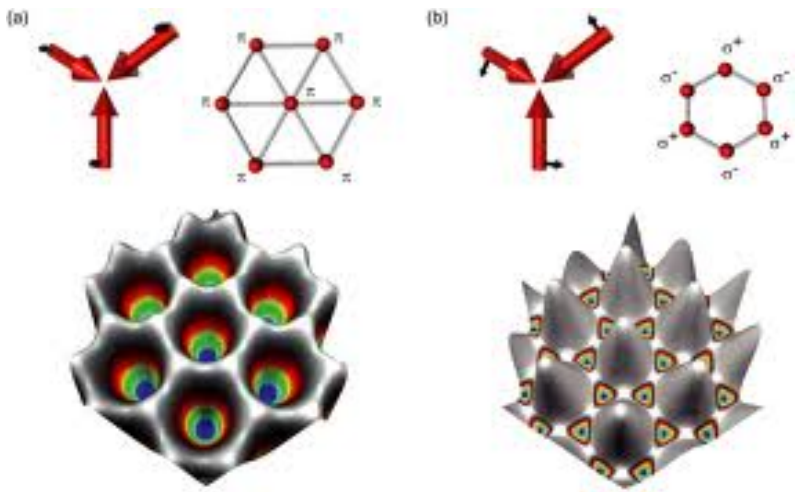

- (a) Polarisations are perpendicular to the plane → **Triangular** lattice
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- [traps differently atoms in different spin states]

Ultracold atoms

- Analogies with the solid state physics
  - Fermionic atoms ↔ Electrons in solids
  - Atoms in optical lattices – Hubbard model
  - Simulation of various many-body effects
- **Advantage :**
  - Freedom in changing experimental parameters that are often inaccessible in standard solid state experiments

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- **Advantage :**
  - Freedom in changing experimental parameters that are often inaccessible in standard solid state experiments
  - e.g. number of atoms, atom-atom interaction, lattice potential

Trapped atoms - **electrically neutral** species

- No direct analogy with **magnetic** phenomena by electrons in solids, such as the Quantum Hall Effect (no Lorentz force)
- A possible method to create an **effective** magnetic field (an **artificial** Lorentz force):

  Rotation

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  Rotation → **Coriolis force** →

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- A possible method to create an **effective** magnetic field:

  Rotation → **Coriolis force**

  (Mathematically equivalent to Lorentz force)

Trap rotation

Hamiltonian in the rotating frame

[see e.g. A. Fetter, RMP 81, 647 (2009)]

Trapping potential

$$H_0' = p^2/(2M) + \tfrac{1}{2} M \omega_\perp^2 r^2 - \boldsymbol{\Omega} \cdot \boldsymbol{r} \times \boldsymbol{p}$$

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or rotation vector

$$H_0' = \frac{(\boldsymbol{p} - M\boldsymbol{\Omega}\times\boldsymbol{r})^2}{2M} + \frac{1}{2}M(\omega_\perp^2 - \Omega^2)r^2$$

Effective vector potential

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(constant $B_{eff} \sim \Omega$)

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Effective vector potential (constant $B_{eff} \sim \Omega$)

Centrifugal potential (anti-trapping)

Coriolis force (equivalent to **Lorentz** force)

Trap rotation:

Summary of the **main features**

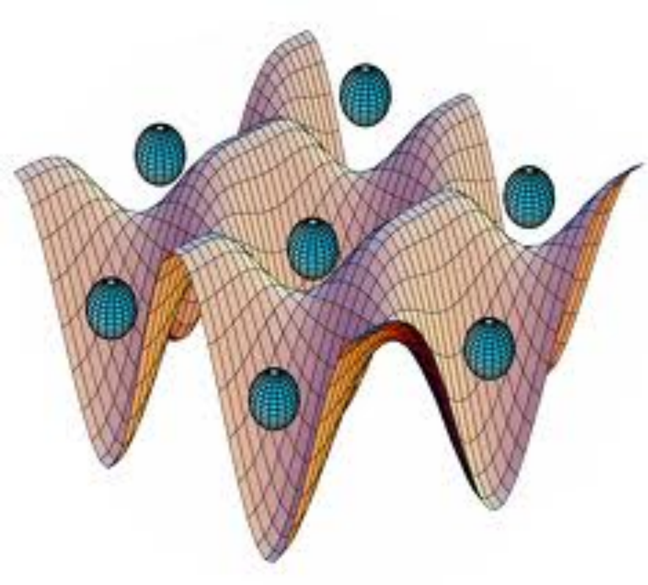
- Constant B_{eff}: $B_{eff} \sim \Omega$
- Trapping frequency: $\omega_{eff} = \sqrt{\omega_\perp^2 - \Omega^2}$
- $\Omega \to \omega_\perp$ ⇨ Landau problem

ROTATION

- Can be applied to utracold atoms both in **usual traps** and also in **optical lattices**

(a) Ultracold atomic cloud (trapped):

(b) Optical lattice:



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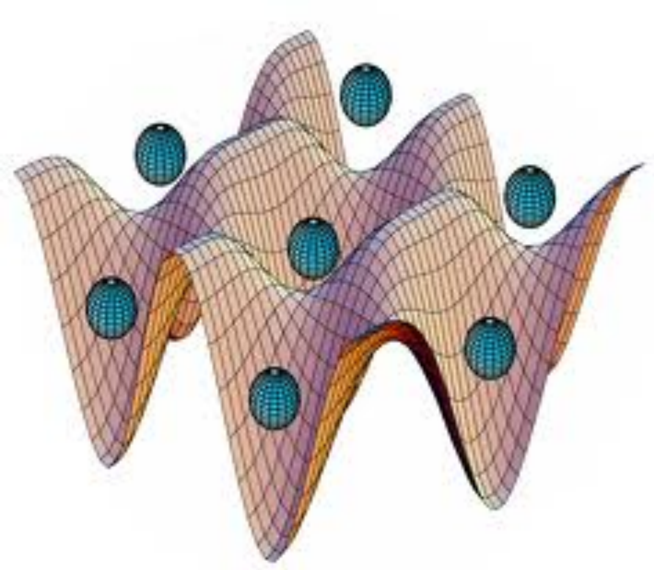

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- Limited magnetic flux

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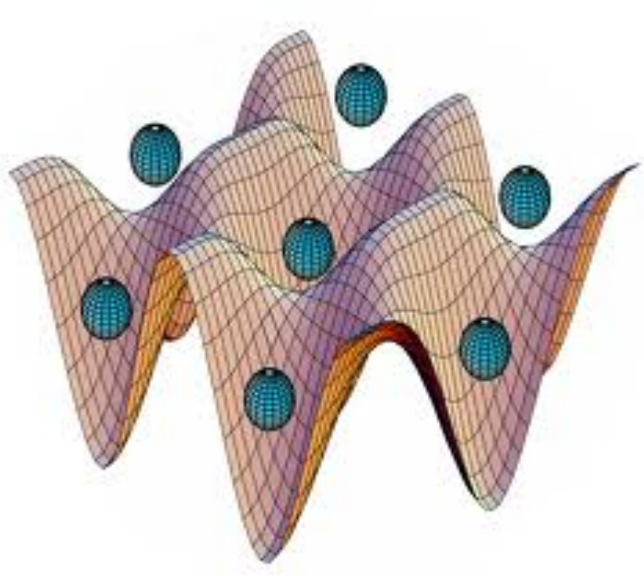

- Not always convenient to rotate an atomic cloud
- Limited magnetic flux → Other methods are desirable

Effective magnetic fields without rotation

- **Using (unconventional) optical lattices**

  Initial proposals:
  - J. Ruostekoski, G. V. Dunne, and J. Javanainen, Phys. Rev. Lett. 88, 180401 (2002)
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - E. Mueller, Phys. Rev. A **70**, 041603 (R) (2004)
  - A. S. Sørensen, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **94**, 086803 (2005)
- B_{eff} *is produced by inducing an* asymmetry in atomic transitions between the lattice sites.
- Non-vanishing phase for atoms moving along a closed path on the lattice (a plaquette)
- → Simulates non-zero magnetic flux → $B_{eff} \neq 0$

Effective magnetic fields without rotation

- **Optical square lattices**
  - D. Jaksch and P. Zoller, New J. Phys. **5**, 56 (2003)
  - J. Dalibard and F. Gerbier, NJP **12**, 033007 (2010).
  - -Ordinary tunneling along *x* direction (J).
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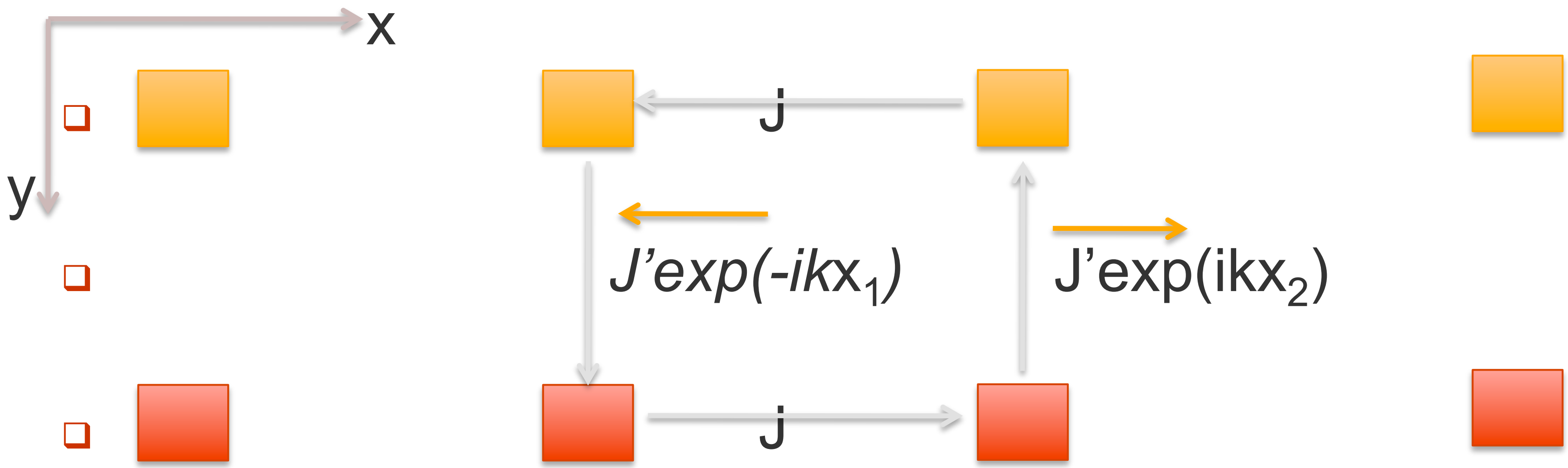

Atoms in different internal states (red or yellow) are trapped at different lattice sites

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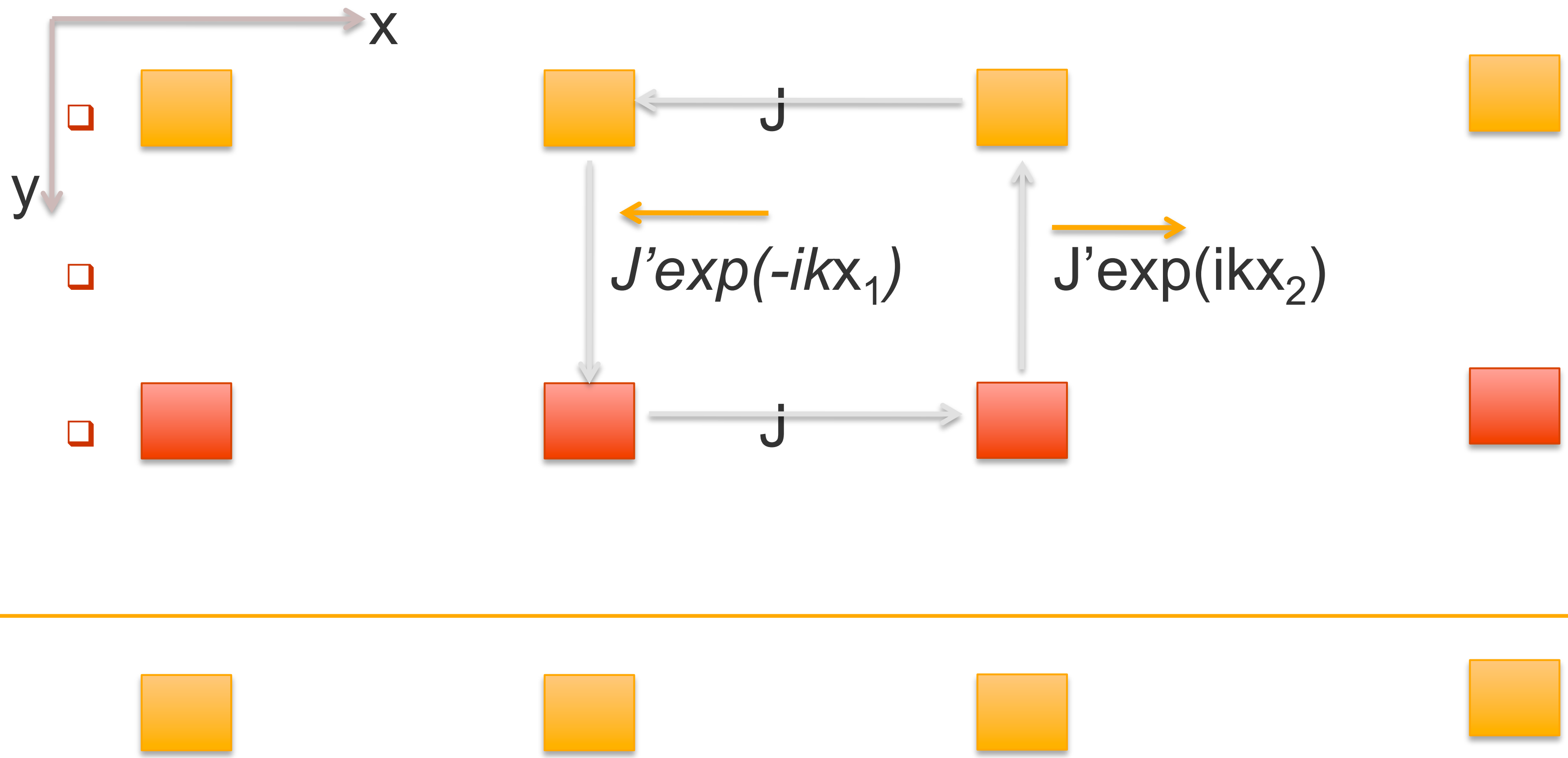

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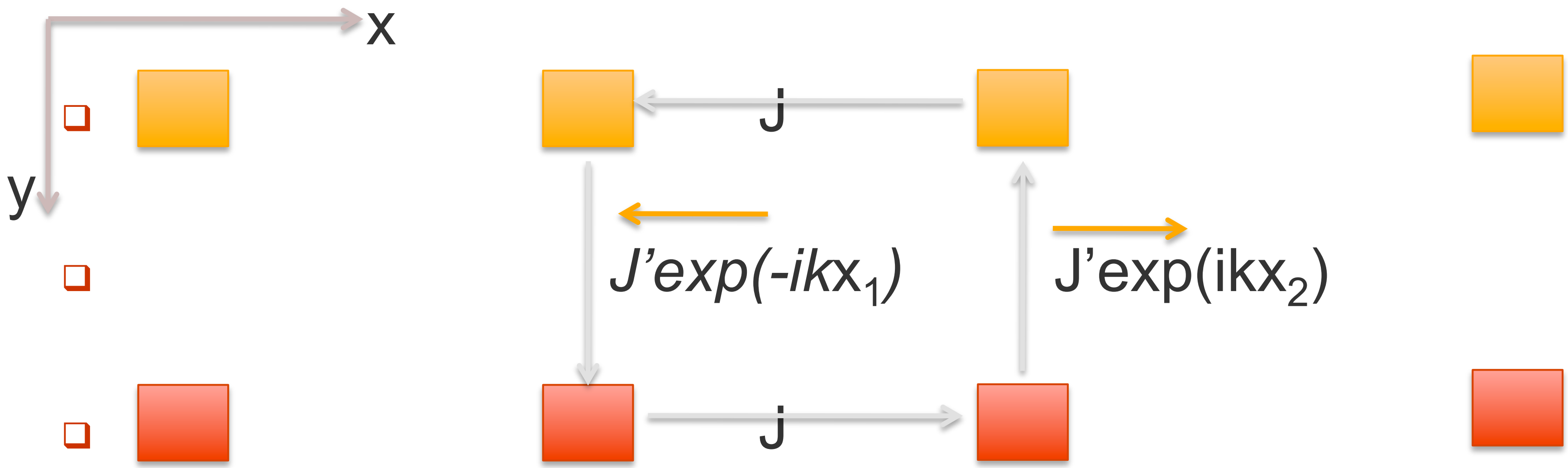

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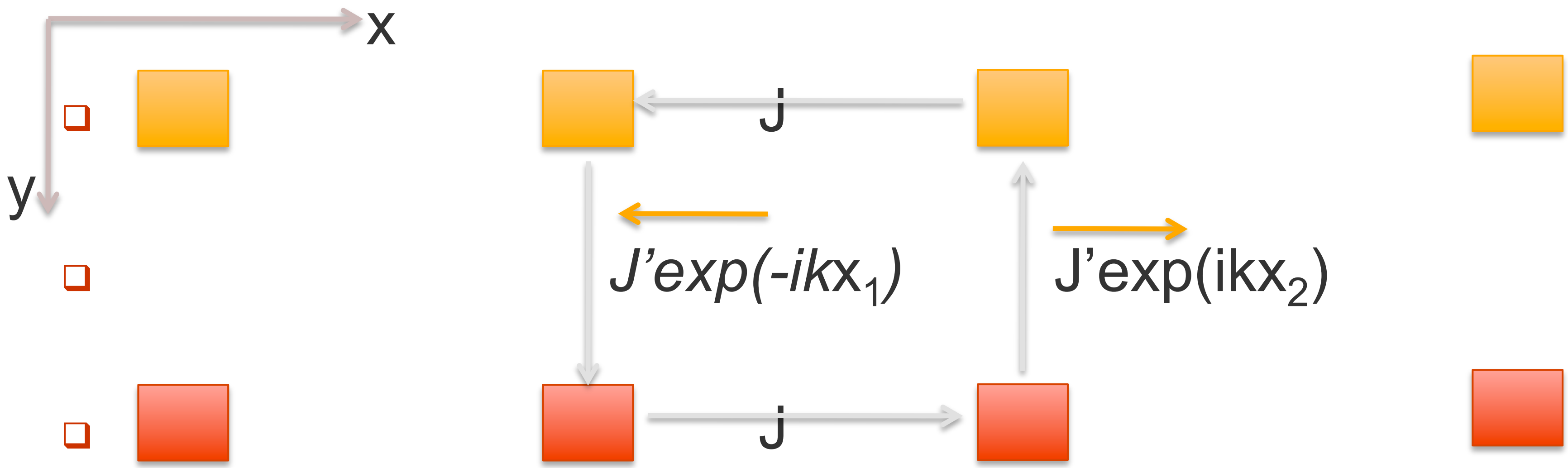

