Theory of Spin-Orbit-Coupled Cold Atomic Systems

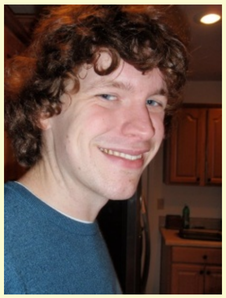
Victor Galitski

Joint Quantum Institute and Physics Department

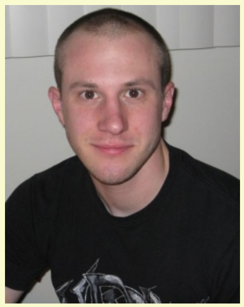
University of Maryland

Acknowledgements: Work supported by NSF-PFC & ARO-MURI

Collaborators

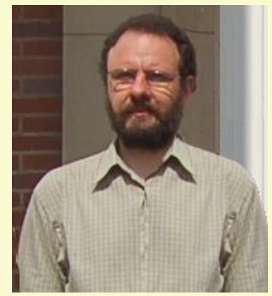


Brandon Anderson (UMD->NIST)

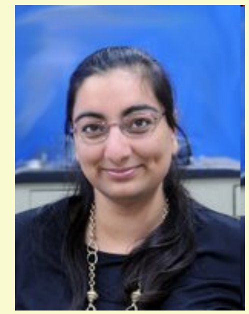


Andrew Robertson (UMD->NRL)

Juraj Radic (UMD)



Tudor Stanescu (UMD->UwVa)



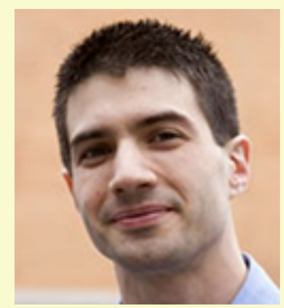
Dr. Jay Vaishnav (NIST)

Charles Clark (JQI-NIST)

Gediminas Juzeliunas (Vilnius)

Sankar Das Sarma (JQI/UMD)

Ian Spielman (JQI/NIST)



Jake Taylor (JQI/NIST)

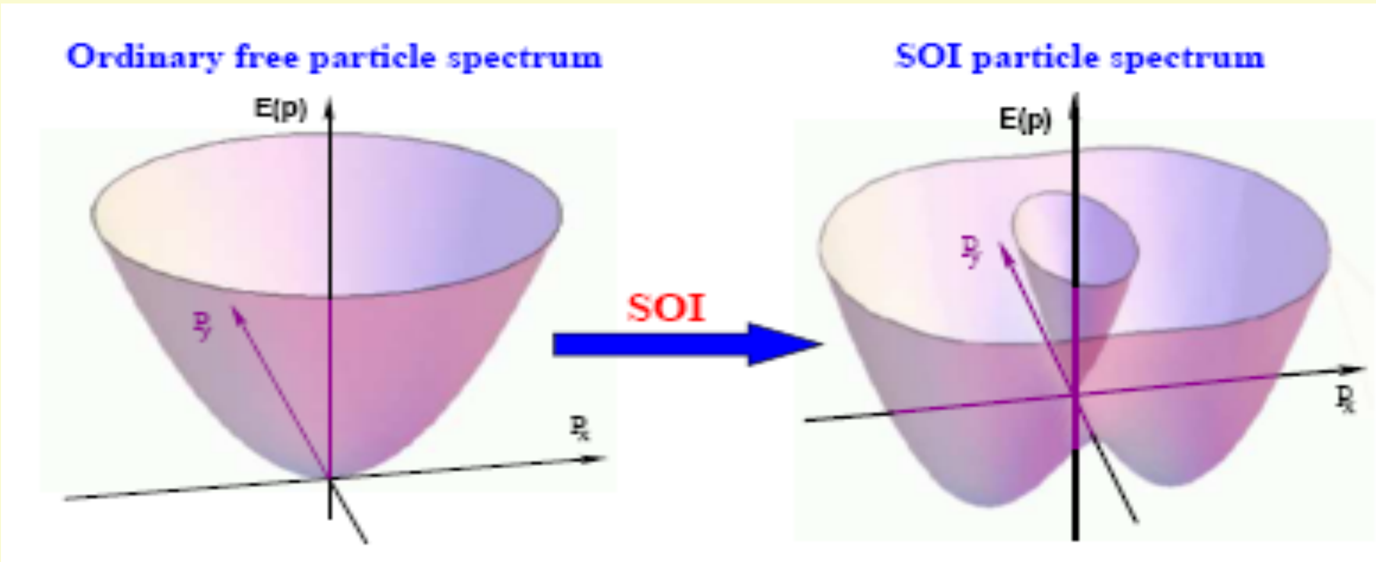
Outline

- Synthetic spin-orbit coupling schemes, old and new and many more
- A zoo of spin-orbit-coupled BECs: (mostly) theory
- How to create vortices in spin-orbit-coupled BECs?
- Topological optical lattices & lattice quantum Hall states
- Practical applications: Quantum gravimetry and interferometry

Spin-Orbit-Coupled Systems

Spin-orbit-coupled 2D electron gas: $\hat{\mathcal{H}} = \frac{p^2}{2m} + \mathbf{h_p} \cdot \hat{\boldsymbol{\sigma}}$

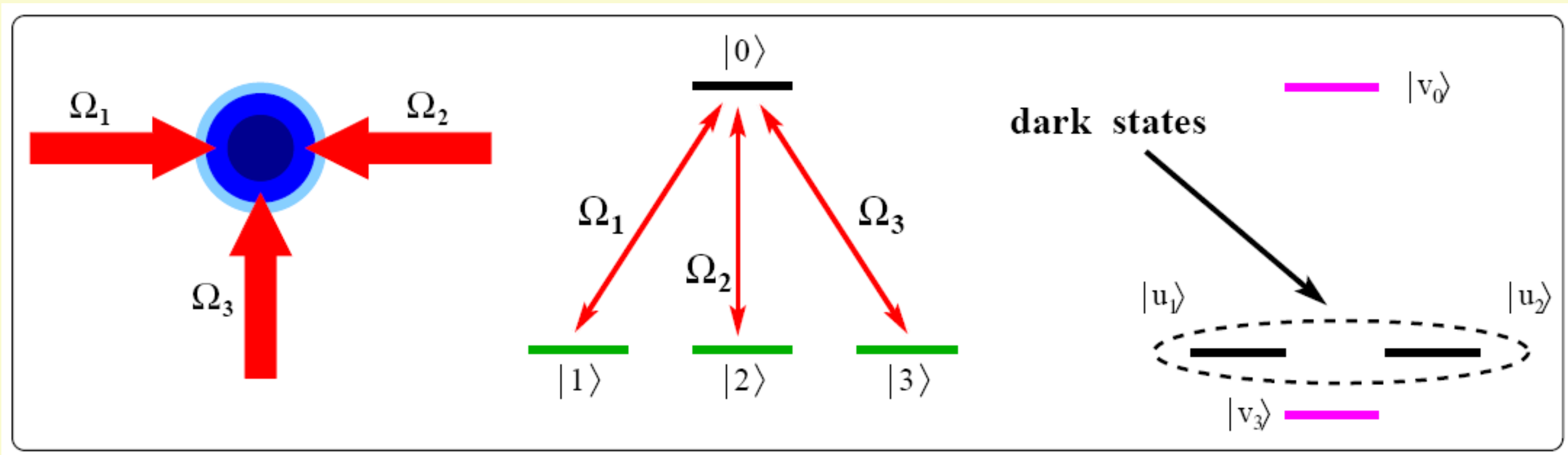
- Rashba: $\mathbf{h}_{\mathbf{p}}^{R} = \alpha \mathbf{p} \times \hat{\mathbf{z}}$; Dresselhaus: $\mathbf{h}_{\mathbf{p}}^{D_1} = -\beta_1 (p_x, -p_y)$



Synthetic Spin-Orbit Couplings

in Cold-Atom Systems

Atom in a $\mathbf{r}$-dependent laser field. Tripod scheme



Hamiltonian: $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\mathsf{kin}} + \hat{V}_{\mathsf{trap}} + \hat{\mathcal{H}}_{\mathsf{a-l}}$

Atom-laser interaction:

$$\hat{\mathcal{H}}_{\mathsf{a-l}} = -\left[\Omega_1(\mathbf{r})|0><1| + \Omega_2(\mathbf{r})|0><2| + \Omega_3(\mathbf{r})|0><3|\right] + \text{h. c.}$$

Refs: Ruseckas, Juzeliunas, Oehberg, & Fleischhauer, Phys. Rev. Lett. **95**, 010404 (2005)
Stanescu, Zhang, & Galitski, Phys. Rev. Lett. **99**, 110403 (2007)

Two equivalent descriptions

To get the effective Hamiltonian...