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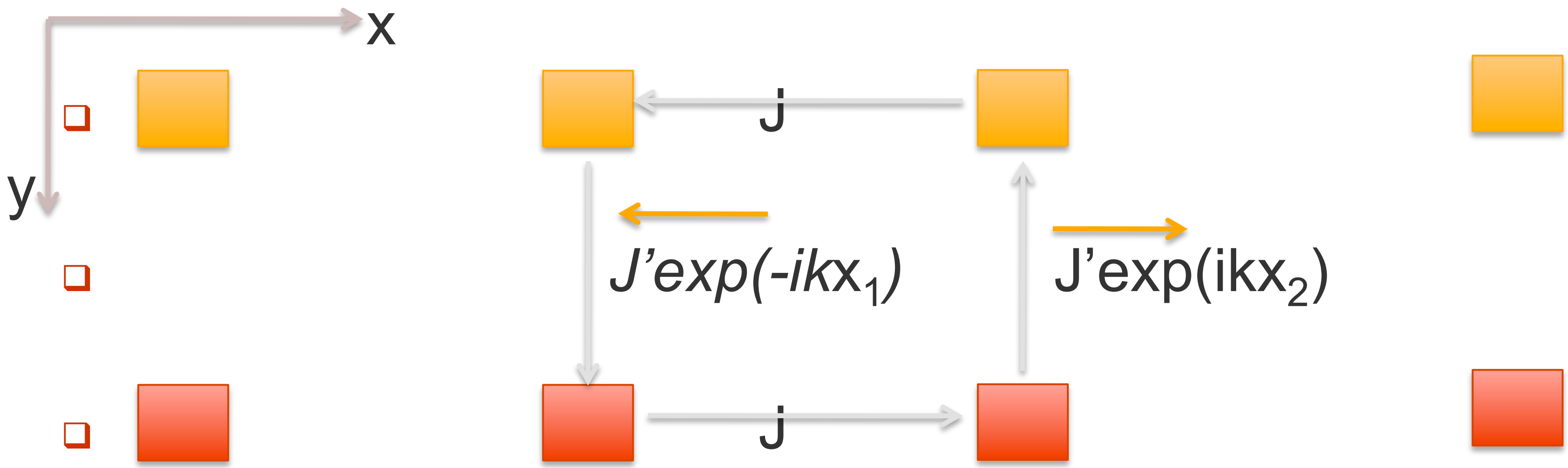

- Non-vanishing phase for the atoms moving over a plaquette: $S=k(x_2-x_1)=ka$
- → Simulates non-zero magnetic flux (over plaquette)

Effective magnetic fields without rotation

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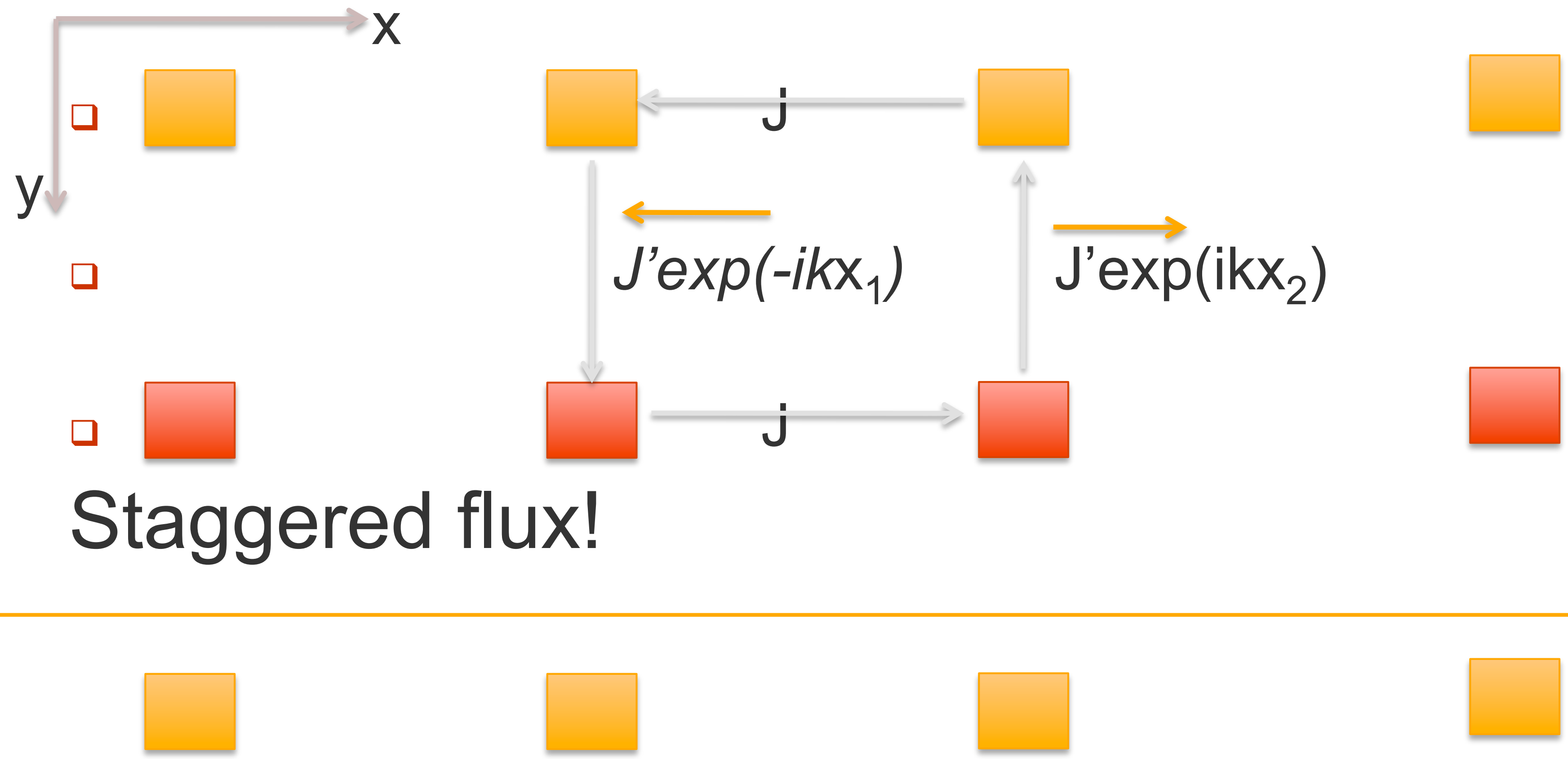

Staggered flux!

Effective magnetic fields without rotation

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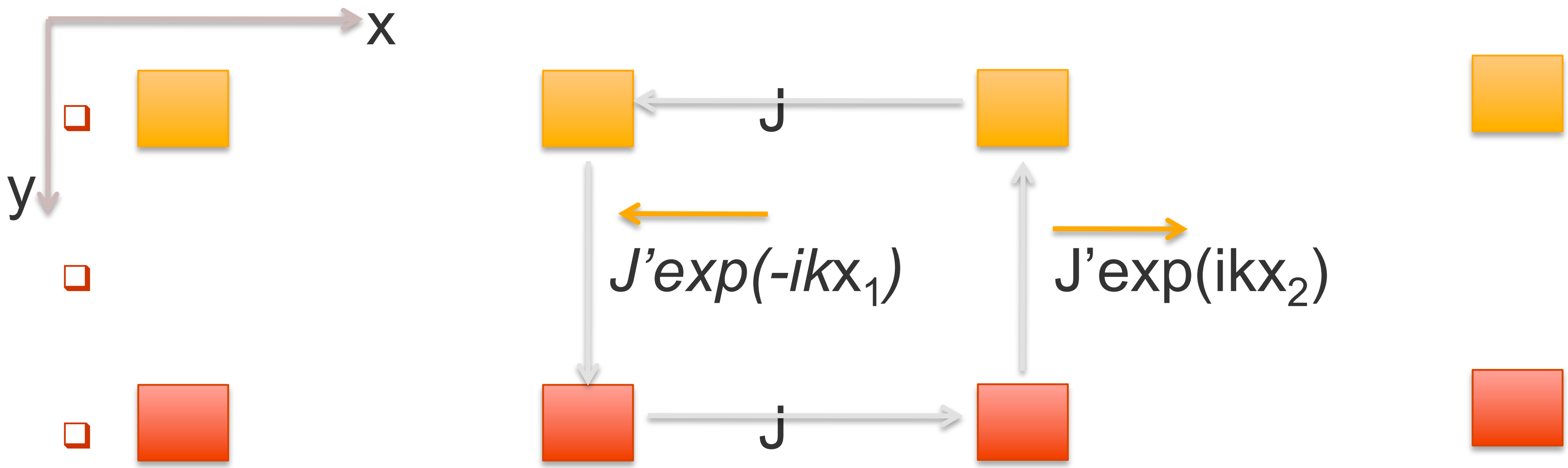

  - Non-vanishing phase for the atoms moving over a plaquette: $S=k(x_2-x_1)=ka$ → non-zero magnetic flux:
  - **Experiment: M. Aidelsburger et al., PRL 107, 255301 (2011)**

Effective magnetic fields without rotation

- **Optical square lattices**
  - **Experiment:** M. Aidelsburger, M. Atala, S. Nascimbène, S. Trotzky, Yu-Ao Chen and I Bloch, PRL 107, 255301 (2011)
  - -Ordinary tunneling along *y* direction.
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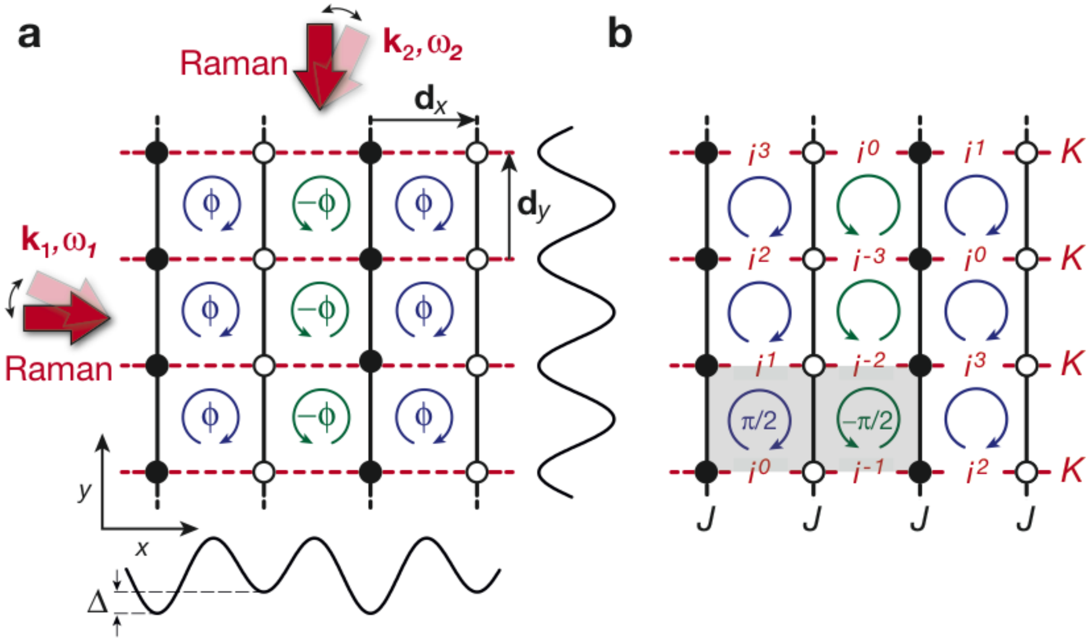
  - 

  - → Non-zero magnetic flux over a plaquette

Effective magnetic fields without rotation

- **Optical lattices**:

The method can be extended to create

Non-Abelian gauge potentials

(Laser assisted, state-sensitive tunneling)

A proposal:

- K. Osterloh, M. Baig, L. Santos, P. Zoller and M. Lewenstein, Phys. Rev. Lett. **95**, 010403 (2005)

Effective magnetic fields without rotation

<u>Optical square lattices</u>

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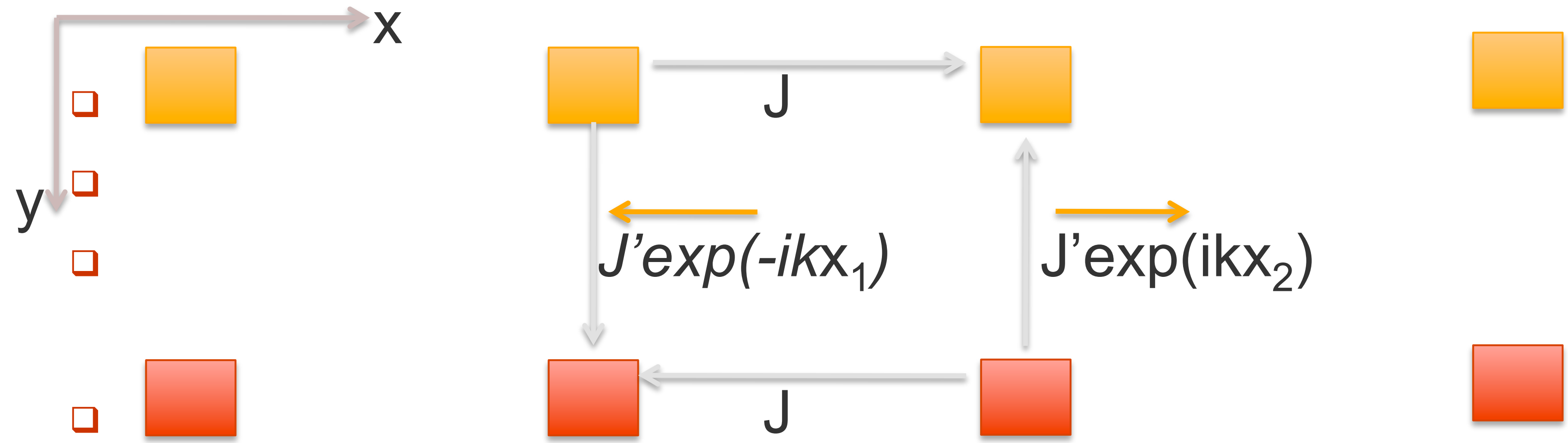
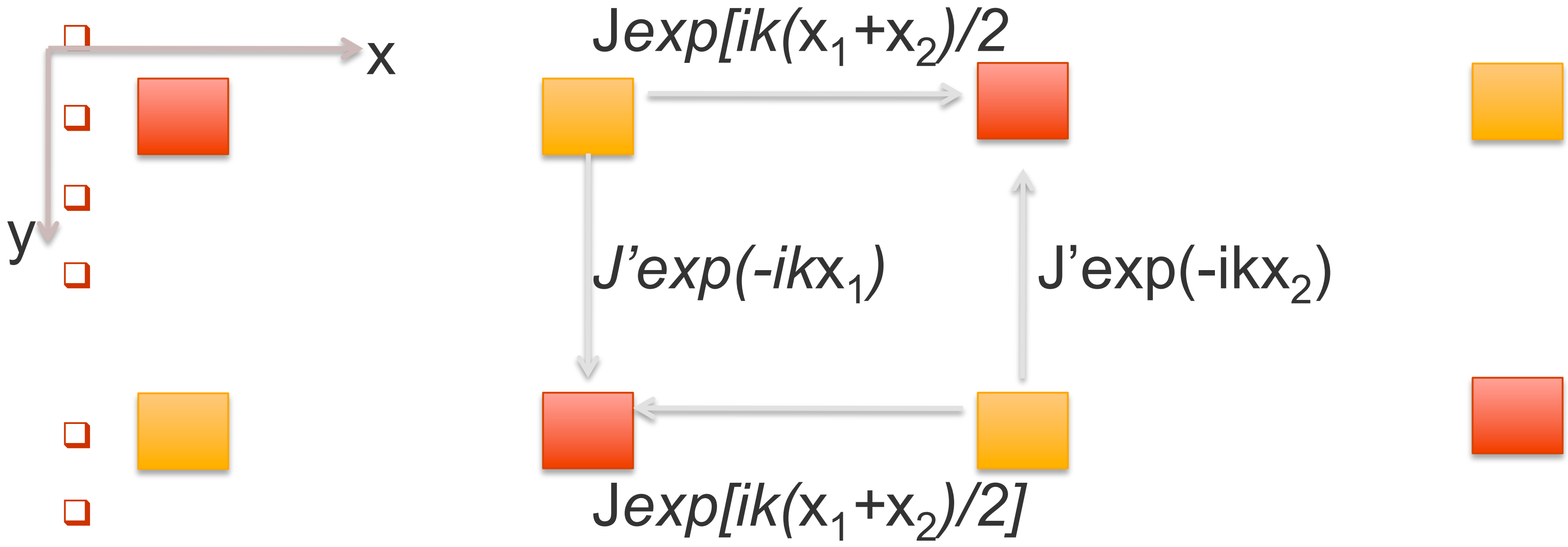


- <u>Non-vanishing phase</u> for the atoms moving over a (square) plaquette: $S=k(x_2-x_1)=ka$
- → Simulates <u>non-zero magnetic flux</u>

Effective magnetic fields without rotation

- **Optical square lattices**
  - **However** if laser-assisted tunneling is along both *x* and *y* axis (**with recoil e.g.** along x)

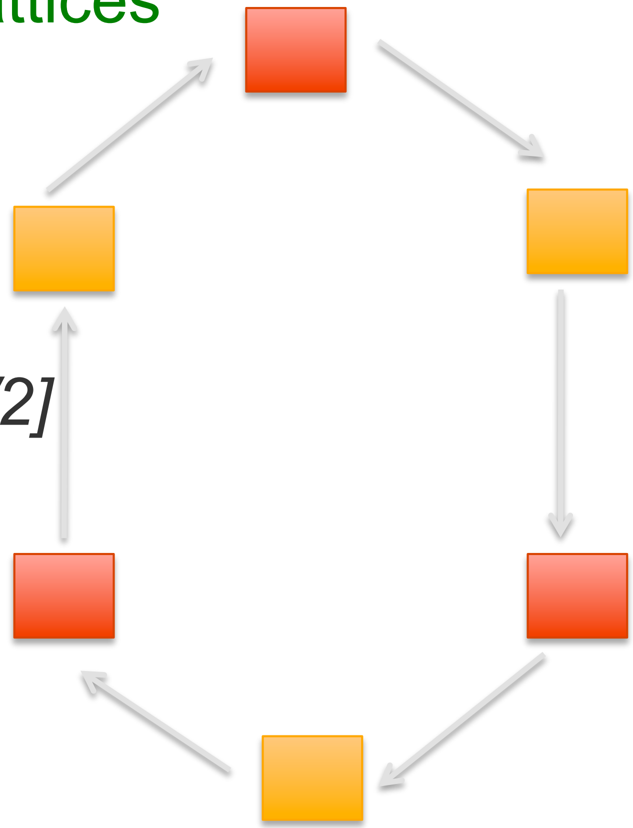


  - Vanishing phase for the atoms moving over a plaquette: $S=0$
  - → Zero magnetic flux

Effective magnetic fields without rotation

- **(Similarly) <u>Optical hexagonal lattices</u>**
  - Laser-assisted tunneling of atoms trapped in two triangular sublattices

$t_{jl}=J exp[i\mathbf{k}(\mathbf{r}_j+\mathbf{r}_l)/2]$



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Effective magnetic fields without rotation

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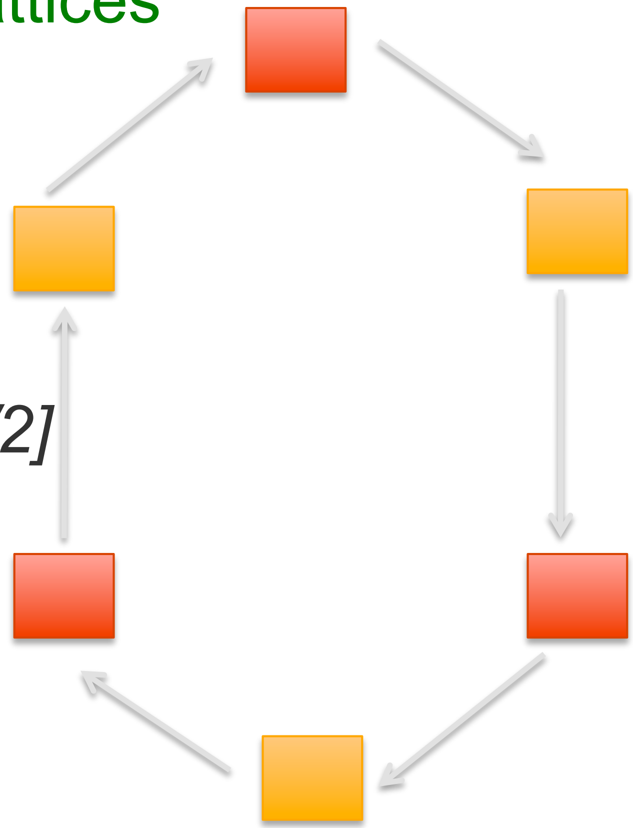

  - Vanishing phase for the atoms moving over a plaquette: *S=0* → Zero magnetic flux
  - unless there is (a real-valued) NNN coupling t_a

Effective magnetic fields without rotation

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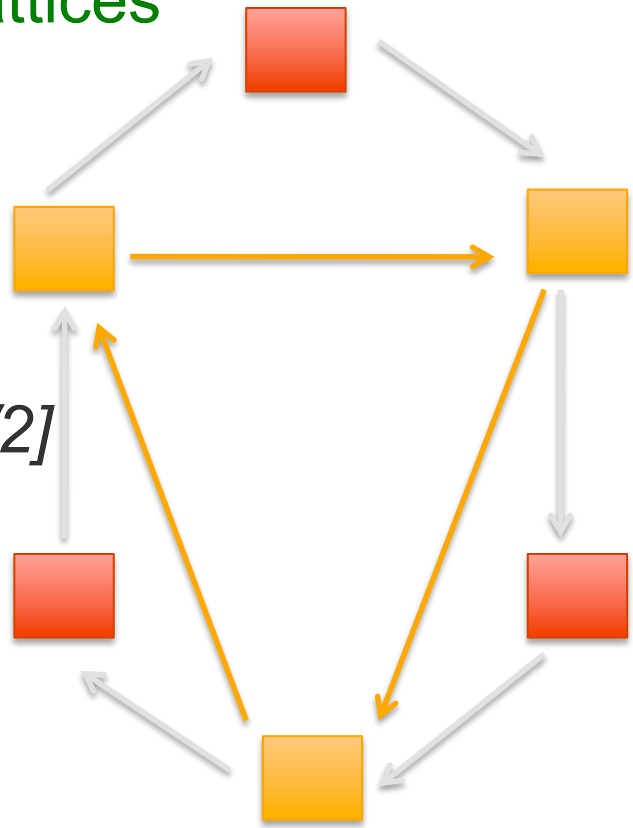

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Effective magnetic fields without rotation

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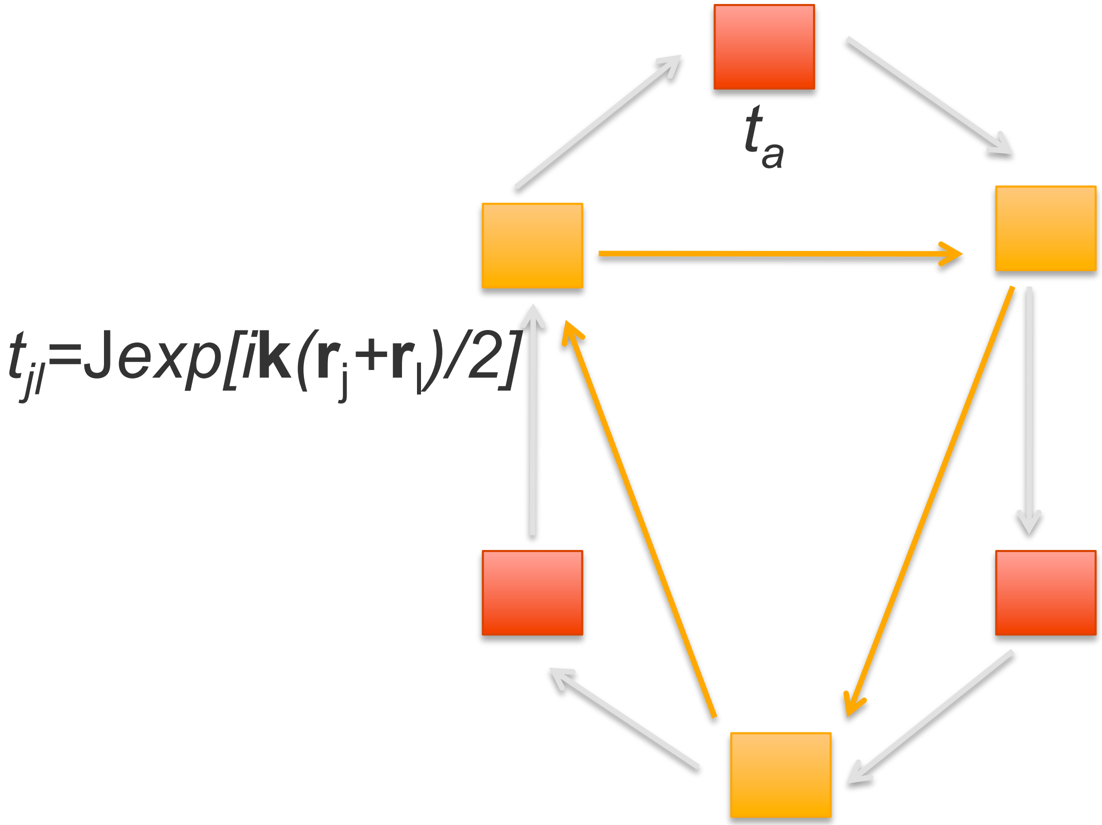
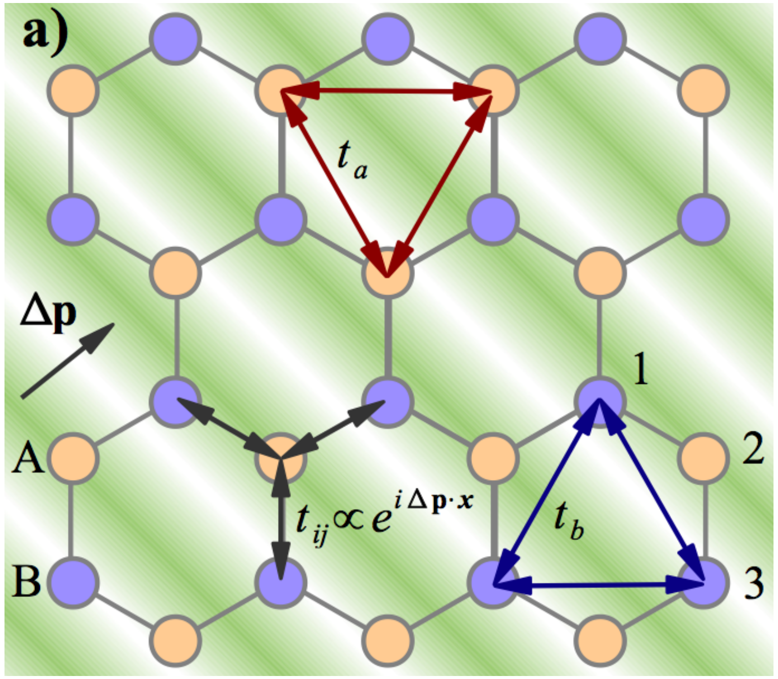


- **Non**-vanishing phase for the atoms moving over a triangular plaquette: $S=0$ → **Non**-zero magnetic flux

Effective magnetic fields without rotation

- **Optical hexagonal lattices**
  - Laser-assisted tunneling of atoms trapped in two triangular sublattices

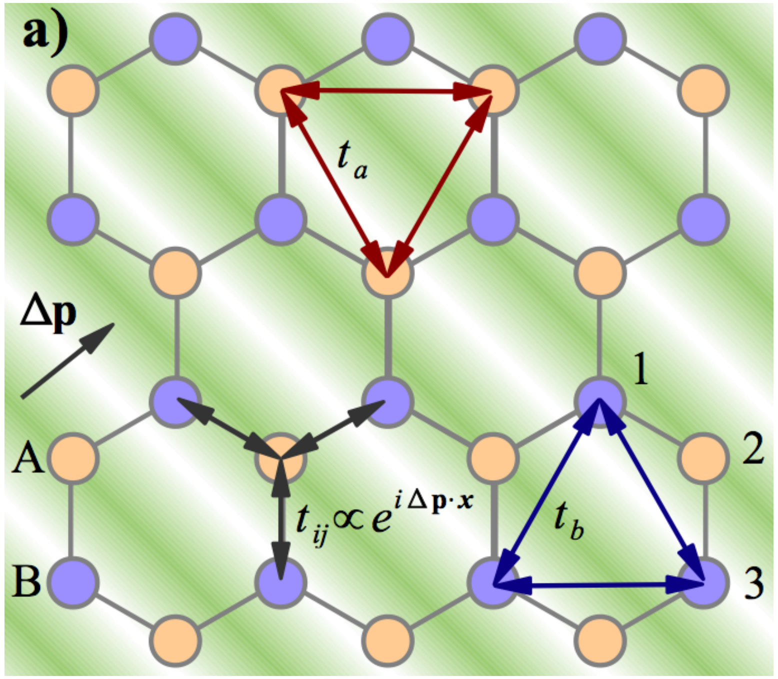


  - Vanishing phase for the atoms moving over hexagonal plaquettes: *S=0* → Zero magnetic flux

Effective magnetic fields without rotation

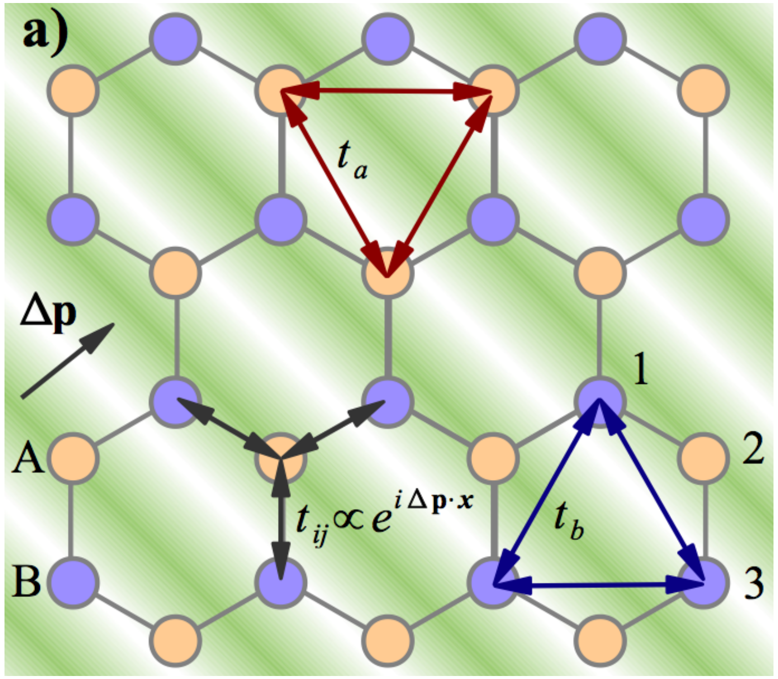
Optical hexagonal lattices

- Laser-assisted tunneling between two triangular sublattices
- If NNN tunneling between the same sub-lattices (t_a and t_b)



Effective magnetic fields without rotation

- **Optical hexagonal lattices**
  - Laser-assisted tunneling between two triangular sublattices
  - If NNN tunneling between the same sublattices (t_a and t_b)



  - → Non-vanishing phase for the atoms moving over (triangular) sub-plaquettes

Effective magnetic fields without rotation

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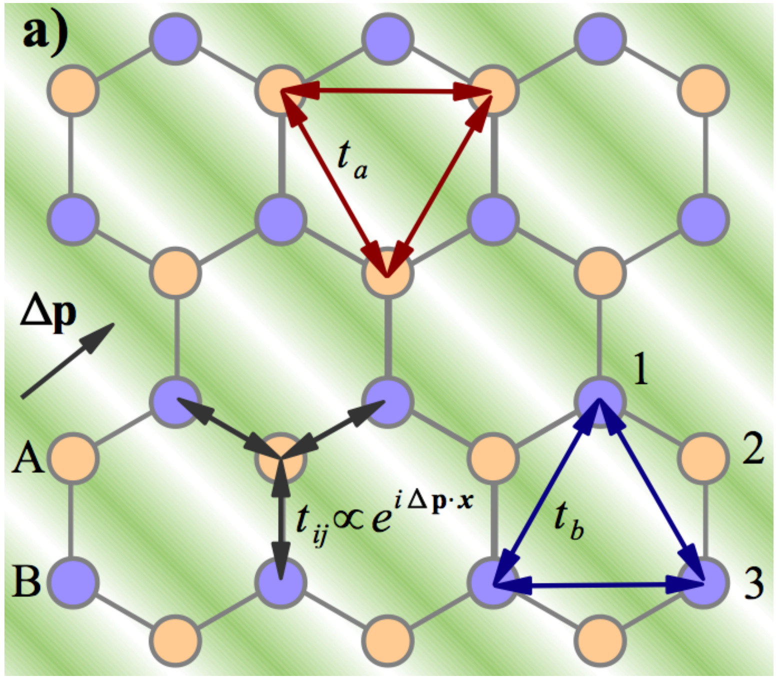

  - → Non-vanishing phase for the atoms moving over sub-plaquettes: → Non-zero Chern number (Haldane, PRL'1988)

Effective magnetic fields without rotation

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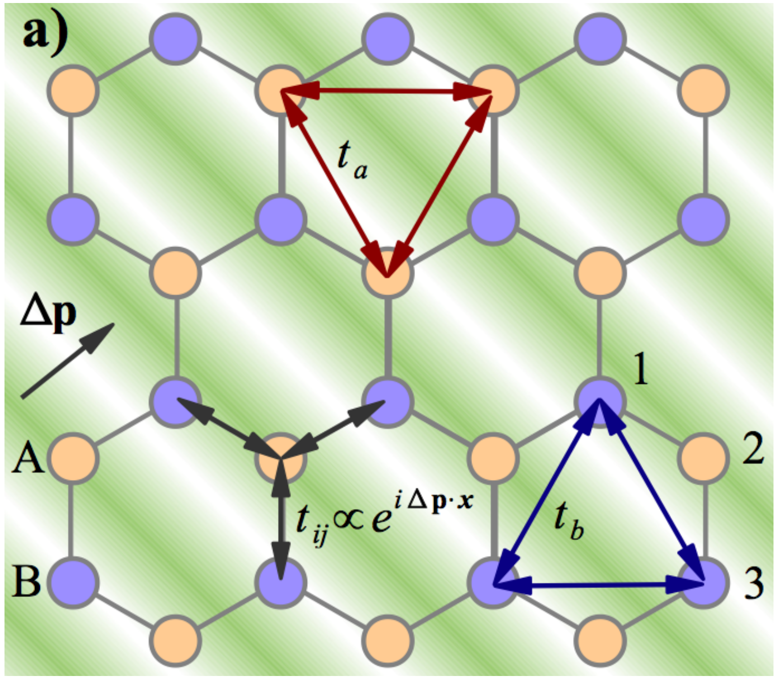
  - If NNN tunneling between the same sublattices (t_a and t_b)



  - → Non-vanishing phase for the atoms moving over sub-plaquettes: → Non-zero Chern numbers

Chern numbers can be found from TFM ! (PRL'2011)

Effective magnetic fields without rotation

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Selected for a Viewpoint in *Physics*
PRL **107**, 235301 (2011) PHYSICAL REVIEW LETTERS week ending 2 DECEMBER 2011

Seeing Topological Order in Time-of-Flight Measurements

E. Alba,[1] X. Fernandez-Gonzalvo,[1] J. Mur-Petit,[1] J. K. Pachos,[2] and J. J. Garcia-Ripoll[1]
[1]*Instituto de Física Fundamental, IFF-CSIC, Calle Serrano 113b, Madrid 28006, Spain*
[2]*School of Physics and Astronomy, University of Leeds, Leeds, LS2 9JT, United Kingdom*
(Received 27 May 2011; revised manuscript received 31 August 2011; published 28 November 2011)

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  - → Non-vanishing phase for the atoms moving over a sub-plaquettes: → Non-zero Chern numbers
  - **Chern numbers can be found from TFM!**
  - **(momentum-dependence of atomic "spin" in TFM)**

Physics

Physics **4**, 99 (2011)

Viewpoint

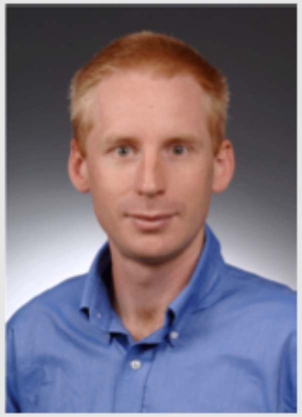
A Viewpoint on:
Seeing Topological Order in Time-of-Flight Measurements
E. Alba, X. Fernandez-Gonzalvo, J. Mur-Petit, J. K. Pachos, and J. J. Garcia-Ripoll
Phys. Rev. Lett. **107**, 235301 (2011) – Published November 28, 2011

Seeing Topological Order

Gediminas Juzeliūnas

Gediminas Juzeliūnas is a principal researcher and a deputy director at the Institute of Theoretical Physics and Astronomy of Vilnius University, Lithuania. He is also professor of physics at Lithuanian University of Educational Sciences in Vilnius. Dr. Juzeliūnas completed his Ph.D. in 1986 in theoretical condensed matter physics at Vilnius University, studying optical properties of excitons in confined geometries. Subsequently he held a two-year postdoctoral appointment at the University of East Anglia, England, shifting his research area towards quantum optics. Dr. Juzeliūnas was a Humboldt Research Fellow at the University of Ulm, Germany (1997–1998), and a Fulbright Scholar at the University of Oregon in the US (2000–2001). Dr. Juzeliūnas received a National State Prize for Science of Lithuania in 2008 and a Vilnius University Rector's award in 2010. His current research focuses on ultracold atomic gases, slow light, metamaterials, and graphene. This includes a pioneering theoretical work on light-induced gauge potential for ultracold atoms.