1. Diagonalize $\widehat{\mathcal{H}}_{\mathsf{a-l}}$ via a unitary rotation, $\widehat{\mathcal{H}}_{\mathsf{a-l}} \to \widehat{U}^\dagger(\mathbf{r})\widehat{\mathcal{H}}_{\mathsf{a-l}}\widehat{U}(\mathbf{r})$
2. Project the result onto the dark subspace

$$\widehat{\mathcal{H}}_{\mathsf{eff}} = \widehat{\mathcal{P}}_{\mathsf{dark}}\widehat{U}^\dagger(\mathbf{r})\left(\frac{-\hbar^2\boldsymbol{\nabla}^2}{2m}\right)\widehat{U}(\mathbf{r})\widehat{\mathcal{P}}_{\mathsf{dark}}$$

- Picture I: Particle in a non-Abelian "gauge field"

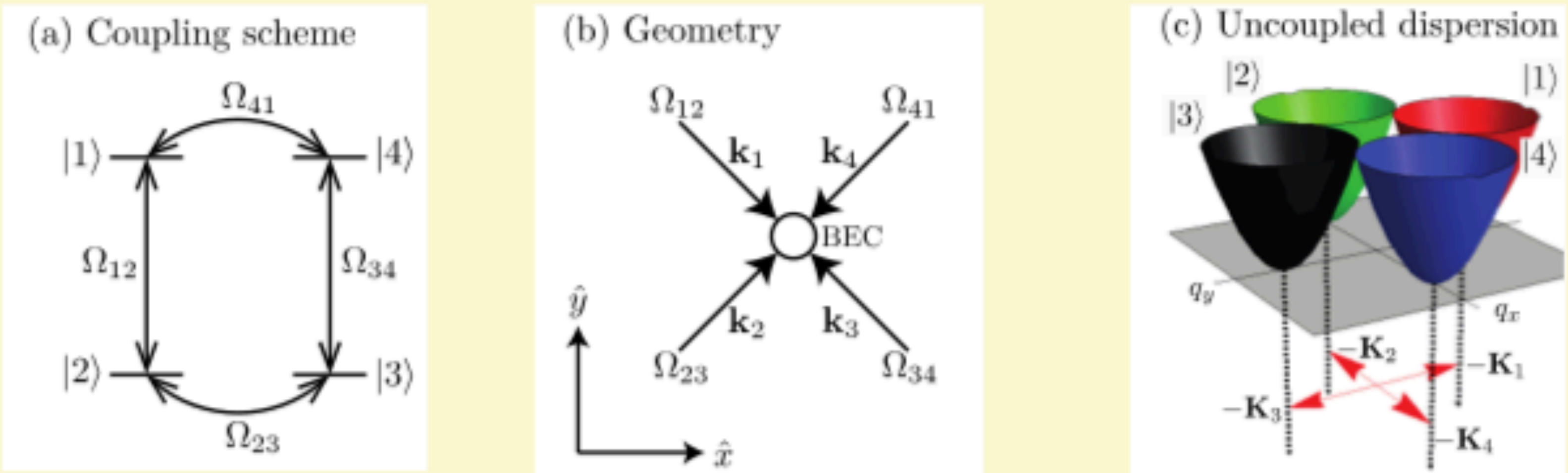
$$\widehat{\mathcal{H}}_{\mathsf{eff}} = \frac{1}{2m}\left[-i\hbar\boldsymbol{\nabla} - \mathbf{A}^i(\mathbf{r})\widehat{\sigma}_i\right]^2$$

- Picture II: Spin-orbit coupled particle

$$\widehat{\mathcal{H}}_{\mathsf{eff}}(\mathbf{p}) = \frac{\mathbf{p}^2}{2m} + \mathbf{b}(\mathbf{p})\cdot\widehat{\boldsymbol{\sigma}}$$

For the tripod scheme in 2D: $\mathbf{b}(\mathbf{p}) = (0, vp_x, v'p_y)$

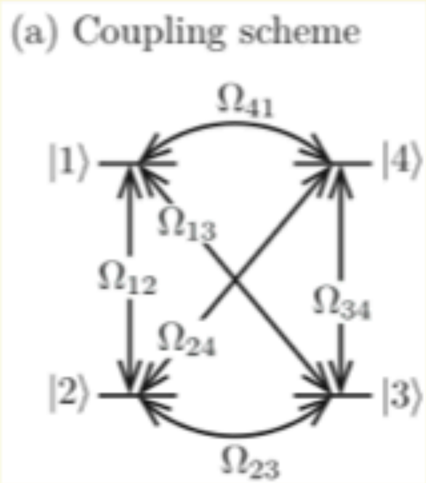
Loop scheme for Rashba SOC

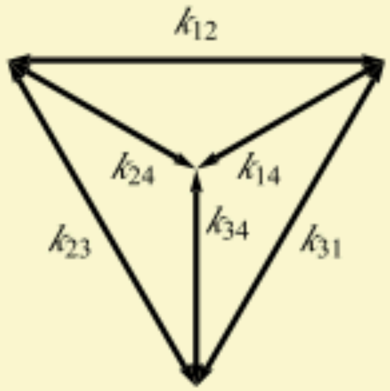


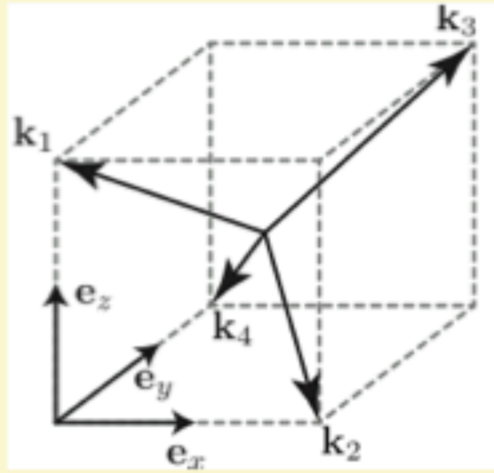
$$\hat{\mathcal{H}}_{\mathrm{SO}} = \frac{\mathbf{p}^2}{2m} + v\left(p_x\hat{\sigma}_x + p_y\hat{\sigma}_y\right)$$

D. Campbell, G. Juzeliunas, & I. Spielman, PRA **84**, 025602 (2011)

Tetrahedron scheme for "Weyl" SOC in 3D







- Chose lasers, so that "momentum flux" through every loop vanishes.

$$\hat{\mathcal{H}}_{\mathrm{SO}} = \frac{\mathbf{p}^2}{2m} + v\left(p_x\hat{\sigma}_x + p_y\hat{\sigma}_y + p_z\hat{\sigma}_z\right)$$

B. Anderson, G. Juzeliunas, I. Spielman, & V. Galitski, tbp

SU(3) "spin"-orbit coupling

A different choice of phases in the tetragonal scheme leads to a Hamiltonian that can NOT be spanned by Pauli matrices alone, but may be spanned by 3×3 Gell-Mann matrices

$$\left[\hat{\lambda}_i, \hat{\lambda}_j\right] = i f_{ij}^{k} \hat{\lambda}_k$$

Gell-mann matrices are generators of $su(3)$.

$su(3)$ spin-orbit coupled system:

$$\hat{\mathcal{H}}_{\mathsf{su}(3)} = \frac{\mathbf{p}^2}{2m} + \sum_{i=1}^{8} b^i(\mathbf{p}) \hat{\lambda}_i$$

No analogue in condensed matter (or any other matter)...