Ian Spielman



Ian Spielman in an experimentalist who received his Ph.D. in physics from the California Institute of Technology in 2004, studying quantum Hall bilayers. From there he moved to NIST in Gaithersburg, Maryland, for a two year NRC postdoc in the Laser Cooling and Trapping group, studying the physics of the superfluid-to-insulator transition in 2D atomic Bose gases. In 2006, he assumed his current position as a NIST physicist and a fellow of the newly founded Joint Quantum Institute (JQI). His research interests focus on using ultracold atomic systems to realize Hamiltonians familiar in condensed matter physics. This includes the pioneering work on creating artificial gauge fields for neutral atoms using laser atom interactions.

Physics

Physics **4**, 99 (2011)

Viewpoint

A Viewpoint on:

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Seeing Topological Order

Calculations by Ian Spielman:

(numerical "experiment")

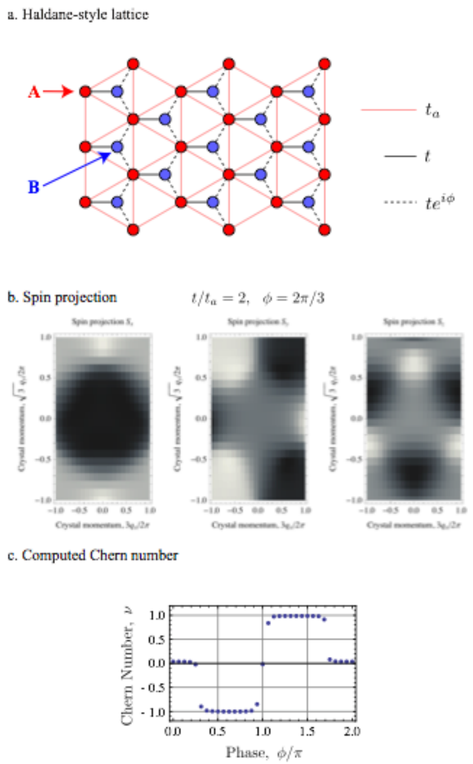

FIG. 1. Model System. a. Geometry of a Haldane-style lattice with two interpenetrating triangular lattices. Laser induced complex valued hopping $t\exp(i\phi)$ (dashed links) allow non-zero Chern numbers. b. Predicted projection of spin along along $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, as could be measured using conventional time-of-flight imaging with ultracold atomic systems. c. Chern number as a function of ϕ computed from 40×40 images such as in b.

Effective magnetic fields without rotation -- using **Geometric Potentials**

- Distinctive features:
  - No rotation is necessary
  - **No lattice is needed**
  - Yet a lattice can be an important ingredient in creating B_{eff} using geometric potentials → **Optical flux lattices**

Geometric potentials

- Emerge in various areas of physics (molecular, condensed matter physics etc.)
- First considered by Mead, Berry, Wilczek and Zee and others in the 80's (initially in the context of molecular physics).
- More recently – in the context of motion of cold atoms affected by laser fields

  (**Currently:** a lot of activities)
- See, e.g.: J.Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg, Rev. Mod. Phys. **83**, 1523 (2011).
- Advantage of such atomic systems: possibilities to control and shape gauge potentials by choosing proper laser fields.

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Creation of B_{eff} using geometric potentials

Atomic dynamics taking into account both internal degrees of freedom and also center of mass motion.

- The full atomic Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hat{V}(\mathbf{r}) + \hat{H}_0(\mathbf{r}, t).$$

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  - $\hat{H}_0(\mathbf{r}, t)$ — the Hamiltonian for the electronic (fast) degrees of freedom, ← (includes **r**-dependent atom-light coupling)
  - $\hat{p}^2/2M + \hat{V}(\mathbf{r})$ — the Hamiltonian for center of mass (slow) degrees of freedom.
  - $\hat{V}(\mathbf{r})$ — the external trapping potential. (for c.m. motion)
- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r}, t)$.

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↖ (**r**-dependent "dressed" eigenstates)

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- $\hat{H}_0(\mathbf{r}, t)$ has eigenfunctions $|\chi_n(\mathbf{r}, t)\rangle$ with eigenvalues $\varepsilon_n(\mathbf{r}, t)$.
- Full atomic wave function ↖ (**r**-dependent "dressed" eigenstates)

$$|\Phi\rangle = \sum_n \Psi_n(\mathbf{r}, t)|\chi_n(\mathbf{r}, t)\rangle.$$

- Adiabatic atomic energies $\varepsilon_n(\mathbf{r})$

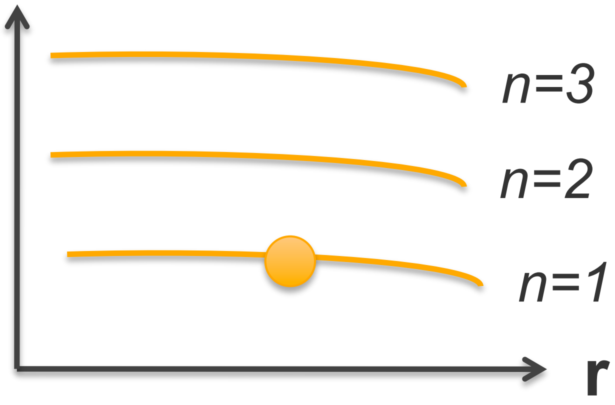
- Full state vector:

$$|\Phi\rangle = \sum_{n=1}^{N} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$

$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal “dressed” state $|\chi_n(\mathbf{r})\rangle$

Non-degenerate state with n=1

- Adiabatic atomic energies $\varepsilon_n(\mathbf{r})$



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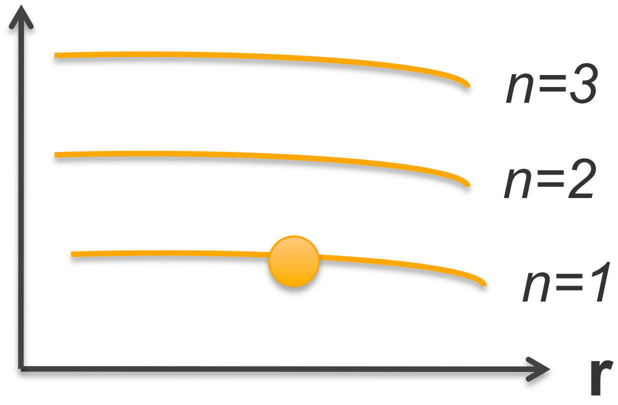

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$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$

(only the atomic internal state with *n=1* is included)

Non-degenerate state with n=1

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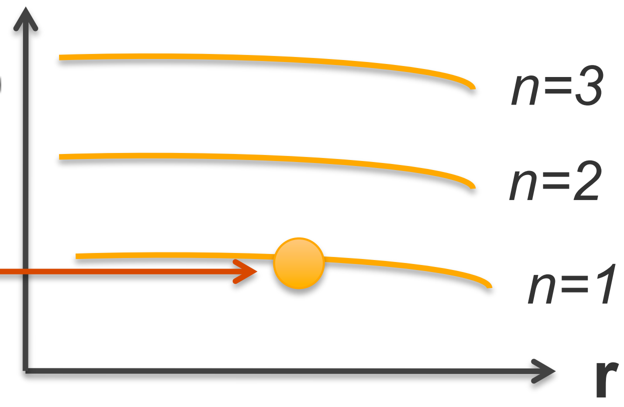

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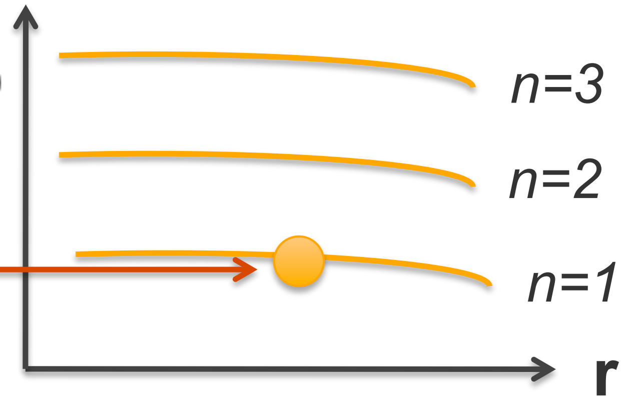

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- Adiabatic approximation

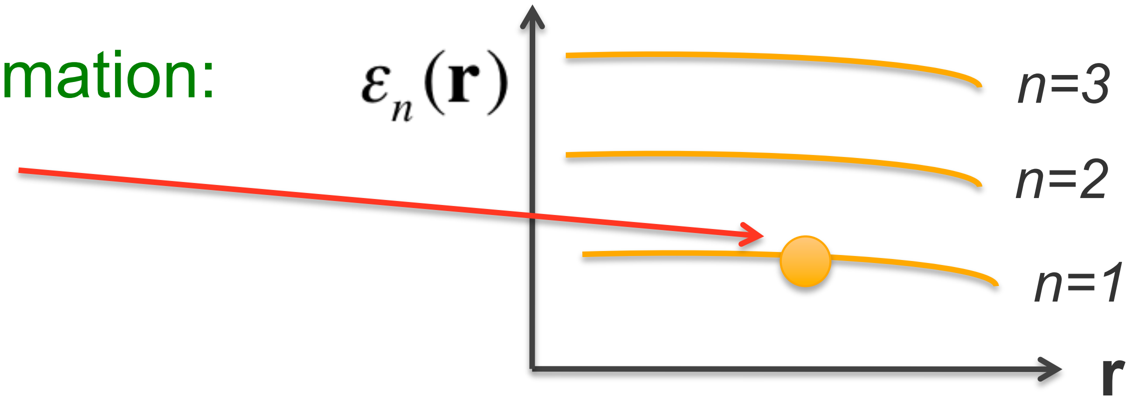
$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$

- What is the equation of motion for $\Psi_1(\mathbf{r},t)$?

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



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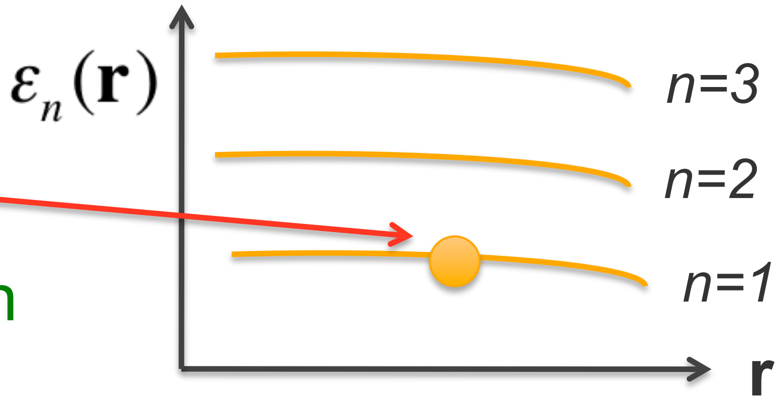

Equation of the atomic motion

in the internal state $|\chi_1(\mathbf{r})\rangle$

$$i\hbar\partial_t \Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\hat{H} = \frac{(\mathbf{p} - \mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) \qquad \mathbf{p} = -i\hbar\nabla$$

$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears

Non-degenerate state with n=1

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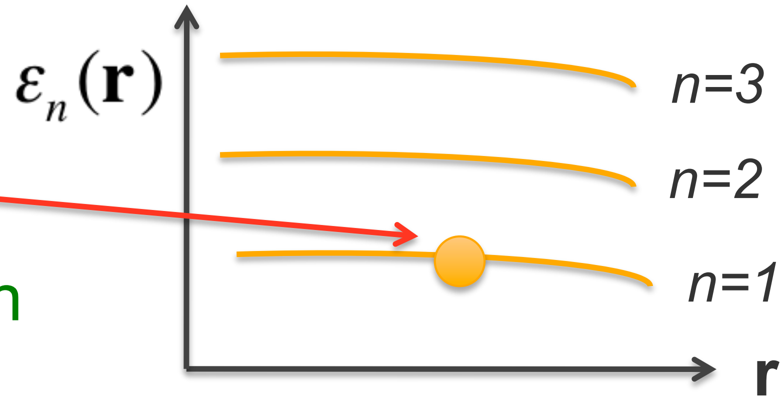

Equation of the atomic motion in the internal state $|\chi_1(\mathbf{r})\rangle$

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Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears (due to the position-dependence of the atomic internal "dressed" state $|\chi_1(\mathbf{r})\rangle$)

Non-degenerate state with n=1

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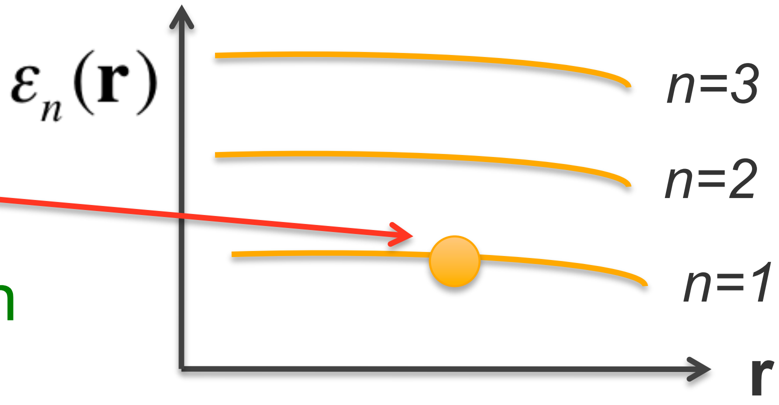

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$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field

Non-degenerate state with n=1

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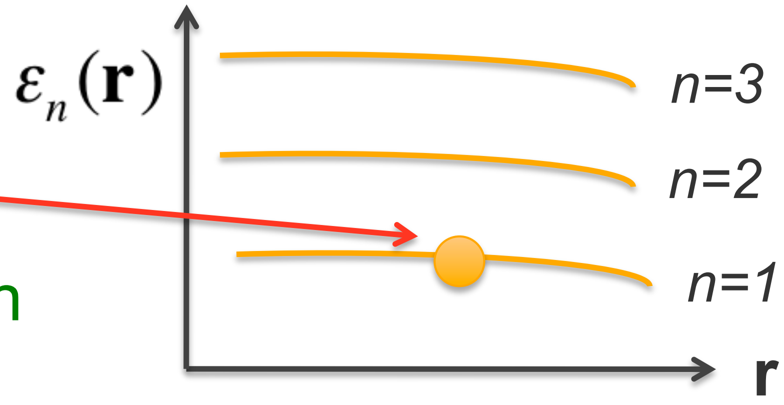

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$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B} \neq 0$)

To summarise

Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears (due to the position-dependence of the atomic internal "dressed" state $|\chi_1(\mathbf{r})\rangle$)

$$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B} \neq 0$)

Large possibilities to **control and shape** the effective magnetic field **B** by **changing** the **light beams**

Light induced effective magnetic field can be due to

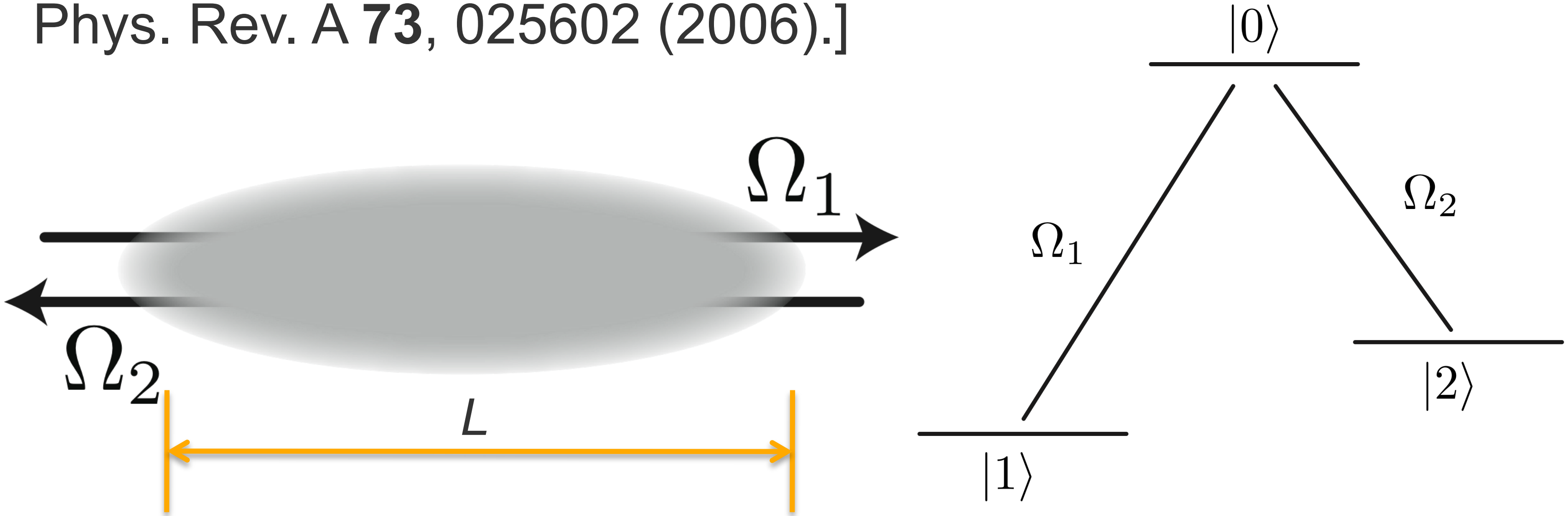
1. ***Spatial dependence*** of laser amplitudes
2. ***Spatial dependence*** of atom-light detuning
3. ***Spatial dependence*** of **both** the laser amplitudes and also atom-light detuning (e.g. optical flux lattices)

Light induced effective magnetic field can be due to

1. ***Spatial dependence*** of laser amplitudes

Counter-propagating beams with spatially shifted profiles

[G. Juzeliūnas, J. Ruseckas, P. Öhberg, and M. Fleischhauer, Phys. Rev. A **73**, 025602 (2006).]



$$\mathbf{B} = \nabla \times \mathbf{A} \neq 0$$

Artificial Lorentz force

Counter-propagating beams with spatially shifted profiles

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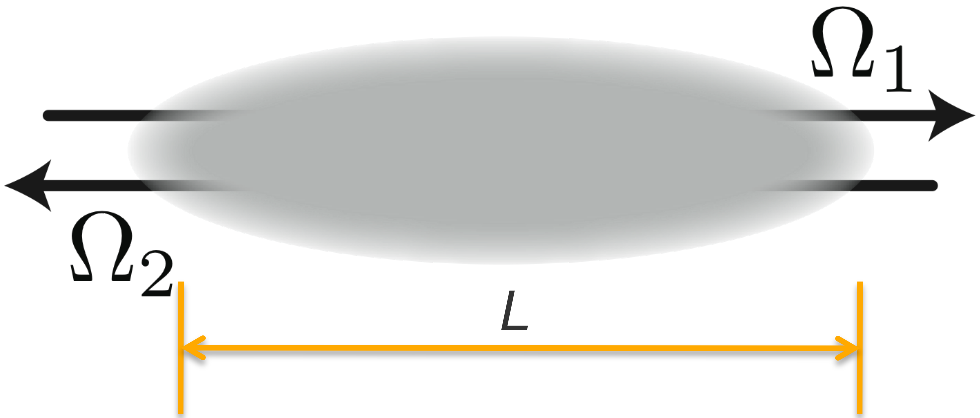