G. Boyd, B. Anderson, & V. Galitski, tbp

Experimental status (see Ian's talk for details)

PRL **102,** 130401 (2009)

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW LETTERS

week ending
3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

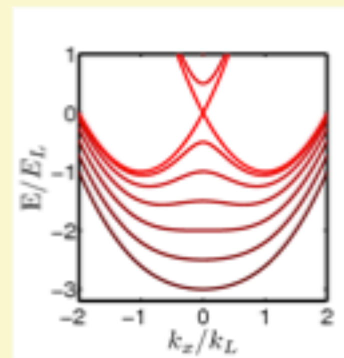
Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*

Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA

(Received 17 September 2008; published 30 March 2009)

- "Only" "Abelian" SOC ($\sim$ persistent spin helix type) has been realized so far

$$\hat{\mathcal{H}} = \frac{\mathbf{p}^2}{2m} + v p_x \hat{\sigma}_z + \Omega \hat{\sigma}_x + \delta \hat{\sigma}_z$$



- The loop SOC ($\sim$ Rashba) may probably be realized soon
- The tetragonal SOC scheme ($\sim$ Weyl or $su(3)$) is realistic

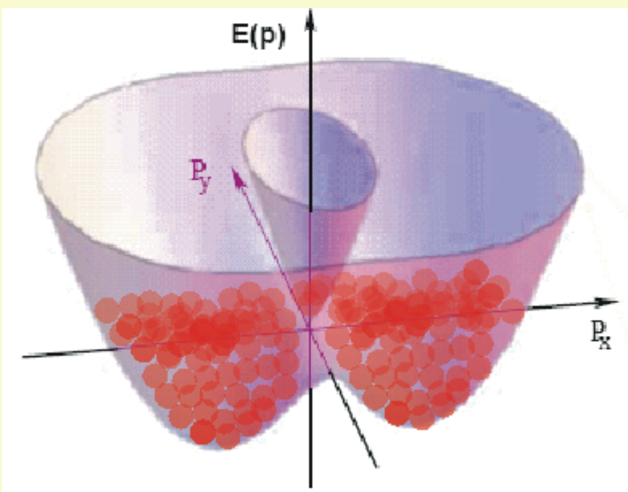
Many-Body Physics
of Spin-Orbit-Coupled Cold Atoms

Spin-Orbit-Coupled
Bose-Einstein Condensates

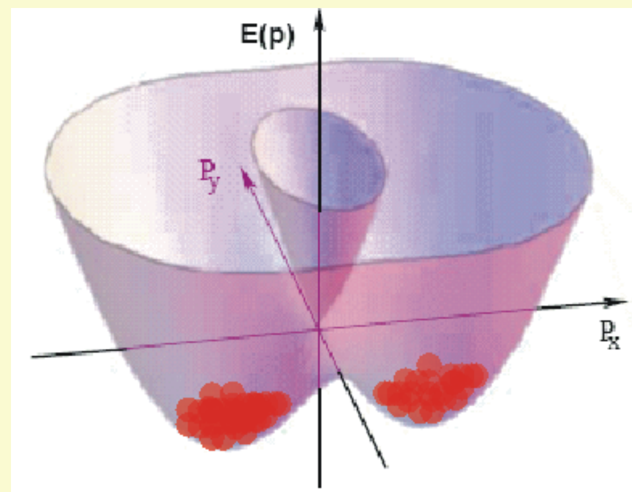
Spin-Orbit-Coupled Fermions and Bosons

- Proposed methods to engineer synthetic spin and synthetic spin-orbit couplings involve single-particle physics. Statistics play no role.

- Spin-orbit-coupled fermions form two Fermi surfaces.



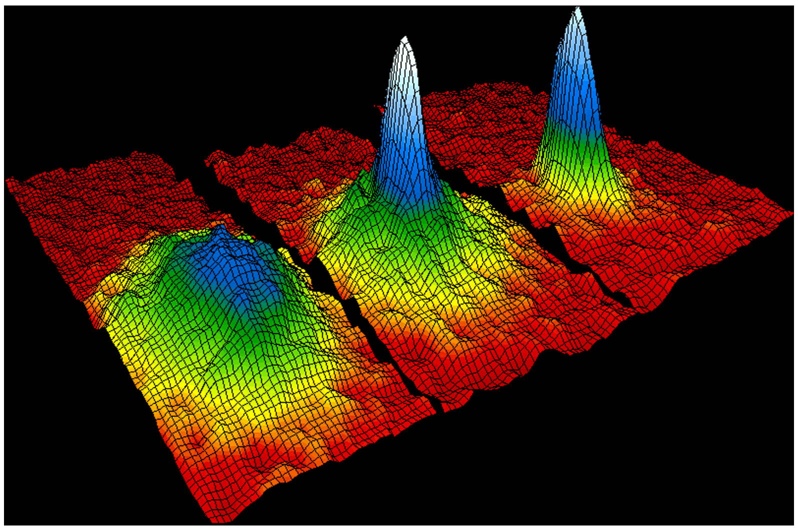
- Spin-1/2 (!) spin-orbit-coupled bosons condense.

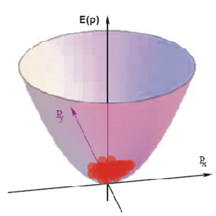


Refs: Stanescu, Zhang, & V.G., *Phys. Rev. Lett.* **99**, 110403 (2007)
Stanescu, Anderson, & V.G., *Phys. Rev. A* **78**, 023616 (2008)

New type of Bose-Einstein condensate

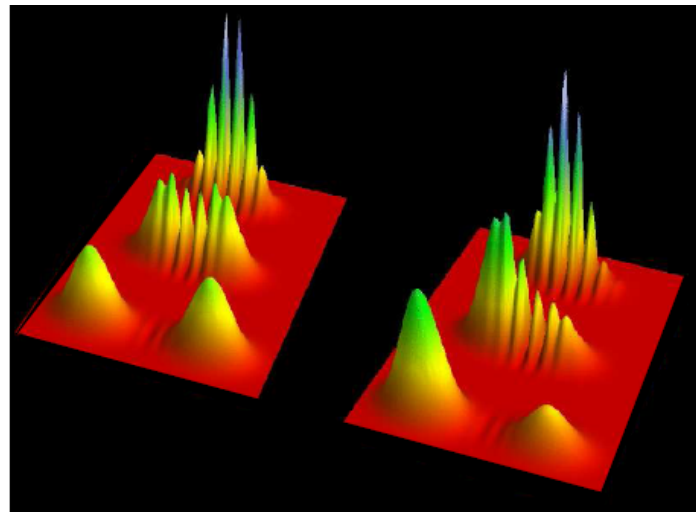
- Time-of-flight for the usual BEC

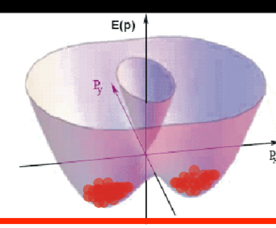




$$|\,\Psi\,\rangle = e^{i\phi_0}\,|\,\text{Zero-momentum state}\,\rangle$$

- ToF for a spin-orbit coupled BEC





$$|\Psi\rangle = \text{Linear combination of } |\text{Left}\rangle \text{ and } |\text{Right}\rangle$$

Selected for a Viewpoint in *Physics*
PRL **102,** 130401 (2009) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2009

Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential

Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman*
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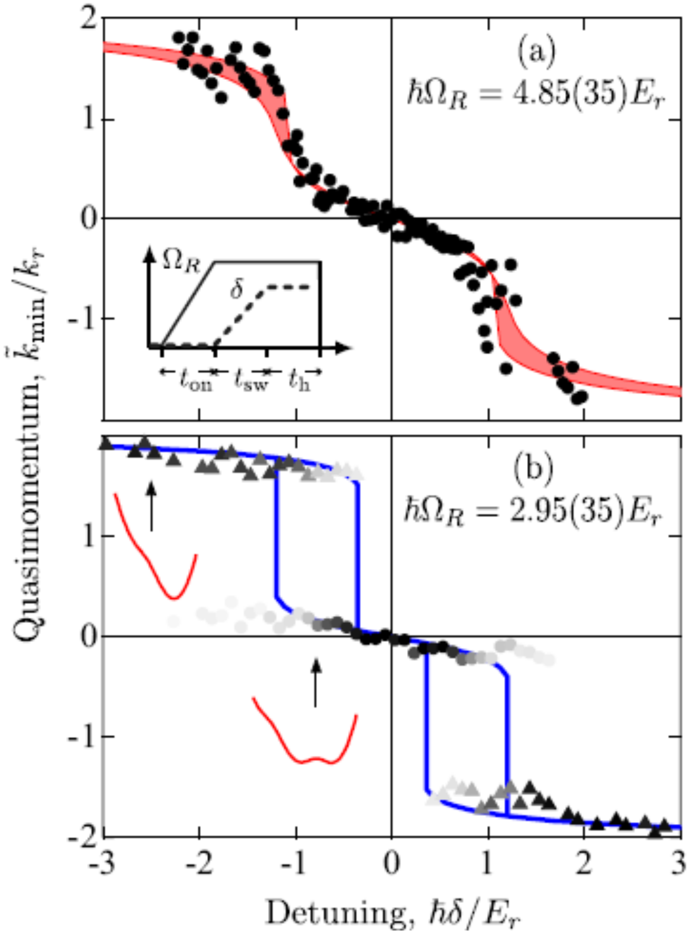
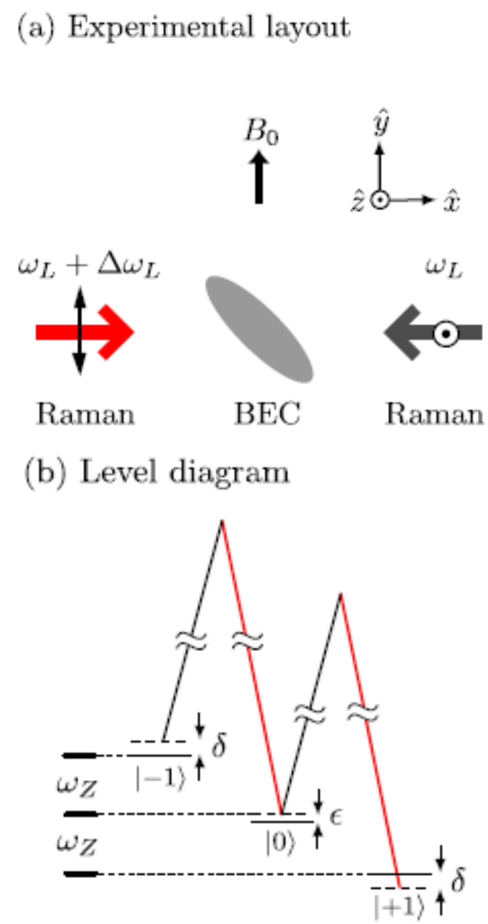
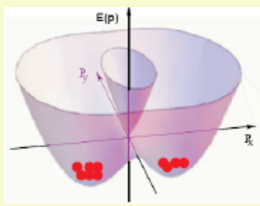
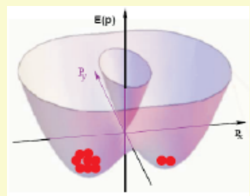


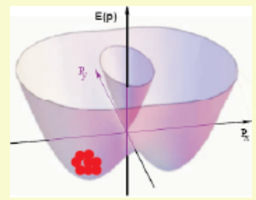


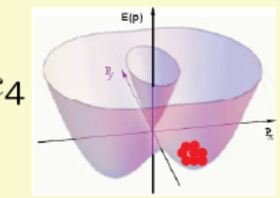
Non-interacting spin-orbit-coupled BEC

If there are no interactions and we do not require momentum to be a good quantum number, then the degeneracy of the many-body ground state is enormous – $(N+1)$-fold.

$||\Psi_N\rangle = c_1$ 

$+ c_2$ 

$+ c_3$ 

$+ c_4$ 

$+ \ldots$

General many-body wave-function

$$||\Psi_N\rangle = \sum_{n=0}^{N} \frac{c_n}{\sqrt{n!(N-n)!}} \left(\hat{B}_L^\dagger\right)^n \left(\hat{B}_R^\dagger\right)^{N-n} ||\text{vac}\rangle,$$

$$\sum_n |c_n|^2 = 1$$

Interactions lift the huge degeneracy at the many-body level and reduce it to a two-fold degenerate state...

“Order-by-disorder:” Selecting a ground state

- Density-density interaction model (in terms of the original bosons)

$$\hat{\mathcal{H}}_{\rm int} = \frac{1}{2V} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} V_{\rm int}(\mathbf{q}) \hat{b}^{\dagger}_{\alpha\mathbf{p}} \hat{b}_{\alpha\mathbf{p}+\mathbf{q}} \hat{b}^{\dagger}_{\beta\mathbf{p}'} \hat{b}_{\beta\mathbf{p}'-\mathbf{q}}$$

- Interaction term for pseudo-spin bosons

$$\hat{\mathcal{H}}_{\rm int} = \frac{1}{2V} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \sum_{\{\sigma_i\}}{}' V_{\rm int}(\mathbf{q}) \hat{B}^{\dagger}_{\sigma_1\mathbf{p}} \hat{B}_{\sigma_2\mathbf{p}+\mathbf{q}} \hat{B}^{\dagger}_{\sigma_3\mathbf{p}'} \hat{B}_{\sigma_4\mathbf{p}'-\mathbf{q}}$$
$$\times U^{\dagger}_{\sigma_1\alpha}(\mathbf{p}) U_{\alpha\sigma_2}(\mathbf{p}+\mathbf{q}) U^{\dagger}_{\sigma_3\alpha'}(\mathbf{p}') U_{\alpha'\sigma_4}(\mathbf{p}-\mathbf{q}),$$

Matrices $\hat{U}$ represent momentum-space rotations (Berry's phases).

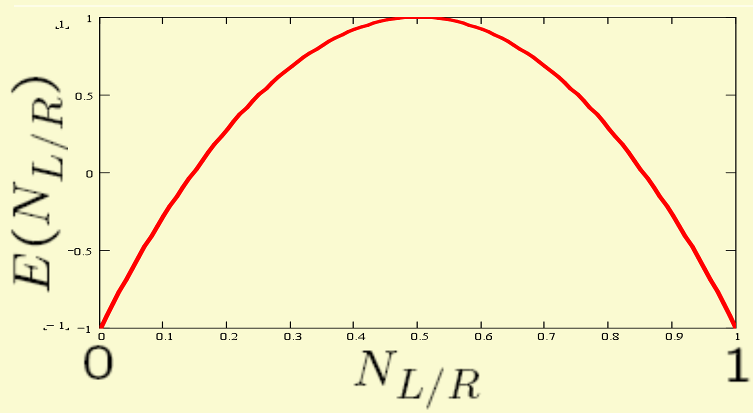
- Bogoliubov theory ($\hat{\beta}$'s below are Goldstone modes)

$$\hat{\mathcal{H}} = \sum_{n=0}^{N} \hat{\mathcal{P}}_{N_L,N_R} \left[\mathcal{E}_0(N_L,N_R) + \sum_{\mathbf{q},\sigma} \Omega_\sigma(n,\mathbf{q}) \hat{\beta}^{\dagger}_{\sigma,\mathbf{q}} \hat{\beta}_{\sigma,\mathbf{q}} \right] \hat{\mathcal{P}}_{N_L,N_R}$$

$\hat{\mathcal{P}}_{N_L,N_R}$ projects on the subspace with N_L left- and N_R right-movers.
The ground state energy is subspace-dependent!

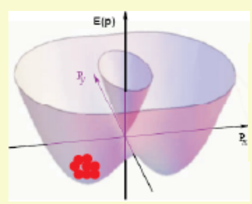
$N00N$ state

Energy of the ground state as a function of the density of the left-movers, N_L/N.

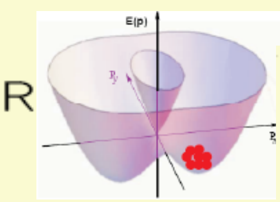


Energy is minimized if all particles are moving to the left or to the right. The ground state

$$||\Psi_N\rangle = \frac{1}{\sqrt{N!}}\Big[\sqrt{w_\mathsf{L}}e^{i\phi_\mathsf{L}}$$



$$+ \sqrt{w_\mathsf{R}}e^{i\phi_\mathsf{R}}$$



$$\Big]$$

Measuring a Spin-Orbit-Coupled "Qubit"

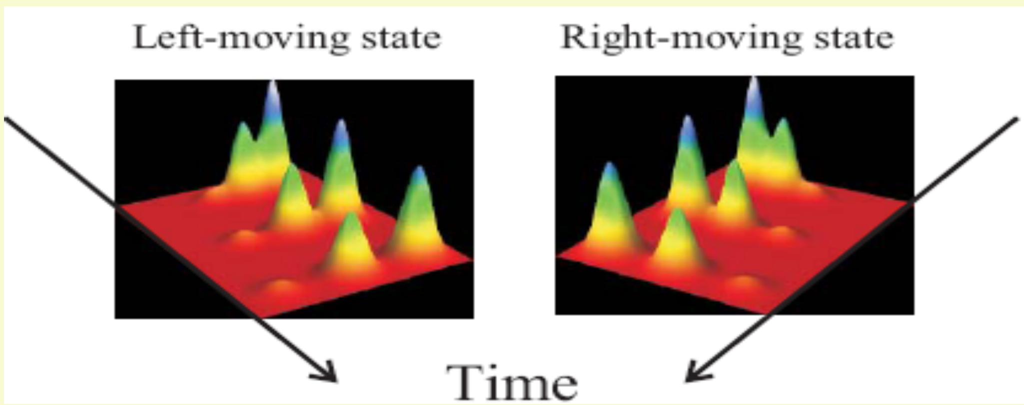
- A way to measure the cat-state ($N00N$-state) of a trapped BEC

$$||\Psi_N\rangle = \frac{1}{\sqrt{N!}}\left[\sqrt{w_\mathsf{L}}e^{i\phi_\mathsf{L}}\,|N\ 0\rangle + \sqrt{w_\mathsf{R}}e^{i\phi_\mathsf{R}}\,|0\ N\rangle\right]$$

  is via time-of-flight expansion.