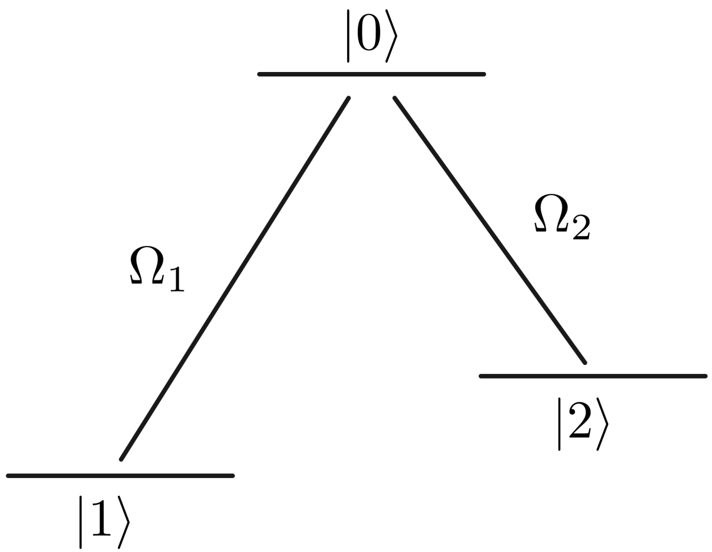

$$\mathbf{B} = \nabla \times \mathbf{A} \neq 0$$

Artificial Lorentz force (due to **photon recoil**)

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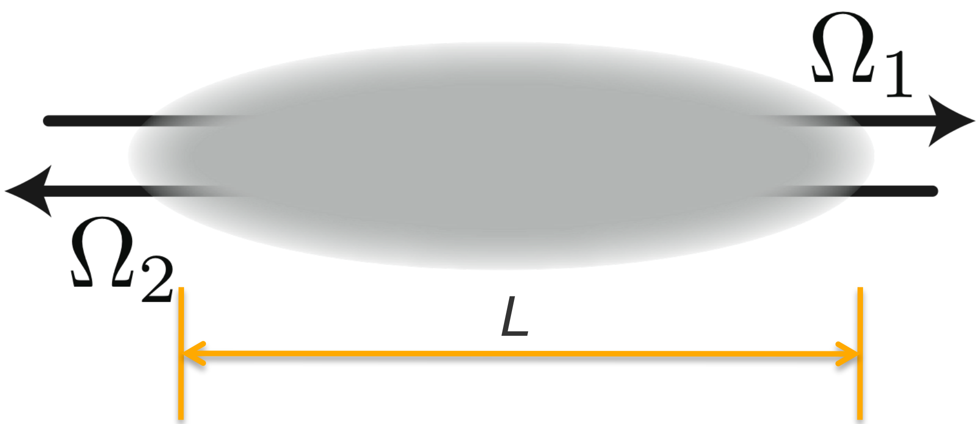

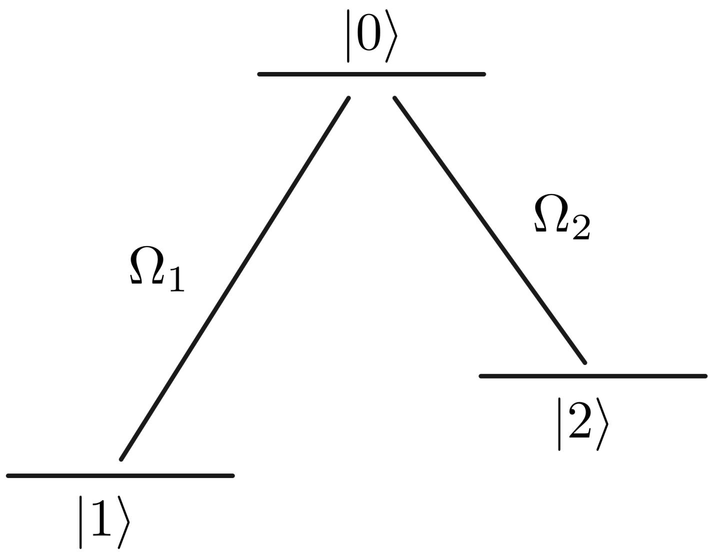

Total magnetic flux is proportional to the **sample length** *L*: $\Phi \approx kL$

(one can not increase the total flux in the transverse direction)

Counter-propagating beams with spatially shifted profiles

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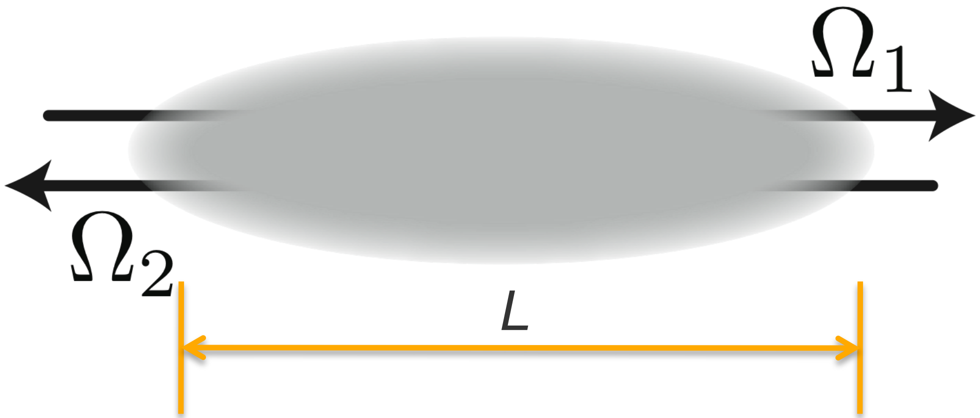

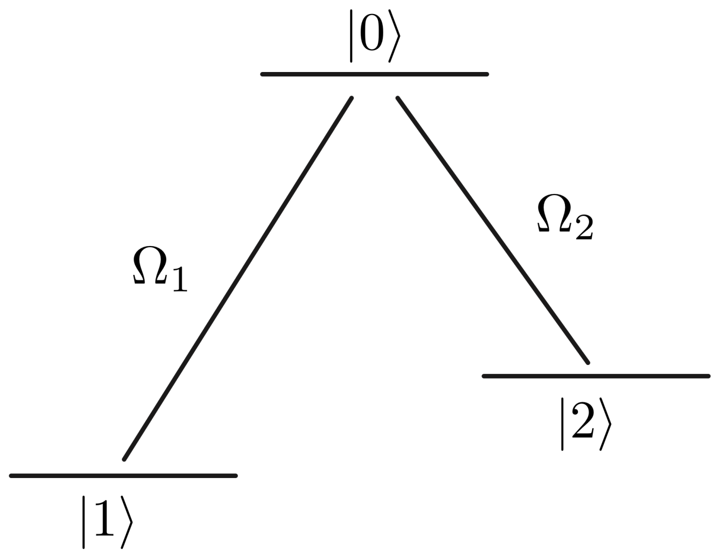

Total magnetic flux is proportional to the **sample length** *L*: $\Phi \approx kL$

(one can not increase the total flux in the transverse direction)

No translational symmetry for shifted beams (in the transverse direction):

→ No lattice

Light induced effective magnetic field due to

- ~~***Spatial dependence*** of laser amplitudes~~
- ***Spatial dependence*** of atom-light detuning

PRL 102, 130401 (2009) | Selected for a Viewpoint in *Physics* | PHYSICAL REVIEW LETTERS | week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*

Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA

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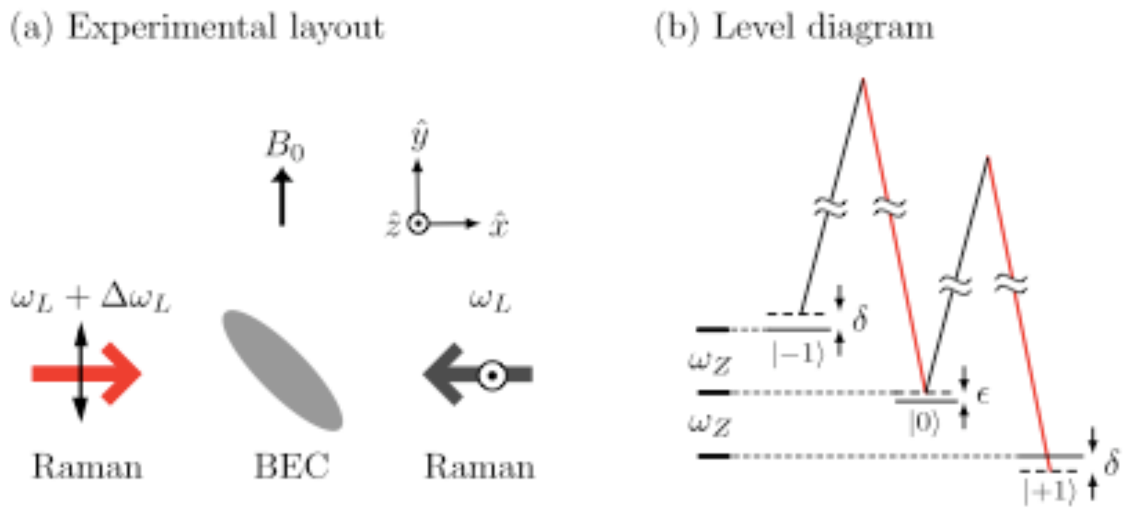

nature Vol 462 | 3 December 2009 | doi:10.1038/nature08609

LETTERS

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

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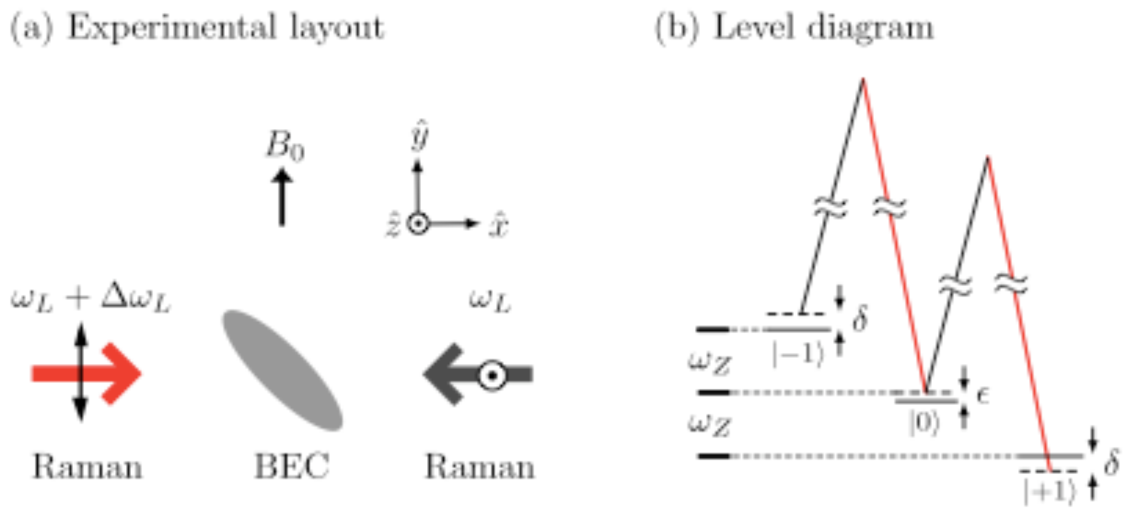

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Position-dependent detuning δ

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PRL 102, 130401 (2009) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*
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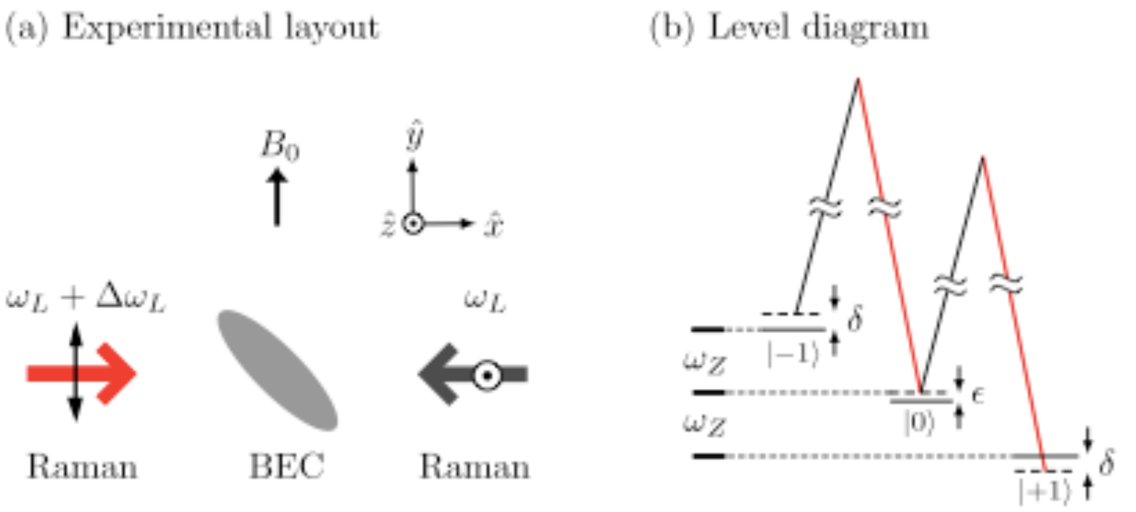
(Received 17 September 2008; published 30 March 2009)



nature Vol 462 | 3 December 2009 | doi:10.1038/nature08609

LETTERS

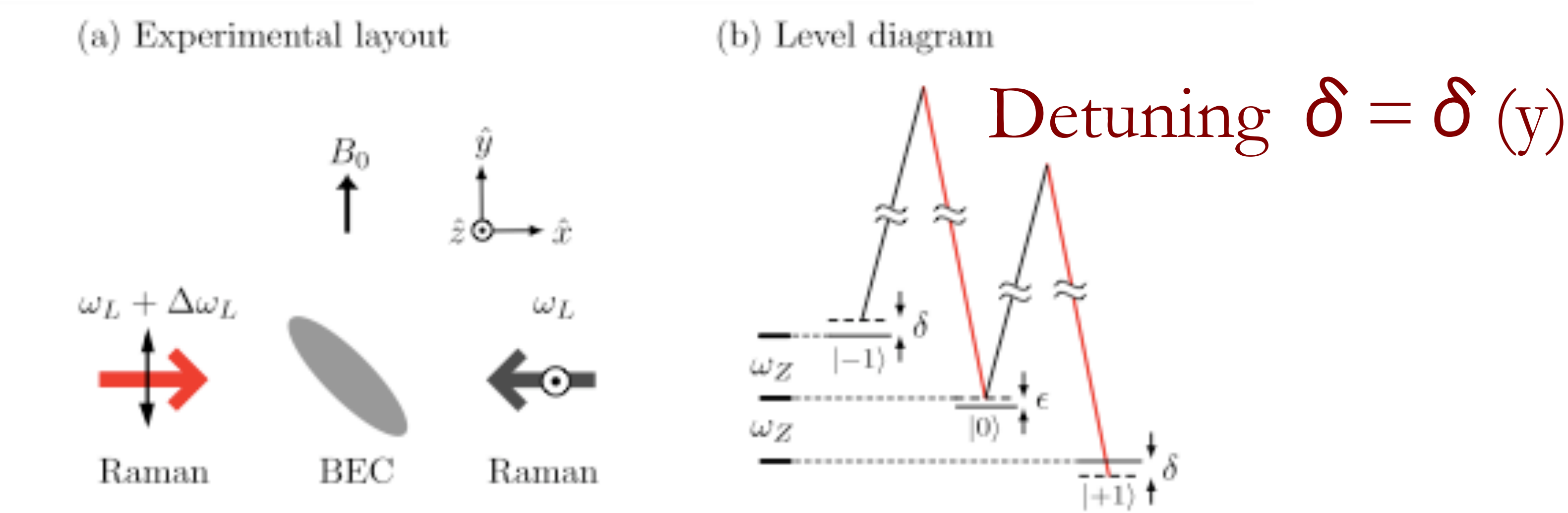
Position-dependent detuning $\delta = \delta(y)$ → $\mathbf{B} \neq 0$

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin[1], R. L. Compton[1], K. Jiménez-García[1,2], J. V. Porto[1] & I. B. Spielman[1]

Light induced effective magnetic field due to

- ***Spatial dependence*** of atom-light detuning



Magnetic flux is again determined by the sample length (**rather than the area**)!

→ One can not create large magnetic flux

Light induced effective magnetic field can be due to

1. ~~***Spatial dependence*** of laser amplitudes~~
2. ~~***Spatial dependence*** of atom-light detuning~~
3. ***Spatial dependence*** of **both** the laser amplitudes and also atom-light detuning

Effective gauge potentials – due to position-dependence of **both**

- A) Detuning *and*
  B) Laser amplitudes
  e.g. **Optical flux lattices**
- N. R. Cooper, Phys. Rev. Lett. **106**, 175301 (2011)

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Magnetic flux is determined by the **<u>area (!!!)</u>** of atomic cloud

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  e.g. **Optical flux lattices**
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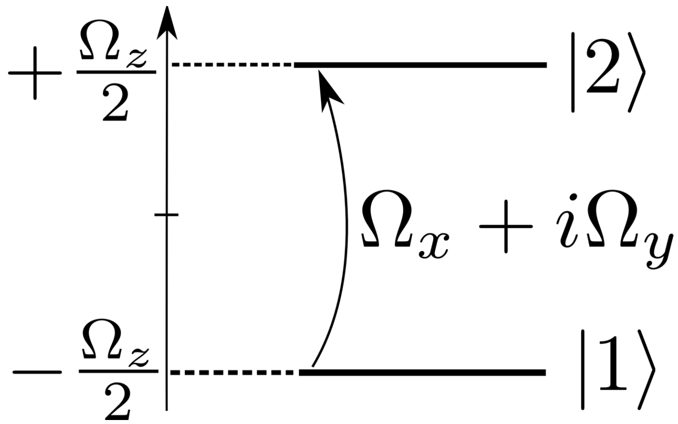
Magnetic flux is determined by the **area (!!!)** of atomic cloud

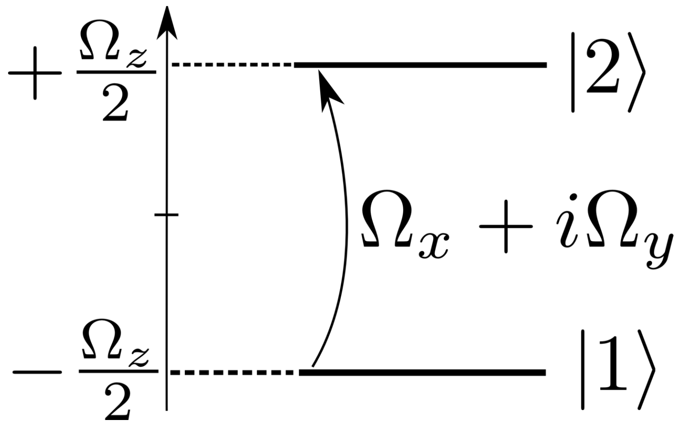
- Related earlier work:
- A. M. Dudarev, R. B. Diener, I. Carusotto, and Q. Niu, Phys. Rev. Lett. 92, 153005 (2004).

Two atomic internal states



- Position-dependent detuning $\Delta(\mathbf{r}) \equiv \Omega_z$
- Position-dependence of the Rabi frequencies of atom-light coupling $\Omega_\pm(\mathbf{r}) \equiv \Omega_x \pm i\Omega_y$

Two atomic internal states



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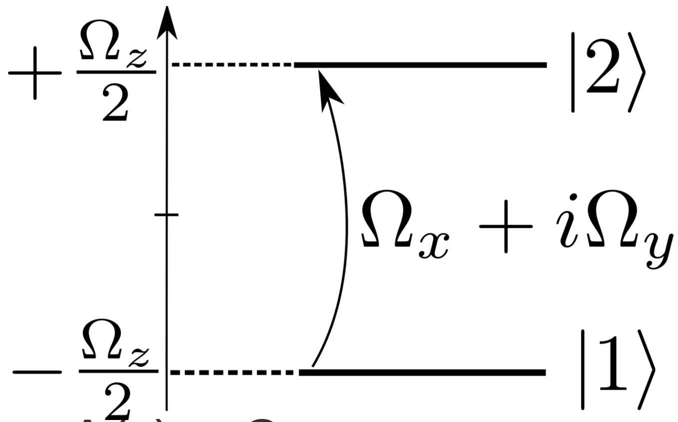
- **Atom-light Hamiltonian:**

$$\hat{H}_0(\mathbf{r}) = -\hbar\begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

(2×2 matrix)

Two atomic internal states



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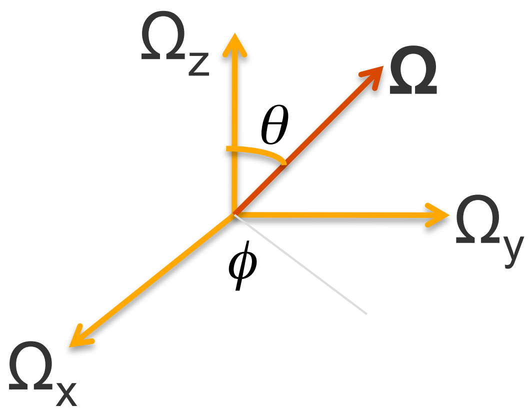
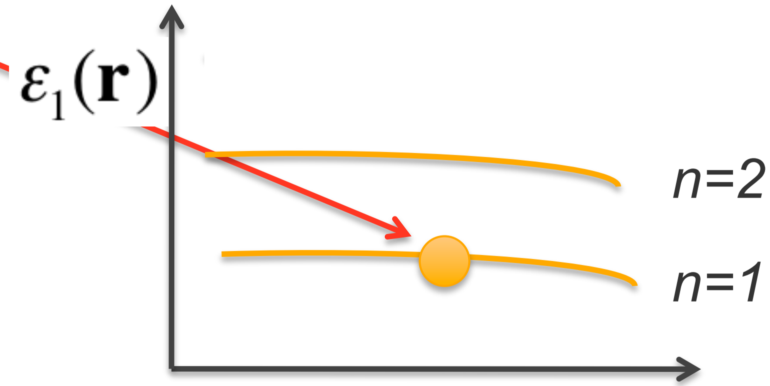
$\Omega_x \neq 0$, $\Omega_y \neq 0$, → **Coupling** between the atomic states → $\hat{H}_0(\mathbf{r})$ has position-dependent eigenstates $|\chi_j(\mathbf{r})\rangle$, $j=1,2$

$$\hat{H}_0(\mathbf{r})|\chi_j(\mathbf{r})\rangle = \varepsilon_j(\mathbf{r})|\chi_j(\mathbf{r})\rangle \quad (j=1,2), \qquad \hat{H}_0(\mathbf{r}) = -\hbar\begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

- Effective vector potential for atomic motion in the lower dressed state $|\chi_1(\mathbf{r})\rangle$:

$$\mathbf{A} \equiv \mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$



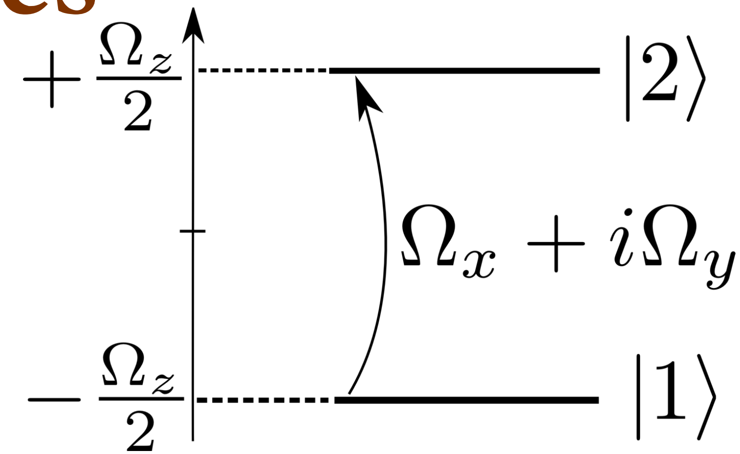


$$\mathbf{A}(\mathbf{r}) = \frac{\hbar}{2}(\cos\theta - 1)\nabla\phi$$

See, e.g.: J.Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg. Rev. Mod. Phys. **83**, 1523 (2011).

→ Optical flux lattices

Two-level system:



$$\hat{H}_0(\mathbf{r}) = -\hbar\begin{pmatrix} \Omega_z/2 & \Omega_x - i\Omega_y \\ \Omega_x - i\Omega_y & -\Omega_z/2 \end{pmatrix}$$

$$\Omega_x = \Omega_\perp \cos(x\pi/a) \quad \Omega_y = \Omega_\perp \cos(y\pi/a)$$

$$\Omega_z = \Omega_{II} \sin(x\pi/a)\sin(y\pi/a)$$

Periodic coupling $\Omega_x + i\Omega_y$ and periodic detuning Ω_z

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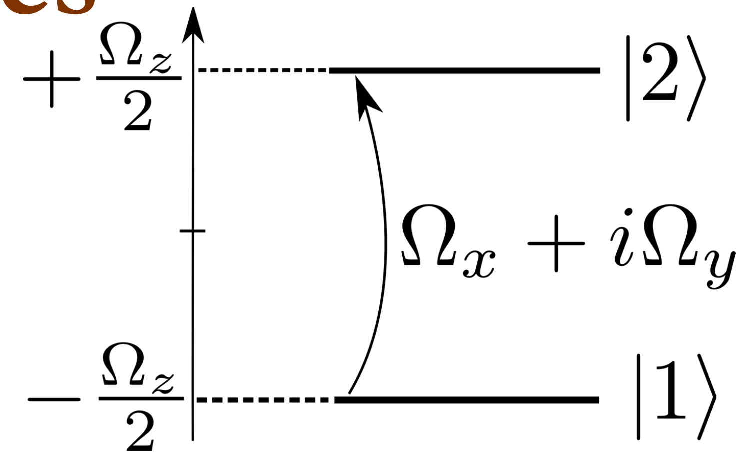

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→Periodic vector potential $\mathbf{A} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$ with $\mathbf{B} = \nabla \times \mathbf{A} \neq 0$

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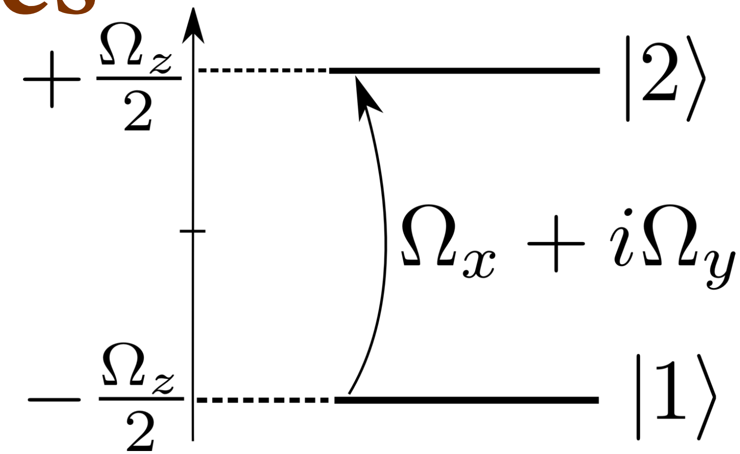

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(B – non-staggered**)!!!**

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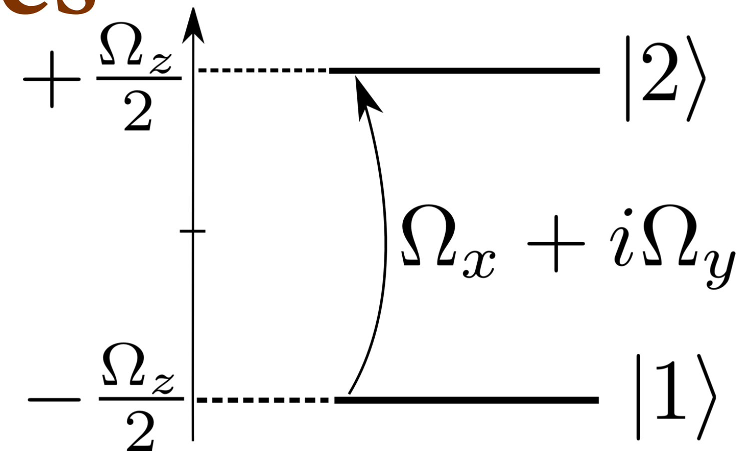

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→**Periodic trapping potential**

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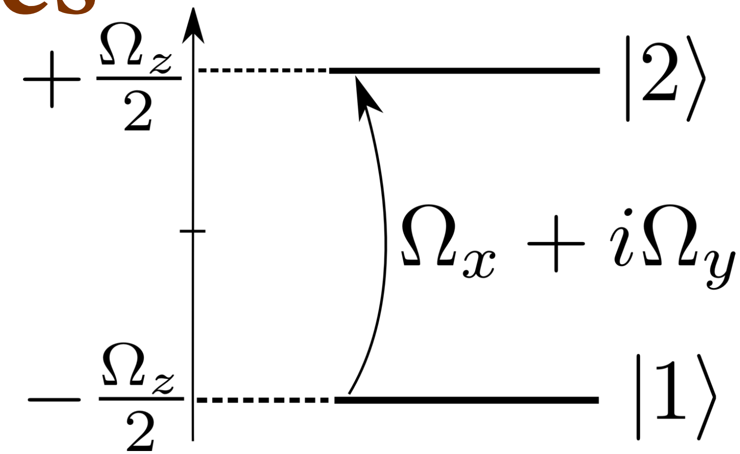

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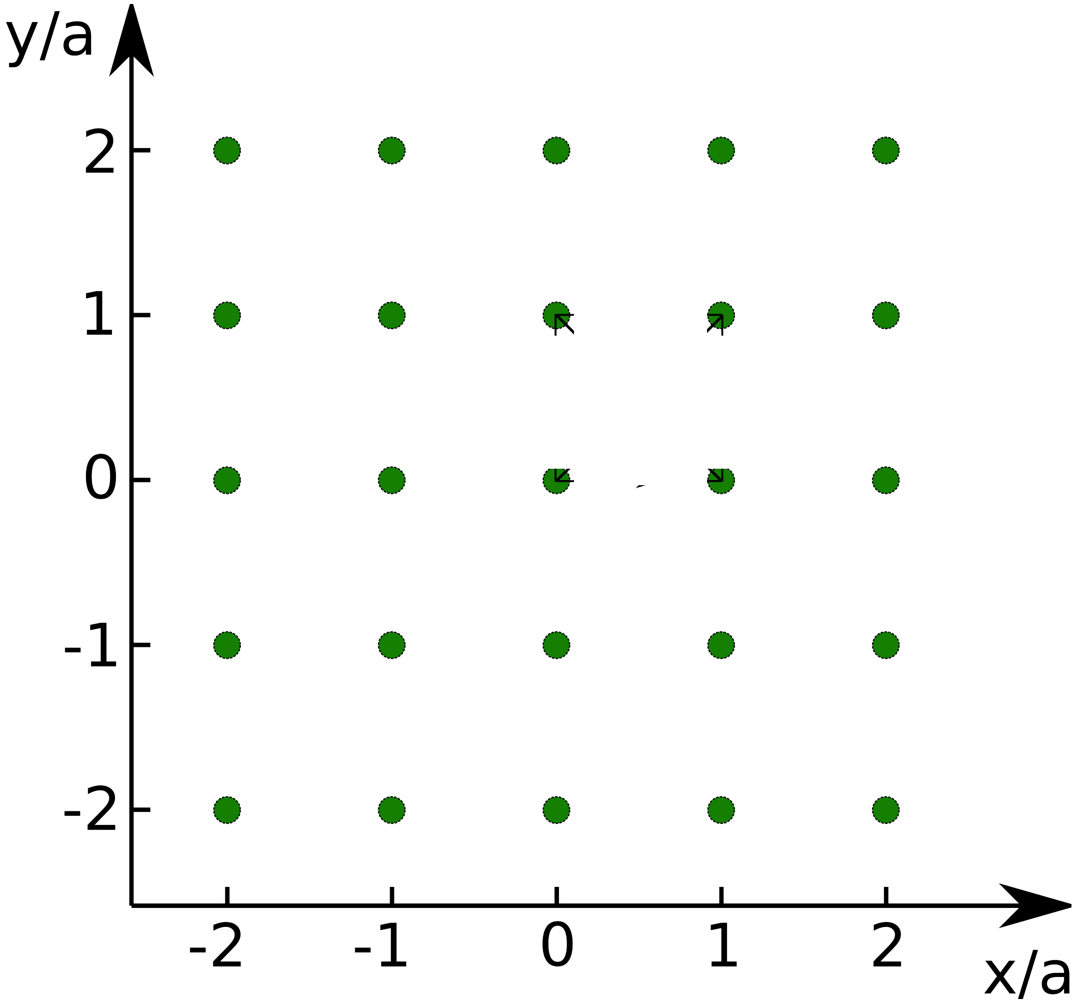
→**Periodic trapping potential**

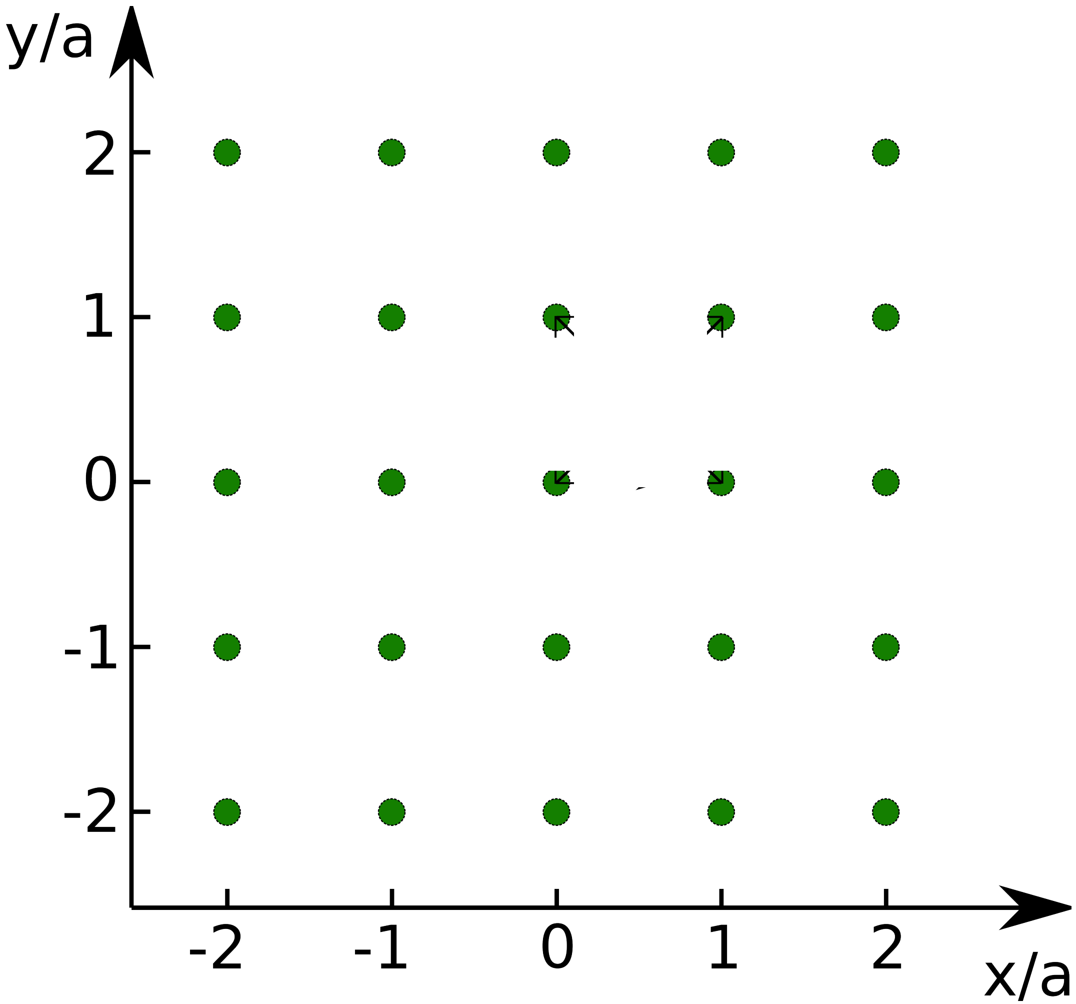
→→ **Optical flux lattice!!!**

→ Optical flux lattices (square)



Non-zero background magnetic flux over an elementary cell

→ Optical flux lattices (square)



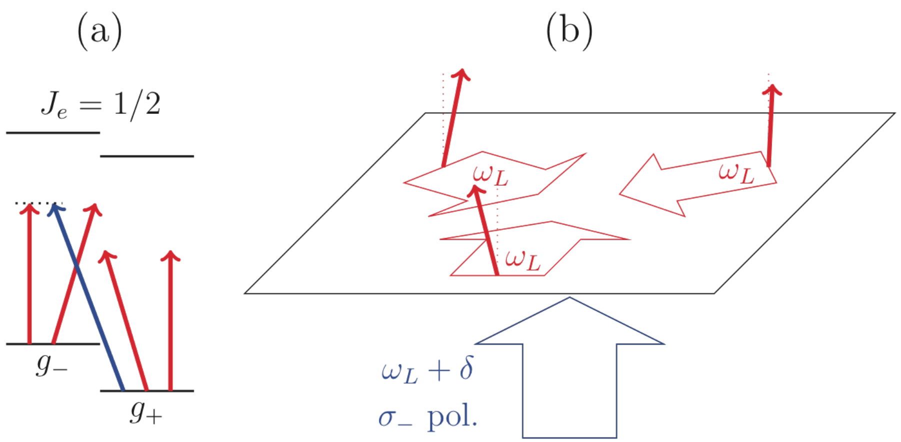
Non-zero background magnetic flux over an elementary cell

Periodic trapping potential (the lattice)

OFL can be produced

- Using Raman transitions between the hyperfine states of alkali atoms (and specially shaped laser fields)

Triangular optical flux lattice
N. R. Cooper and J. Dalibard, EPL, 95 (2011) 66004.