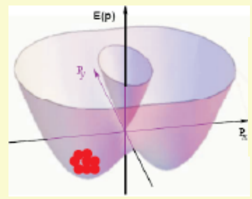
- Velocities at the minima *vanish*, $\left\langle \frac{\partial \hat{\mathcal{H}}}{\partial \mathbf{p}} \right\rangle_{\pm \mathbf{p}_0} = \mathbf{0}$. To observe the condensate(s), we need to turn off both the trap and SO-couplings.

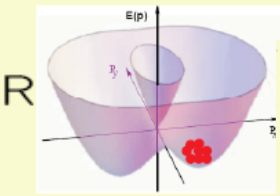
- The result of the measurement is intrinsically impossible to predict with certainty. There are two possibilities:



Topological Bose-Einstein Condensates

- Energetics analysis selects a "double-degenerate" BEC:

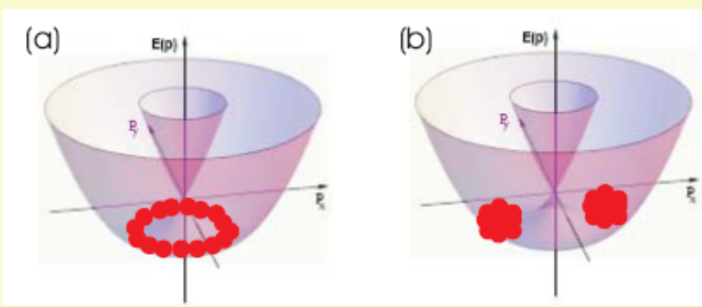
$||\Psi_N\rangle = \frac{1}{\sqrt{N!}}\Big[\sqrt{w_{\mathsf{L}}}\,e^{i\phi_{\mathsf{L}}}$ 

$+ \sqrt{w_{\mathsf{R}}}\,e^{i\phi_{\mathsf{R}}}$ 

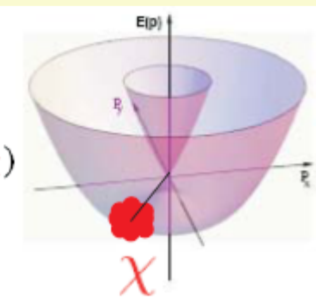
$\Big]$

- The Heisenberg uncertainty principle, $\delta x\,\delta p \gtrsim \hbar$, provides intuition: Bosons repel each other in $\mathbf{r}$-space ("attract" each other in $\mathbf{p}$-space).

- More exotic states appear for infinite-degenerate (Rashba) BEC:



(c) $\propto \int_0^{2\pi} \frac{d\chi}{2\pi}\sqrt{w(\chi)}e^{i\phi(\chi)}$



(b) Spontaneous symmetry breaking ("Higgs" physics)

(c) Topologically distinct states

Vortices in Spin-Orbit

Bose-Einstein Condensates

PHYSICAL REVIEW A **84**, 063604 (2011)

Vortices in spin-orbit-coupled Bose-Einstein condensates

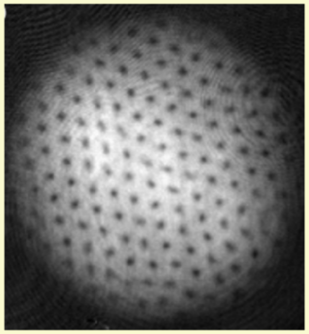
J. Radić,[1] T. A. Sedrakyan,[1] I. B. Spielman,[1,2] and V. Galitski[1]
[1]*Joint Quantum Institute, University of Maryland, College Park, Maryland 20742-4111, USA*
[2]*National Institute of Standards and Technology, Gaithersburg, Maryland 20899, USA*
(Received 24 August 2011; published 5 December 2011)

How to create vortices in a BEC?

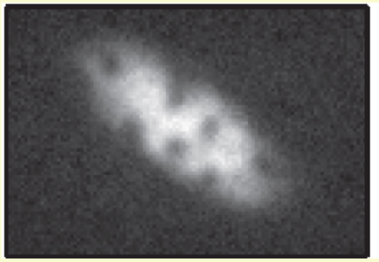
- Rotation: If an ordinary BEC is stirred by a laser "spoon" or the *anisotropic* trap is rotated:
  - The Hamiltonian in the rotating frame is *time-independent*:

$$H_{\mathrm{RF}} = H_0 - \mathbf{\Omega} \cdot \mathbf{L}$$

  - Equilibrium stat. mechanics apply
  - Vortex lattice appears
- Synthetic magnetic field for neutral atoms leads to an effective magnetic field for dressed states



Ketterle, MIT (2001)



Spielman, JQI-NIST (2009)

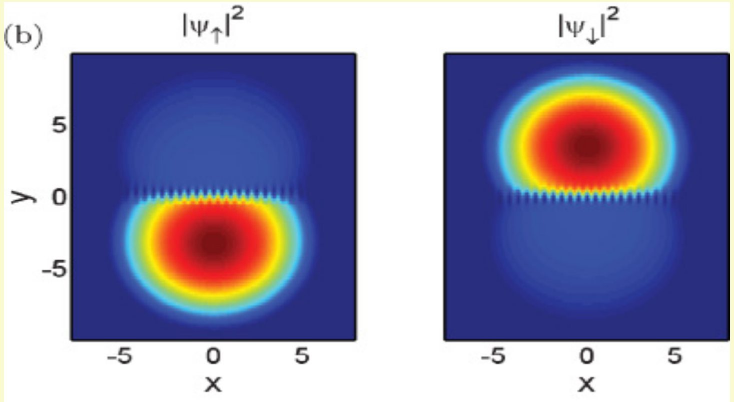
Rotating the trap generally does NOT work for spin-orbit BECs, as there is no rotating frame where the Hamiltonian is time-independent (non-equilibrium physics, heating). However, to combine synthetic spin-orbit coupling with synthetic magnetic field should work.

Spin-orbit coupling + spatially-dependent detuning

- Introducing spatially-dependent detuning $\delta(\mathbf{r})$ in I. Spielman's existing scheme creates an effective gauge field

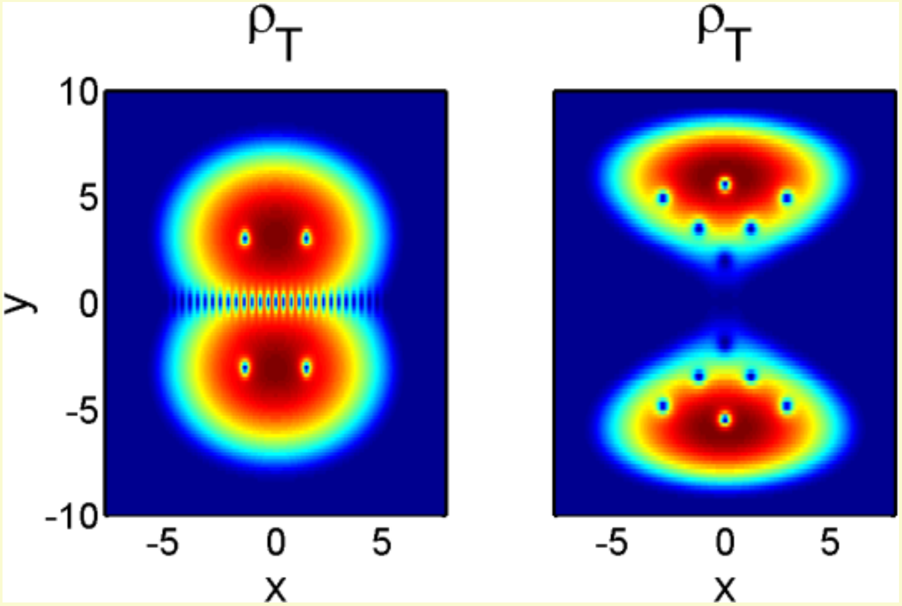
$$H_{\text{eff}} = \frac{\mathbf{p}^2}{2m} + v p_x \hat{\sigma}_z + \Omega \hat{\sigma}_x + \delta(y) \hat{\sigma}_z$$