Square optical flux lattice:
G. Juzeliunas and I.B. Spielman, in preparation

Characteristic features of light-induced gauge potentials

- No rotation of atomic gas
- Effective magnetic field can be shaped by choosing proper laser beams
- The magnetic flux can be made proportional to the area using the optical flux lattices

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- **Generation of dynamical gauge fields???**
- **Extension to the non-Abelian case**

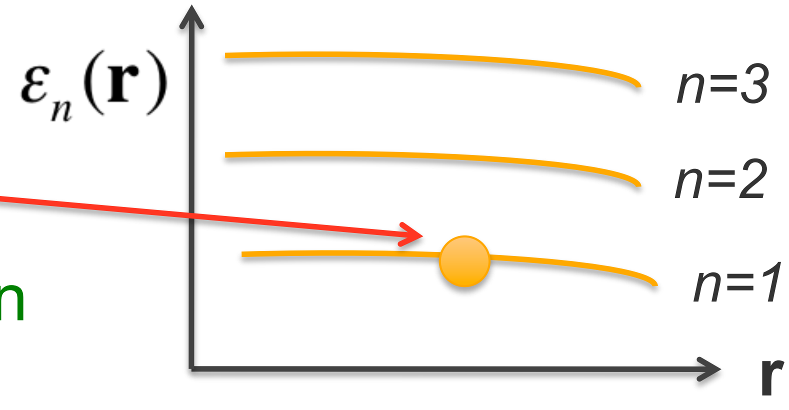
If **one** degenerate atomic internal dressed state

-- **Abelian gauge potentials**

Non-degenerate state with n=1

Adiabatic approximation:

$$|\Phi\rangle \approx |\chi_1(\mathbf{r})\rangle \Psi_1(\mathbf{r},t)$$



Equation of the atomic motion in the internal state $|\chi_1(\mathbf{r})\rangle$

$$i\hbar\partial_t \Psi_1(\mathbf{r},t) = H\Psi_1(\mathbf{r},t)$$

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A}_{11})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r}) \qquad \mathbf{p} = -i\hbar\nabla$$

$$\mathbf{A}_{11} = i\hbar\langle\chi_1(\mathbf{r})|\nabla\chi_1(\mathbf{r})\rangle$$

Effective Vector potential $\mathbf{A}_{11} \equiv \mathbf{A}$ appears

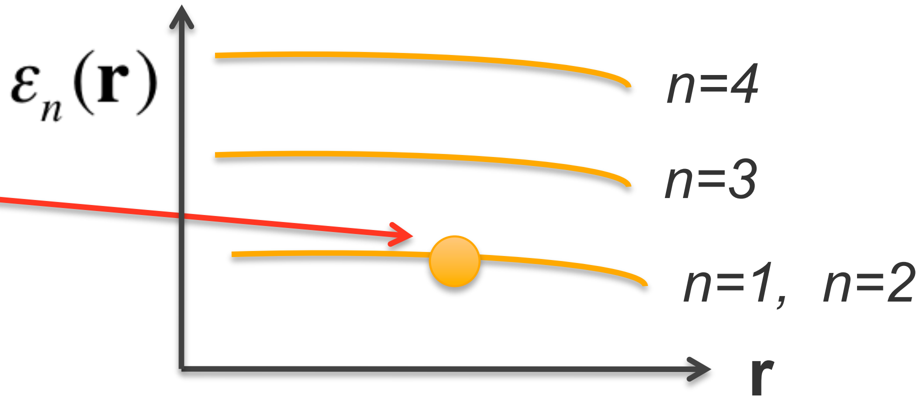
$\mathbf{B} = \nabla \times \mathbf{A}$ - effective magnetic field (non-trivial situation if $\mathbf{B} \neq 0$)

If **more than one** degenerate atomic internal dressed state --**Non-Abelian gauge potentials**

Degenerate states with n=1 and n=2

Adiabatic approximation:

$$|\Phi\rangle \approx \sum_{n=1}^{2} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$



$\Psi_n(\mathbf{r},t)$ – wave-function of the atomic centre of mass motion in the n-th atomic internal “dressed” state (*n=1,2*)

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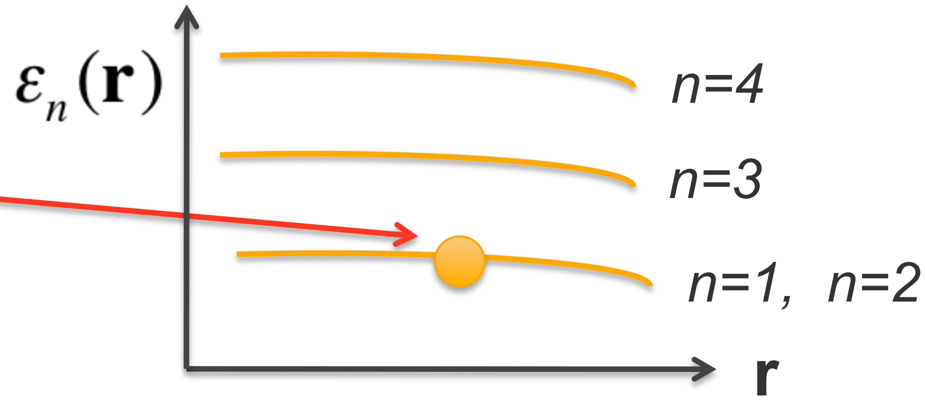

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$$\Psi(\mathbf{r},t) = \begin{pmatrix} \Psi_1(\mathbf{r},t) \\ \Psi_2(\mathbf{r},t) \end{pmatrix}$$

-two-component atomic wave-function
(spinor wave-function)
→ **Quasi-spin 1/2**

Repeating the same procedure …

Degenerate states with n=1 and n=2

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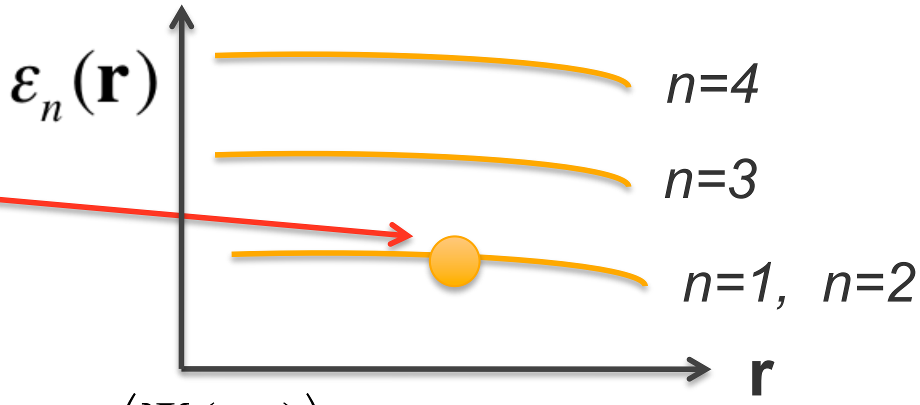
$$|\Phi\rangle \approx \sum_{n=1}^{2} |\chi_n(\mathbf{r})\rangle \Psi_n(\mathbf{r},t)$$



Equation of motion:

$$\hat{H} = \frac{(\mathbf{p}-\mathbf{A})^2}{2M} + V(\mathbf{r}) + \varepsilon_1(\mathbf{r})$$

$$\Psi(\mathbf{r},t) = \begin{pmatrix} \Psi_1(\mathbf{r},t) \\ \Psi_2(\mathbf{r},t) \end{pmatrix}$$ - two-comp. atomic wave-function

2x2 matrix

$$\mathbf{A}_{lj} = i\hbar\langle\chi_l(\mathbf{r})|\nabla\chi_j(\mathbf{r})\rangle, \quad (l,j=1,2)$$ - effective vector potential

2x2 matrix

$$\mathbf{B} = \nabla\times\mathbf{A} + \frac{1}{i\hbar}\mathbf{A}\times\mathbf{A}$$ - effective magnetic field (non-trivial situation if $\mathbf{B}\neq 0$)

$\mathbf{A}$ appears due to position-dependence of $|\chi_n(\mathbf{r})\rangle$

Degenerate states with n=1 and n=2

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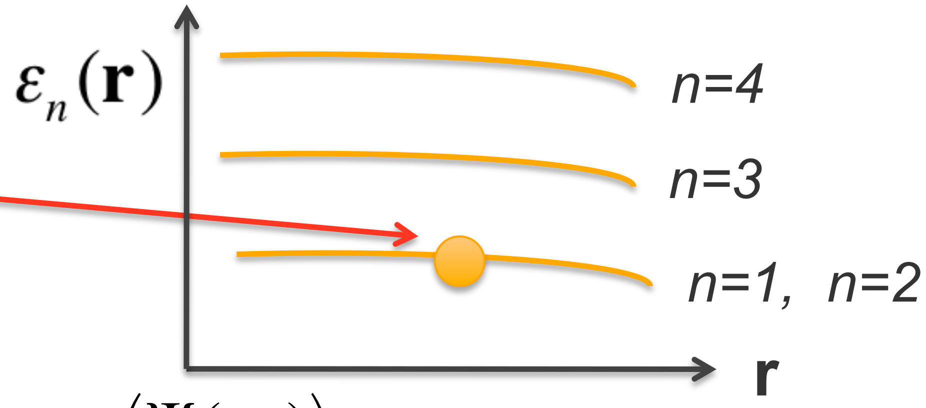

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$\mathbf{A}$ appears due to position-dependence of $|\chi_n(\mathbf{r})\rangle$

If A_x, A_y, A_z do not commute, **B**≠0 even if **A** is constant !!!

Degenerate states with n=1 and n=2

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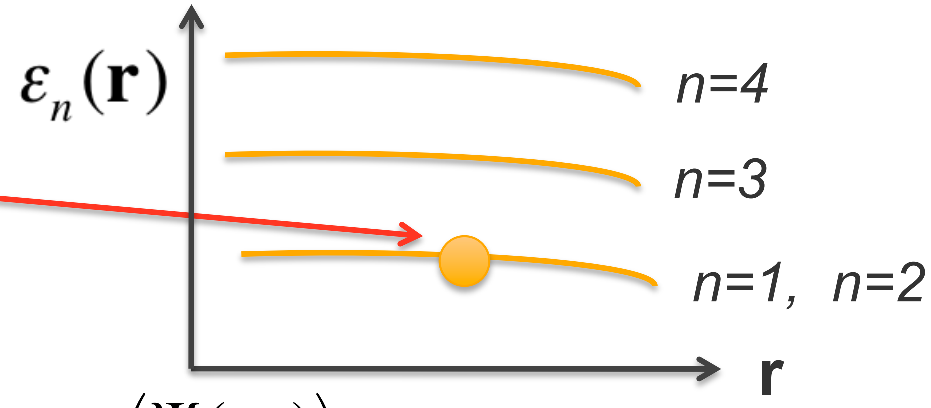

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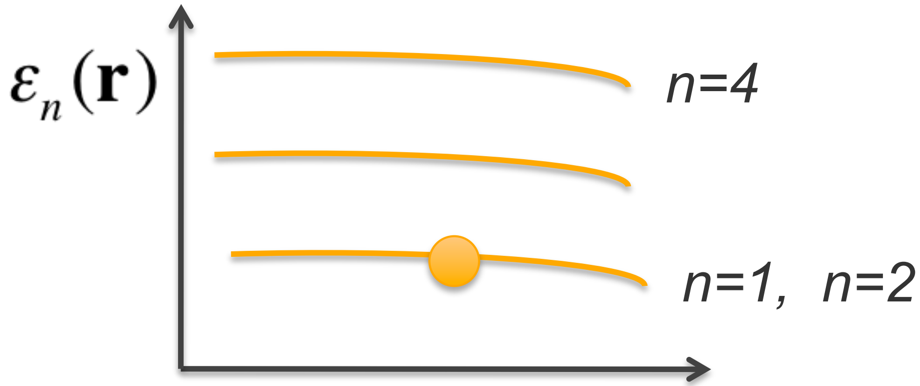
If A_x, A_y, A_z do not commute, **B**≠0 even if **A** is constant !!

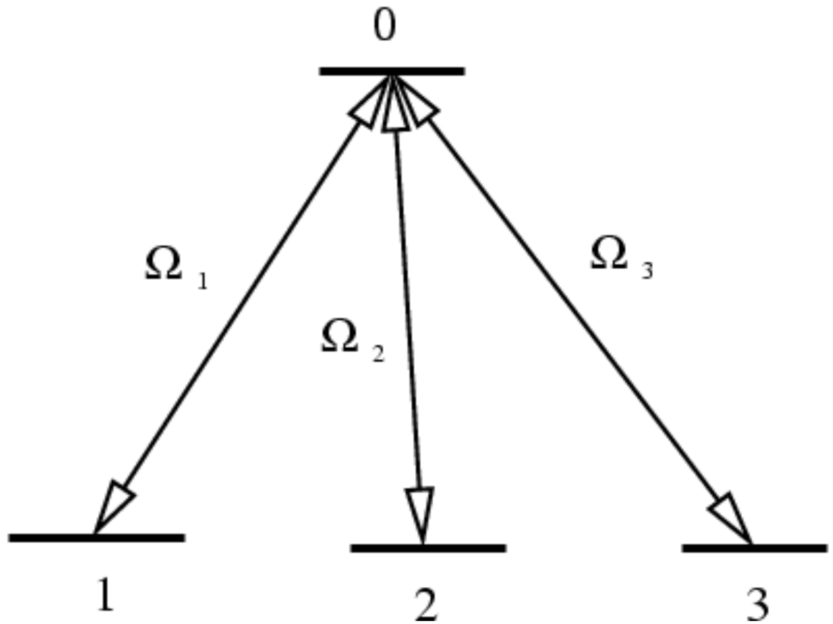
→ Non-Abelian gauge potentials are formed

Non-Abelian gauge potentials

More than one degenerate dressed state



Tripod configuration



- M.A. Ol'shanii, V.G. Minogin, Quant. Optics 3, **317** (1991)
- *R. G. Unanyan, M. Fleischhauer, B. W. Shore, and K. Bergmann, Opt. Commun.* ***155****, 144 (1998)*
- ***Two*** *degenerate* ***dark states***
- *(Superposition of atomic ground states* ***immune*** *of the atom-light coupling)*

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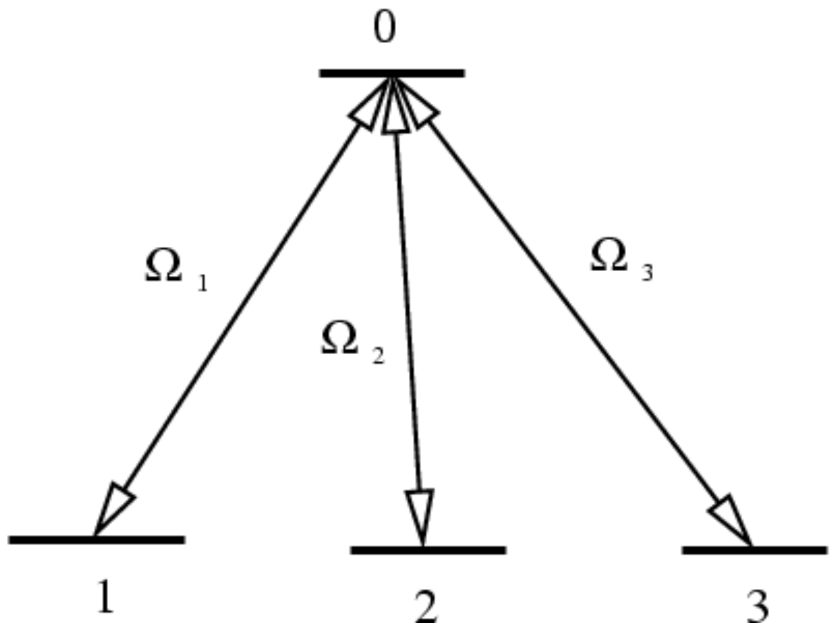
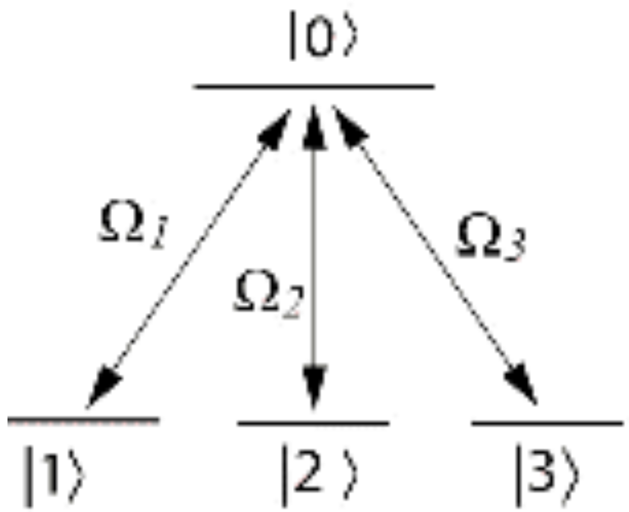


- ***Two*** *degenerate* ***dark states***
- *(Superposition of atomic ground states* ***immune*** *of the atom-light coupling)*
- ***Dark states:*** *destructive interference for transitions to the excited state*
- *Lasers keep the atoms in these* ***dark*** *(dressed)* ***states***

(Non-Abelian) light-induced gauge potentials



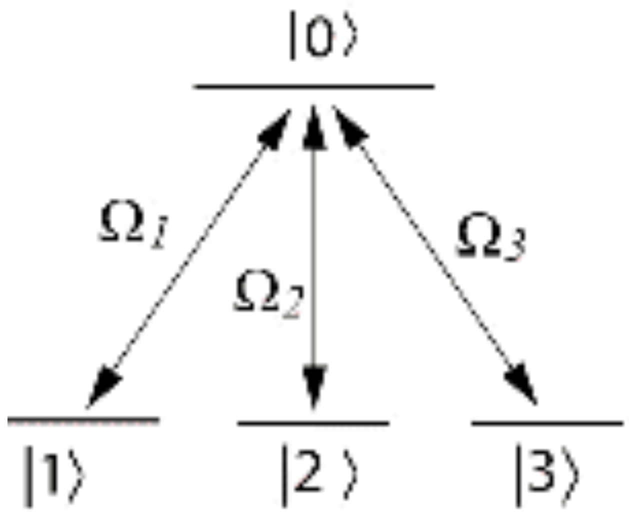
for centre of mass motion of dark-state atoms:

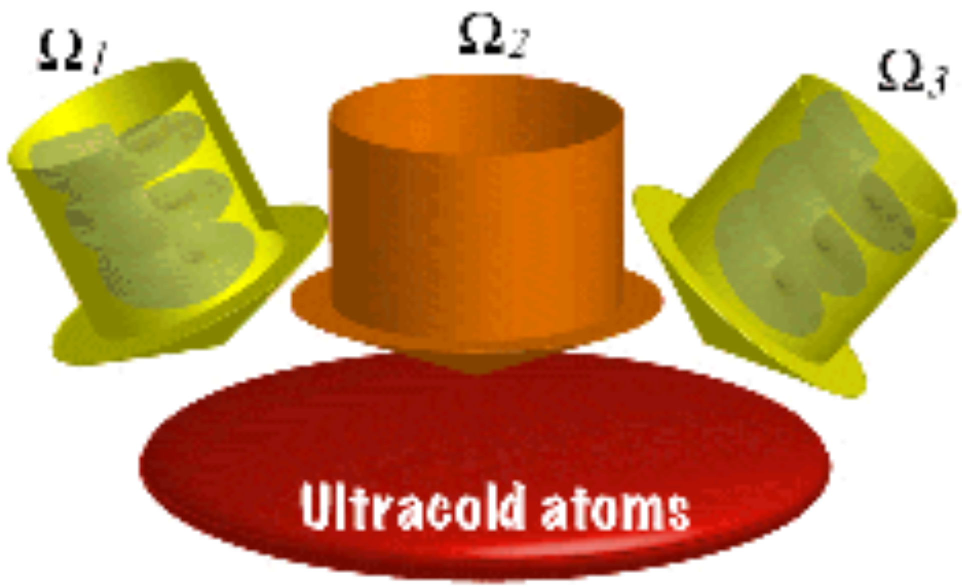
(Due to the spatial dependence of the dark states)