- The combination of the SOC and synthetic gauge field yields two main effects:
  1. Spatial separation of the left- and right-movers
  2. Synthetic mag. field for each component and vortex nucleation



Parity effect in vortex nucleation

Due to symmetry of the effective gauge field with respect to reflection about the $y = 0$ axis and almost spin-independent interactions, an interesting parity effect is observed (in GPE simulations):
the number of vortices is the same in both components.



Topological Optical Lattices

PHYSICAL REVIEW A **79**, 053639 (2009)

Topological insulators and metals in atomic optical lattices

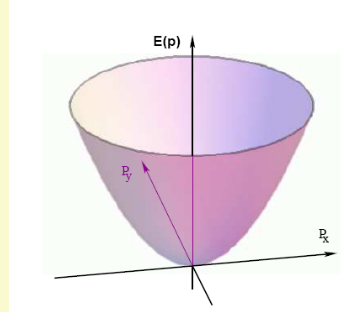
Tudor D. Stanescu,[1] Victor Galitski,[1] J. Y. Vaishnav,[2] Charles W. Clark,[2] and S. Das Sarma[1]

PHYSICAL REVIEW A **82**, 013608 (2010)

Topological states in two-dimensional optical lattices

Tudor D. Stanescu,[1,2] Victor Galitski,[1] and S. Das Sarma[1]

Hidden Topology in a Piece of Solid



- Kinetic energy of a moving particle: $E = \frac{m\mathbf{v}^2}{2} = \frac{\mathbf{p}^2}{2m}$.
  Free particle plane-wave wave-function, $\psi_{\mathbf{p}} \propto e^{i\mathbf{p}\cdot\mathbf{r}}$

- Now consider electrons moving in a crystal lattice.
  1. Band structure changes. I.e., $E \neq p^2/(2m)$ in a solid
  2. Discrete translational symmetry demands that $\psi(x) = \psi(x + na)$ and lattice momentum is defined modulo $2\pi\hbar/a$. Topologically momentum space is a torus in 2D with complex wave-functions $\psi_{\mathbf{p}}$ associated with each point. Complicated "topological" stuff!

Quantum wave-functions on a torus

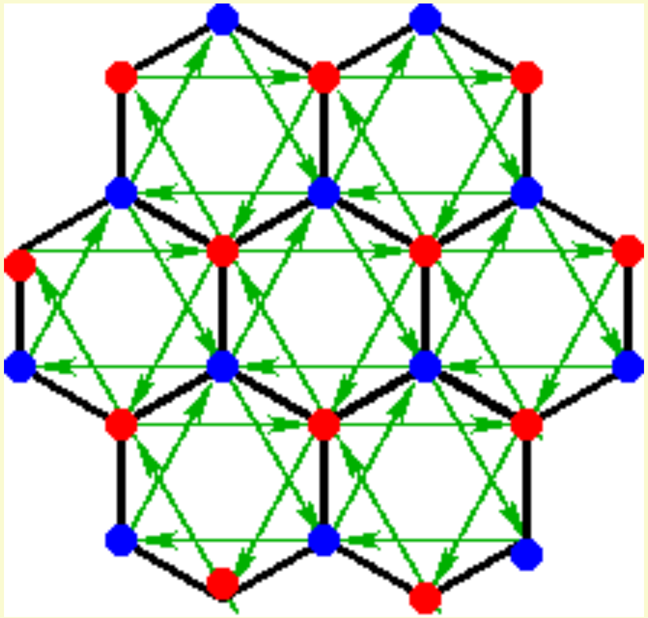
→ Classical world, where we measure things (resistance, etc.)

This correspondence can be classified by integer topological indices (Chern numbers associated with bands).

Topological Optical Lattices

- $\mathbb{T}$-invariant topological insulators are akin having two replicas of QHE. They are of interest in solids, because no $\mathbf{B}$-field is needed. It is not relevant in AMO: Lattice QHE is easier to realize.

- Canonical Haldane model of a top. insulator with broken $\mathbb{T}$-reversal:



$$\hat{\mathcal{H}} = \frac{1}{2m}\left[\hat{\mathbf{p}} - \mathbf{A}_{\mathrm{synt}}(\mathbf{r})\right]^2 + V_0 \sum_{i=1}^{3} \cos^2(\mathbf{k}_i \cdot \mathbf{r}) + U_{\mathrm{trap}}(\mathbf{r})$$

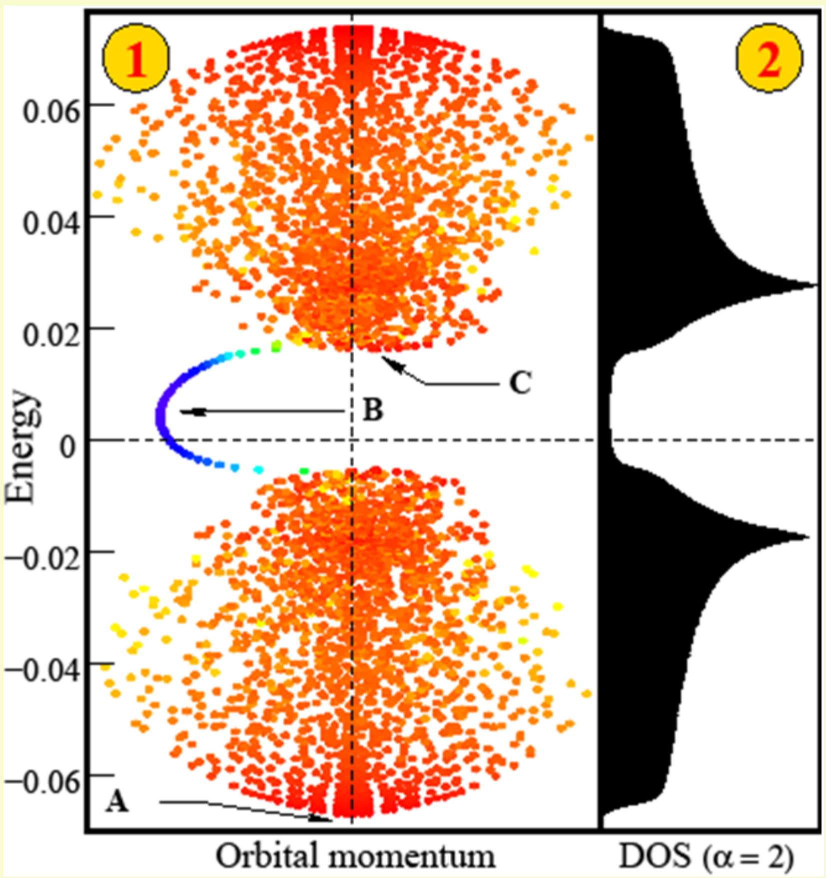
Synthetic gauge fields

Lattice formed by 3 standing waves

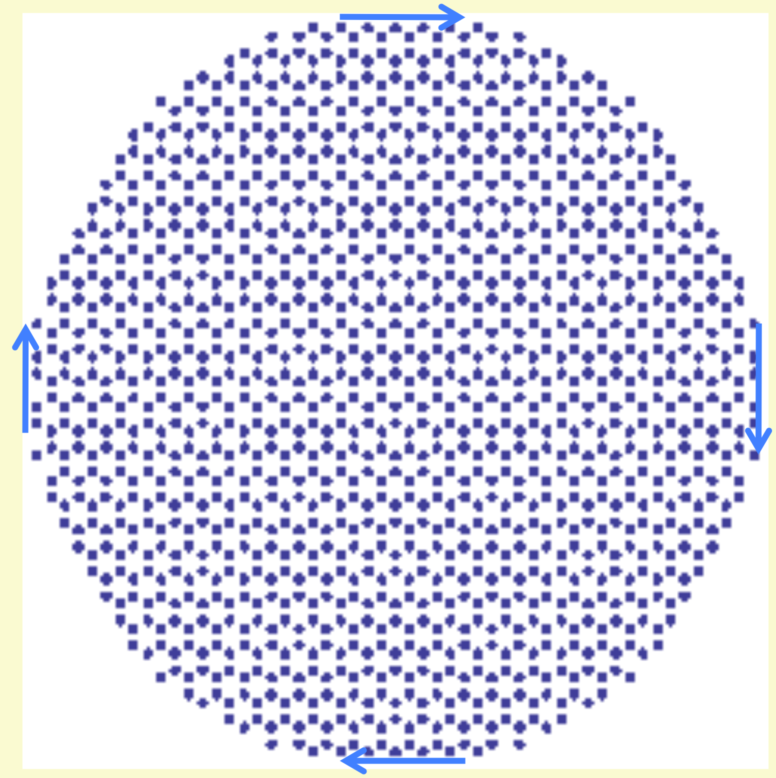
Trap

Topological Spectrum and Wave-Function: an Example

Exact single-particle spectrum and density of states

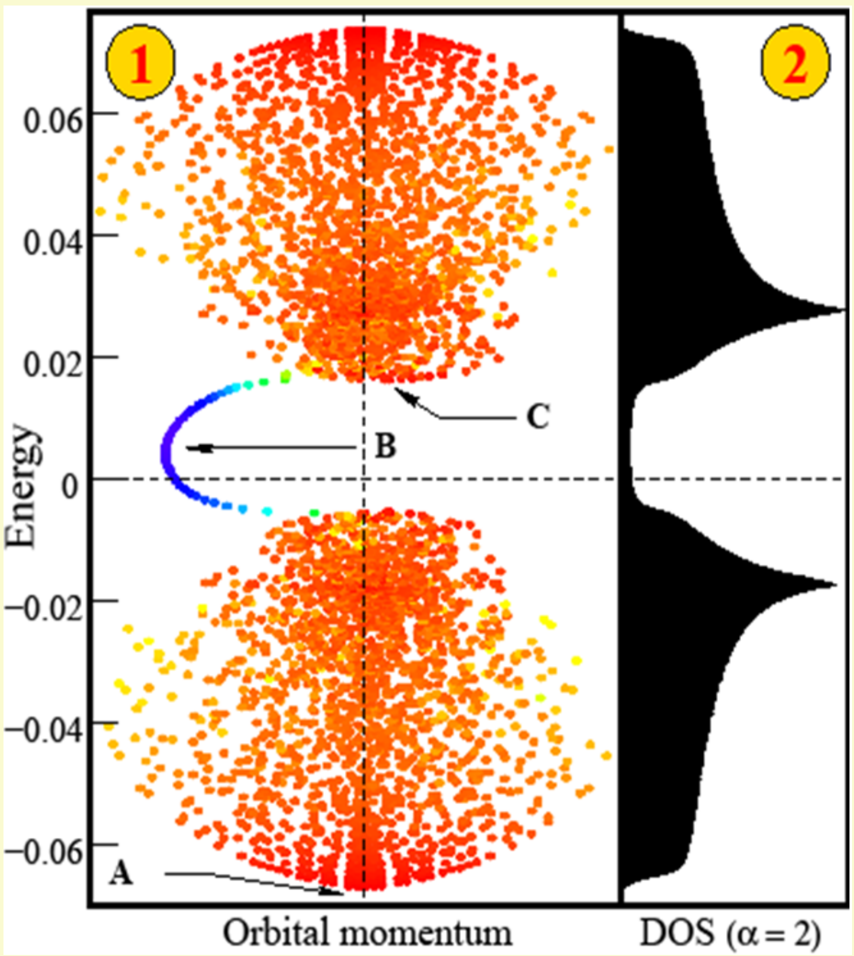


Geometry: A finite-size disk sample

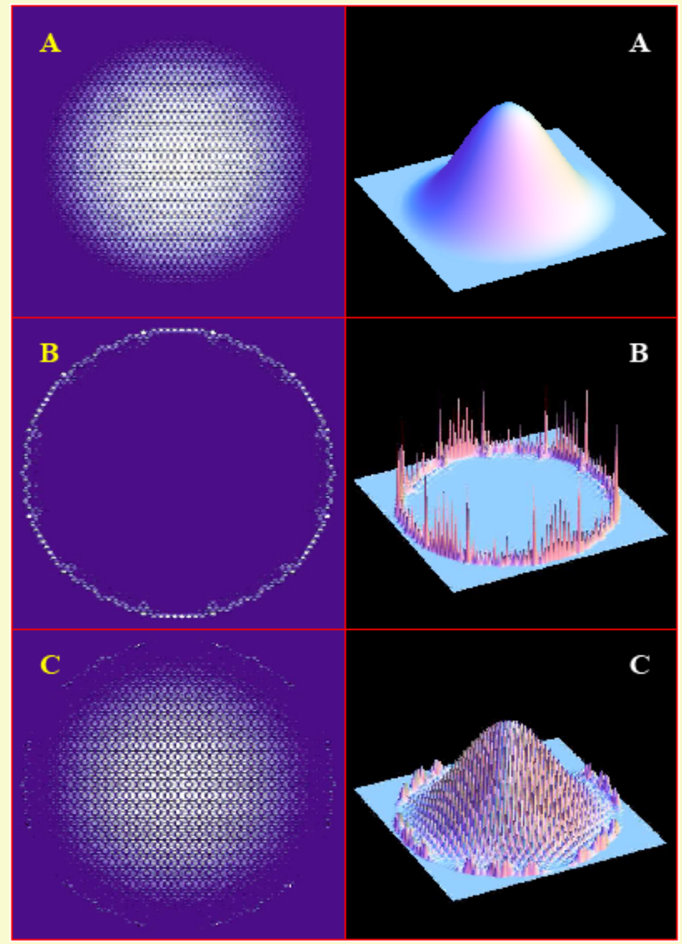


Topological Spectrum and Wave-Function: an Example

Exact single-particle spectrum and density of states



Exact |wave-function| profile



Physics

Physics **4**, 62 (2011)

Viewpoint

Quantum liquids move to a higher dimension

Tameem Albash and Stephan Haas
Department of Physics and Astronomy, University of Southern California, Los Angeles, CA 90089-0484,

Selected for a Viewpoint in *Physics*

PRL **107,** 077201 (2011) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2011

Kaleidoscope of Exotic Quantum Phases in a Frustrated *XY* Model

Christopher N. Varney,[1,2] Kai Sun,[1,3] Victor Galitski,[1,3] and Marcos Rigol[2]

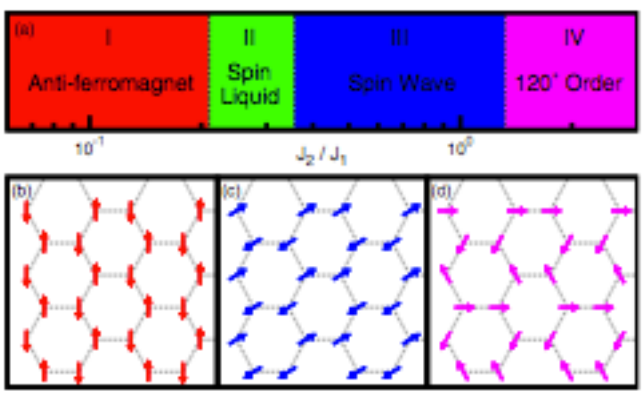
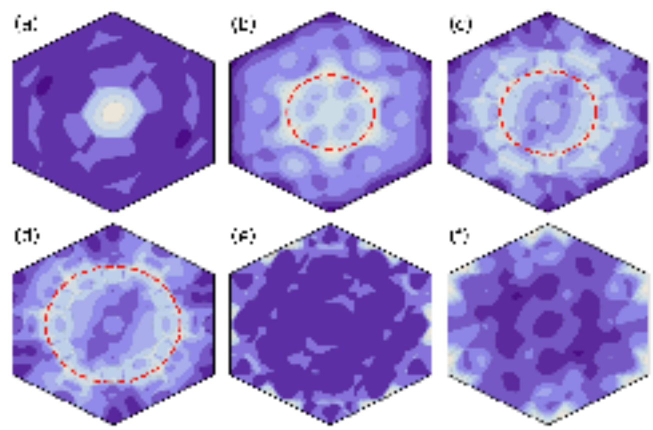