⇖ (two dark states)

J. Ruseckas, G. Juzeliūnas and P.Öhberg, and M. Fleischhauer, Phys. Rev. Letters 95, 010404 (2005).

(Non-Abelian) light-induced gauge potentials





for centre of mass motion of dark-state atoms:

(Due to the spatial dependence of the dark states)

$$\mathbf{A}_{n,m} = i\hbar\langle D_n(\mathbf{r})|\nabla D_m(\mathbf{r})\rangle$$

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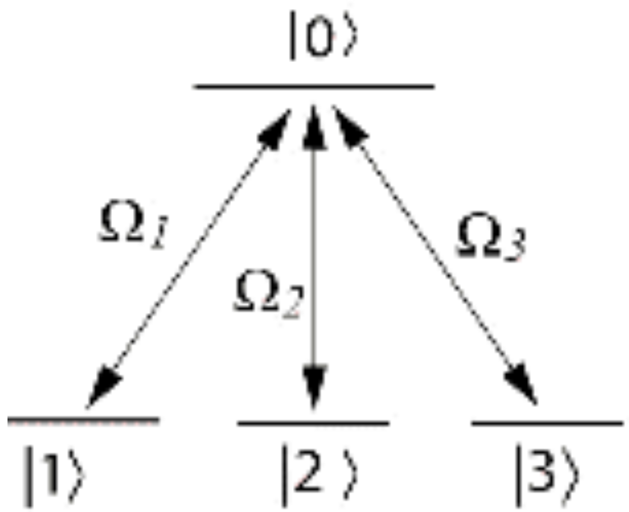
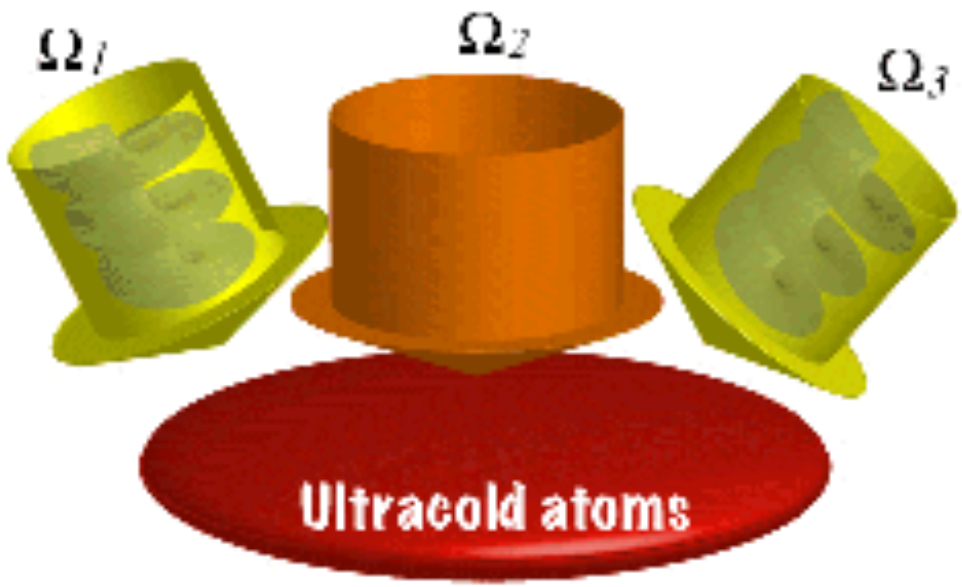




Centre of mass motion of dark-state atoms:

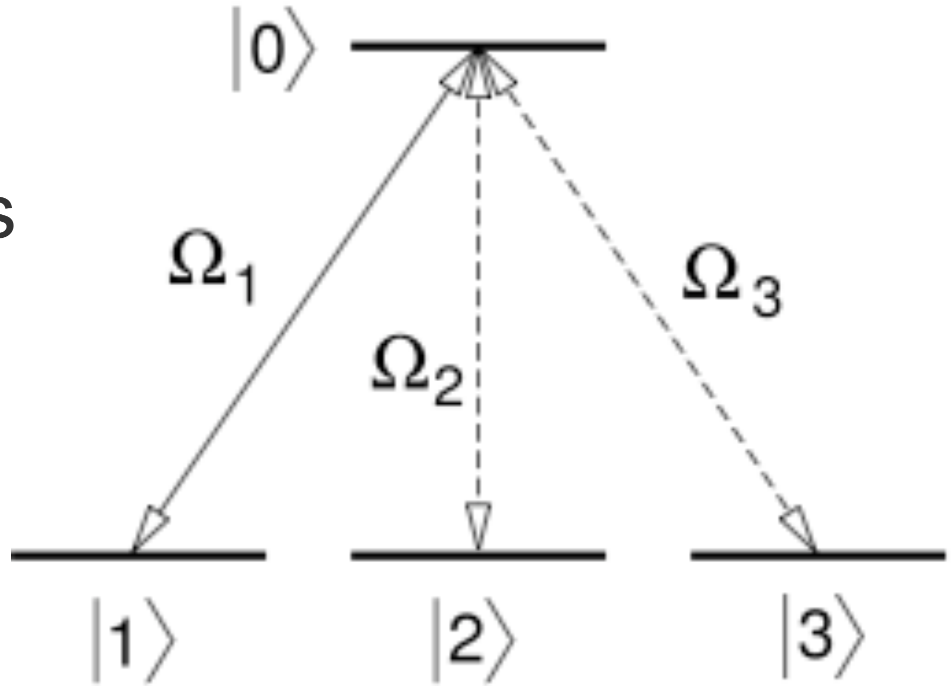
$$i\hbar\frac{\partial}{\partial t}\Psi = \left[\frac{1}{2m}(-i\hbar\nabla - \mathbf{A})^2 + V + \Phi\right]\Psi,$$ ← Two component atomic wave-function

$$\mathbf{A}_{n,m} = i\hbar\langle D_n(\mathbf{r})|\nabla D_m(\mathbf{r})\rangle$$ - 2x2 matrix

A - effective vector potential (Mead-Berry connection)

Tripod scheme

Two degenerate dark states



- **A** is 2×2 matrix

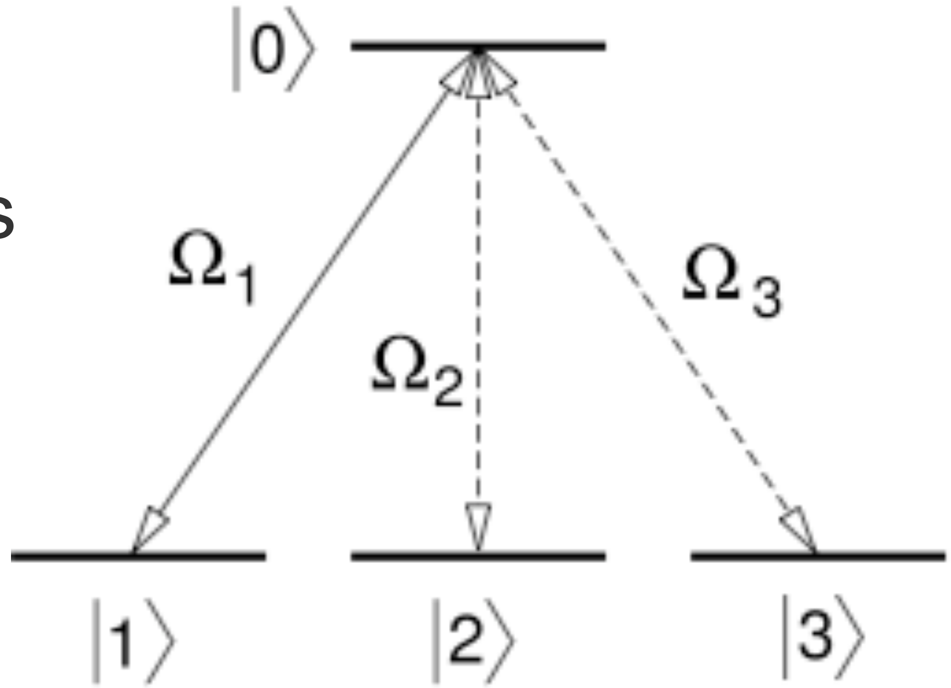
- Non-Abelian case if A_x, A_y, A_z do not commute

$$B_i = \frac{1}{2}\epsilon_{ikl}F_{kl}, \qquad F_{kl} = \partial_k A_l - \partial_l A_k - \frac{i}{\hbar}[A_k, A_l].$$

B – curvature

Tripod scheme

Two degenerate dark states



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B – curvature

- Can be achieved using a plane-wave setup

Three plane wave setup

T. D. Stanescu,, C. Zhang, and V. Galitski,
Phys. Rev. Lett 99, 110403 (2007).
A. Jacob, P. Öhberg, G. Juzeliūnas and
L. Santos, *Appl. Phys. B.* **89,** 439 (2007).

$$\Psi(\mathbf{r},t) = \begin{pmatrix} \Psi_1(\mathbf{r},t) \\ \Psi_2(\mathbf{r},t) \end{pmatrix}$$

(centre of mass motion, dark-state atoms):

$$H = \frac{1}{2m}(-i\hbar\nabla - \hbar\kappa\sigma_\perp)^2 \;\text{-}$$ (→ Rashba-type Hamilltonian)

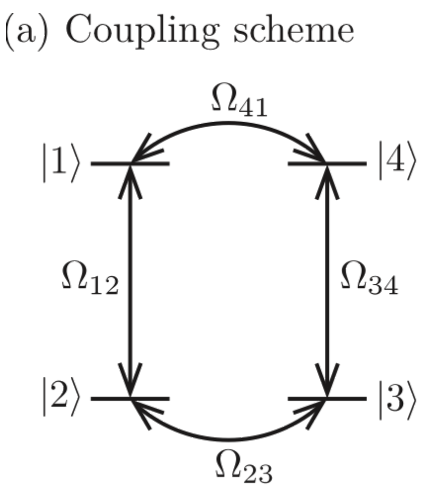
$$H = \frac{1}{2M}(-i\hbar\nabla + \hbar\kappa\sigma_\perp)^2 + V_1$$ ½ operator

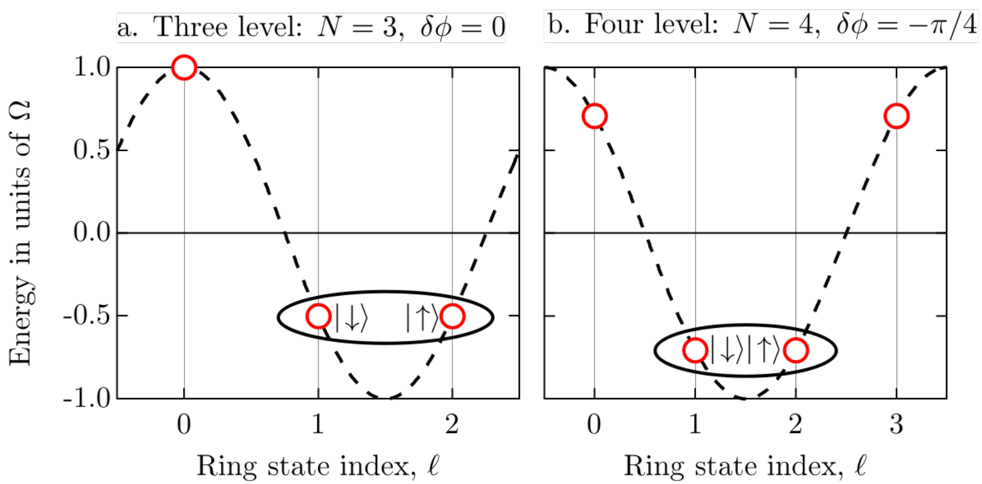
(acting on the subspace of atomic dark states)

→Spin-Orbit coupling of the Rashba-Dresselhaus type

Constant non-Abelian **A** with $[A_x,A_y]\sim\sigma_z$ → **B** ~ **e**$_z$

- <u>Drawback of the tripod scheme</u>: degenerate dark states are not the ground atomic dressed states → collision-induced loses
- Closed loop setup overcomes this drawback:





D. L. Campbell, G. Juzeliūnas and I. B. Spielman, Phys. Rev. A 84, 025602 (2011)

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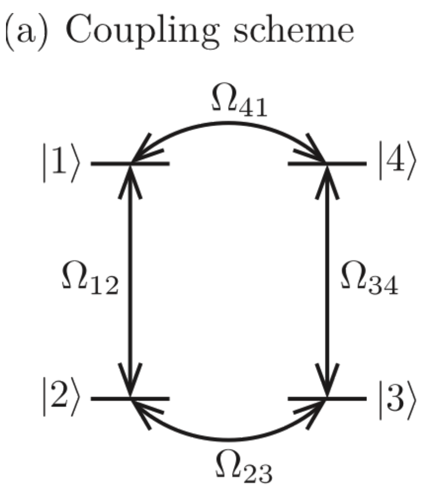
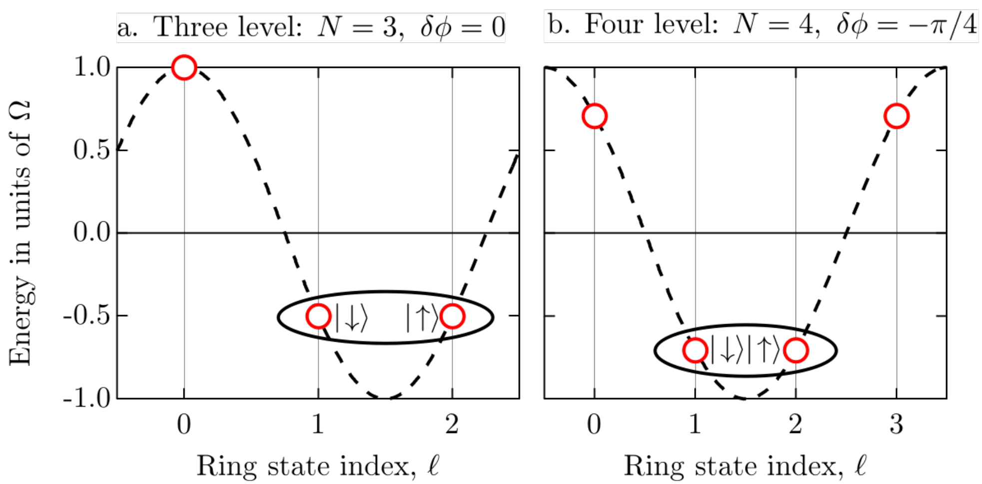
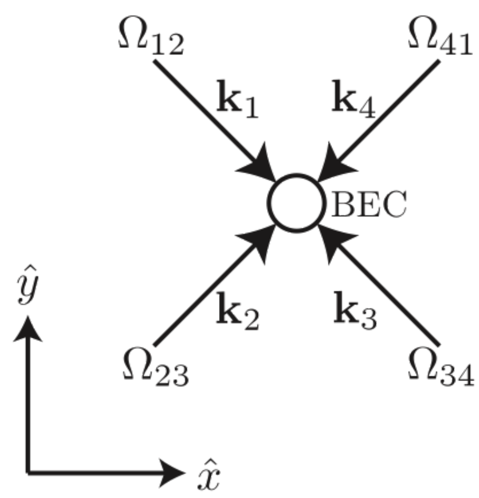


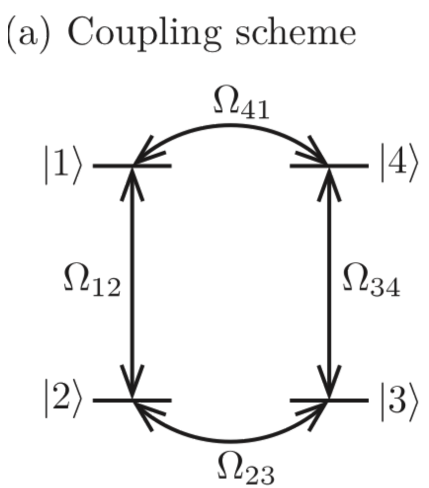


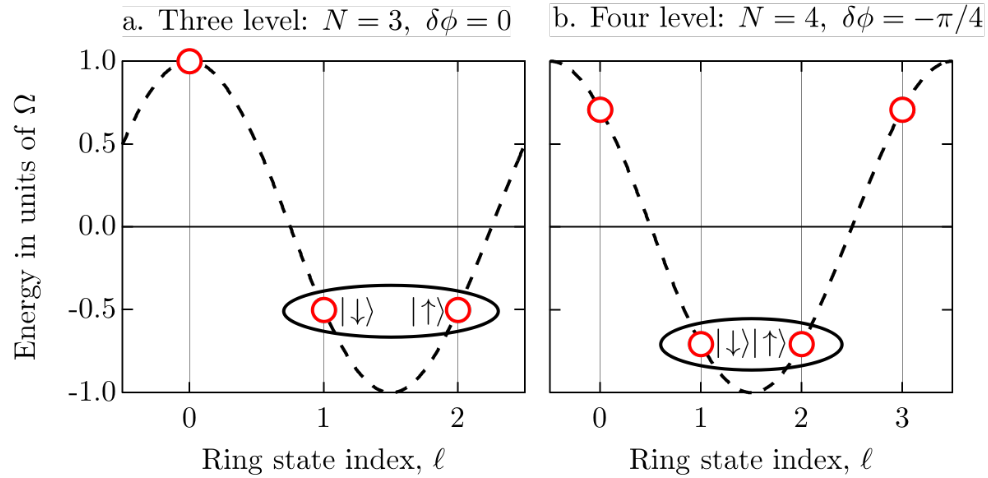
D. L. Campbell, G. Juzeliūnas and I. B. Spielman, Phys. Rev. A 84, 025602 (2011)

- **Two degenerate** internal ground states
- → **non-Abelian gauge fields** for ground-state manifold

- Laser fields represent Counter-propagating plane waves:







→ Closed loop setup produce Rashba-Dresselhaus SO coupling for cold atoms

D. L. Campbell, G. Juzeliūnas and I. B. Spielman, Phys. Rev. A 84, 025602 (2011)

Conclusions

- Abelian gauge potentials appear if there is non-trivial spatial dependence of amplitudes or phases of laser fields (or spatial variation of atomic levels).
- Non-Abelian fields can be formed using even the plane-wave setups. They can simulate the spin 1/2 Rashba-type Hamiltonian for cold atoms.
- Spin 1 Rashba coupling can also be generated

  [G.J., J. Ruseckas and J. Dalibard, PRA 81, 053403 (2010)].
- For more see: J.Dalibard, F. Gerbier, G. Juzeliūnas and P. Öhberg. *Colloquium: Artificial gauge potentials for neutral atoms*, Rev. Mod. Phys. 83, 1523 (2011)

Thank you!