FIG. 1 (color online). (a) Phase diagram of the model in Eq. (1) as a function of J_2/J_1, (b) antiferromagnetic ordering



Practical Applications of the Synthetic Gauge Fields for Precision Quantum Interferometry

RAPID COMMUNICATIONS

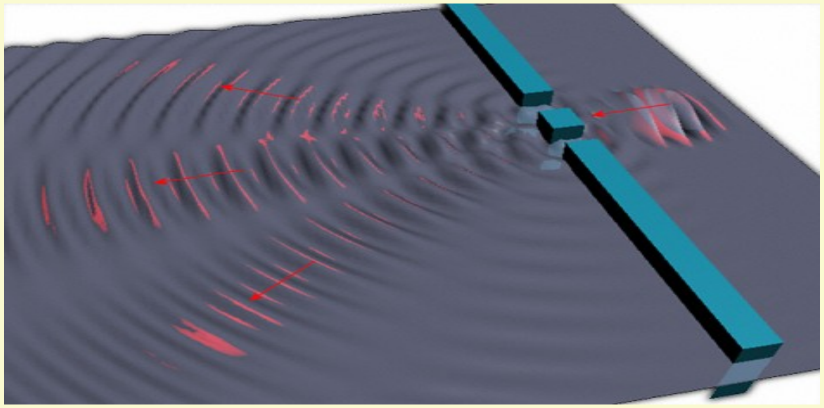
PHYSICAL REVIEW A **83**, 031602(R) (2011)

Interferometry with synthetic gauge fields

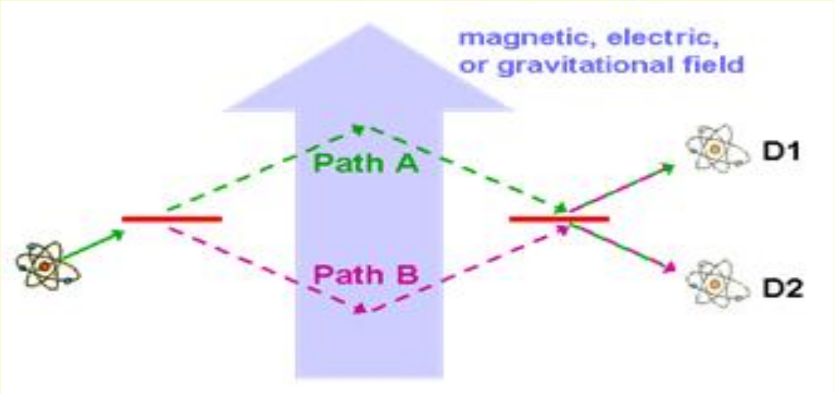
Brandon M. Anderson, Jacob M. Taylor, and Victor M. Galitski
Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA

Existing quantum devices: Atomic interferometers

- Key idea: To take advantage of the wave-like nature of particles



- Separating and recombining particle beams leads to interference



- Dynamics of QM phases depends on external fields (gravity, acceleration, magnetic field, etc.). So, we can measure them!

letters to nature

Measurement of gravitational acceleration by dropping atoms

Achim Peters, Keng Yeow Chung & Steven Chu

Physics Department, Stanford University, Stanford, California 94305-4060, USA

Laser-cooling of atoms and atom-trapping are finding increasing application in many areas of science[1]. One important use of laser-cooled atoms is in atom interferometers[2]. In these devices, an atom is placed into a superposition of two or more spatially separated atomic states; these states are each described by a quantum-mechanical phase term, which will interfere with one another if they are brought back together at a later time. Atom

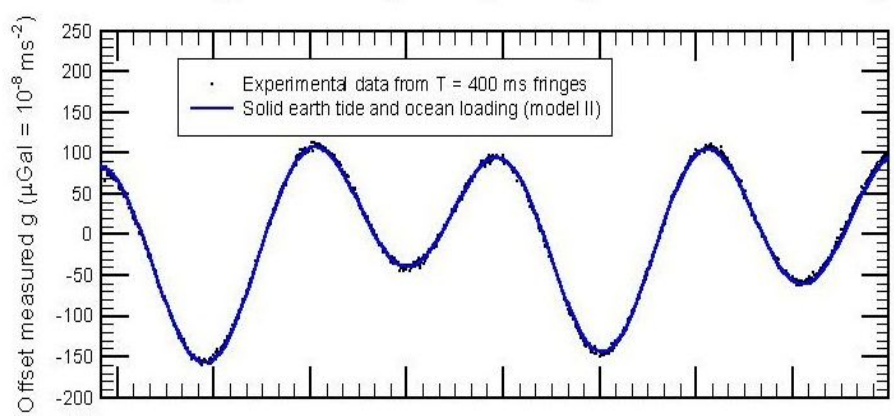
NATURE | VOL 400 | 26 AUGUST 1999 | www.nature.com

Steven Chu

From Steven Chu group web-site at Stanford:

http://www.stanford.edu/group/chugroup/

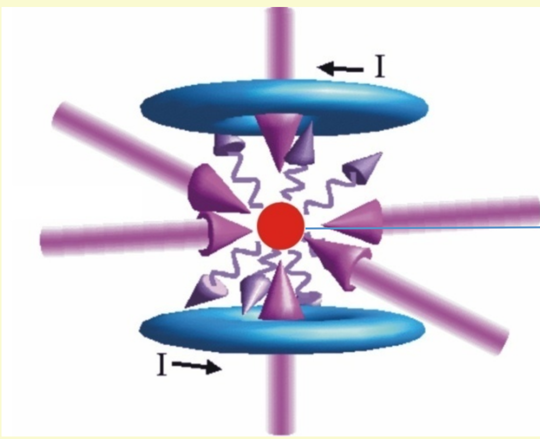
Monitoring of local gravity using T = 400 ms fringes



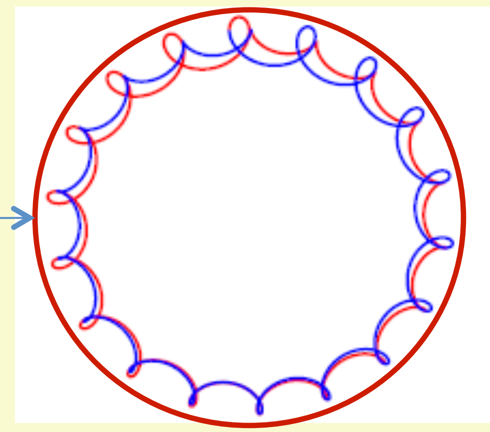
AC Interferometer Through Synthetic Spin

Existing gravimeters do not probe time-dependent gravity/acceleration, $\dot{\mathbf{a}}(t)$. But it is possible with synthetic gauge fields [B. Anderson, J.Taylor, & VG, Phys. Rev. A **83**, 031602(R) (2011)].

Atoms with $e \pm 1$ in a field: $H = \dfrac{(\mathbf{p} - \sigma m \omega_c \mathbf{e}_z \times \mathbf{r})^2}{2m} + \dfrac{1}{2} m \omega_0^2 \mathbf{r}^2 - m \delta \mathbf{g}(t) \cdot \mathbf{r}$



Trajectories for “spin-up” and “spin-down” differ depending on time-dependent gravity!



Phase-difference: $\langle \hat{S}_z \rangle \propto \sin\left(2 \int_0^T dt \mathbf{r}(t) \cdot \mathbf{g}(t)\right)$

Summary of the recent progress

- Dressed states with cold atoms potentially host a much richer variety of spin-orbit structures than that available in solids (Rashba, Dresselhaus, "Weyl," su(3)-SOC, ...)
- Synthetic spin-structures may be practically useful for quantum interferometry
- Abelian spin-orbit BECs have already been observed. Theory predicts macroscopically-entangled states in non-Abelian SO-BECs (staying tuned for new experiments...)
- Synthetic SOC + synthetic magnetic field = new vortex structures
- SO coupling on a lattice may lead to lattice quantum Hall effect
- Future work: Non-equilibrium physics + synthetic fields. Cold atoms may be ideal candidates to realize Floquet topological insulators.