# Spintronics with cold atoms

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*Phys. Rev. Lett.* ***103**, 170401 (2009)*
*Phys. Rev. Lett.* ***104**, 220403 (2010)*
*Phys. Rev. Lett.* ***105**, 155301 (2010)*

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# Collaborators

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Acknowledgements:
Gerrit Bauer (*Delft*)
Hiroshi Kohno (*Osaka*)
Maxim Mostovoy (*Groningen*)

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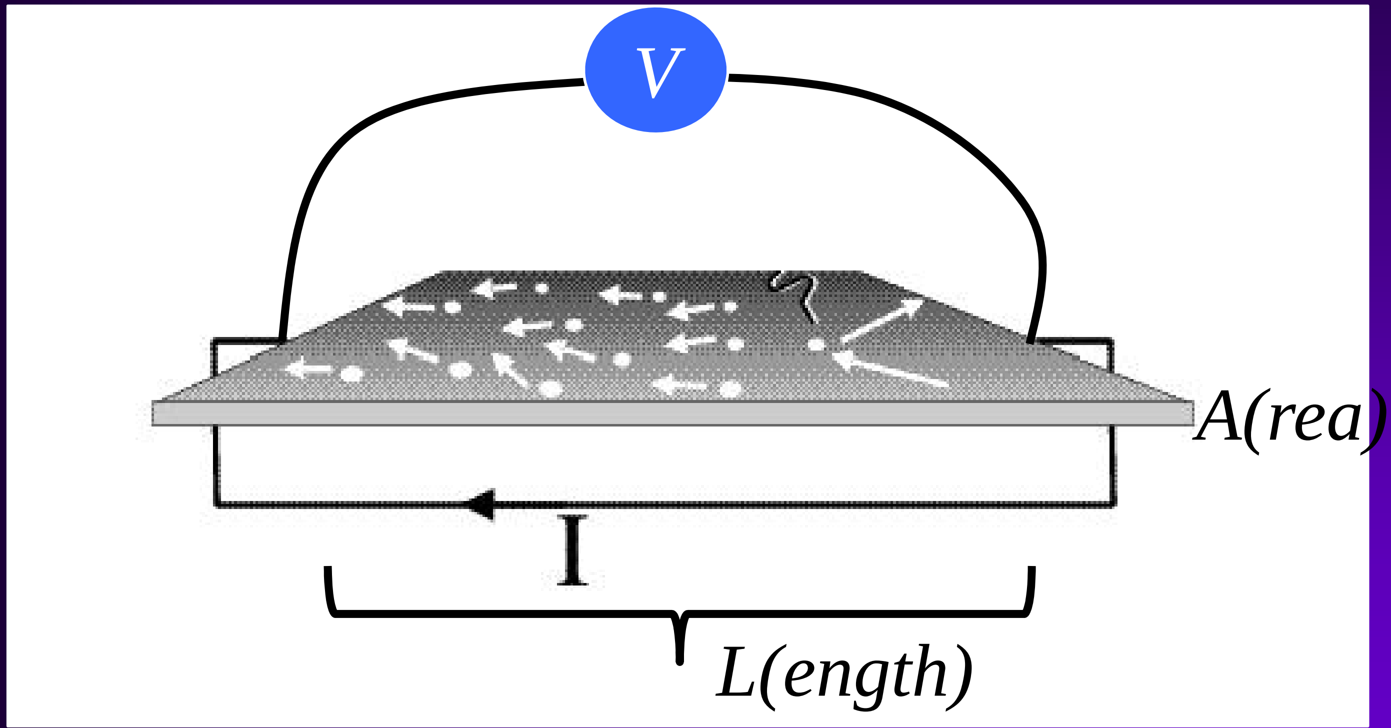
# Spintronics with cold atoms

- Does it exist?
- What can we learn from it?
- What is it good for?

# Lecture - outline

- Solid-state electronics and spintronics
- Cold atoms
- Spintronics with cold-atom systems

# Electronic transport (I)

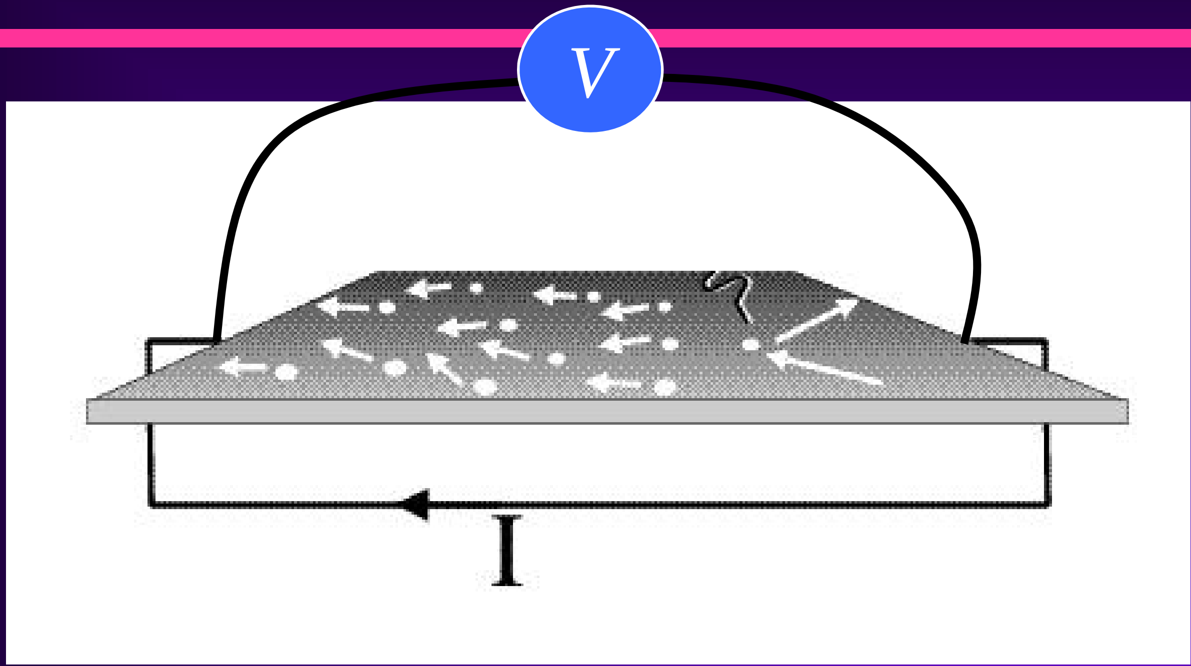



$$I = \frac{V}{R} \Rightarrow j_e A = \frac{EL}{R} \Rightarrow j_e = \frac{E}{RA/L} \equiv \frac{E}{\rho_e} \equiv \sigma_e E$$

$\rho$(esistivity)

$\sigma$: conductivity

# Electronic transport (II)




*Drift velocity:*

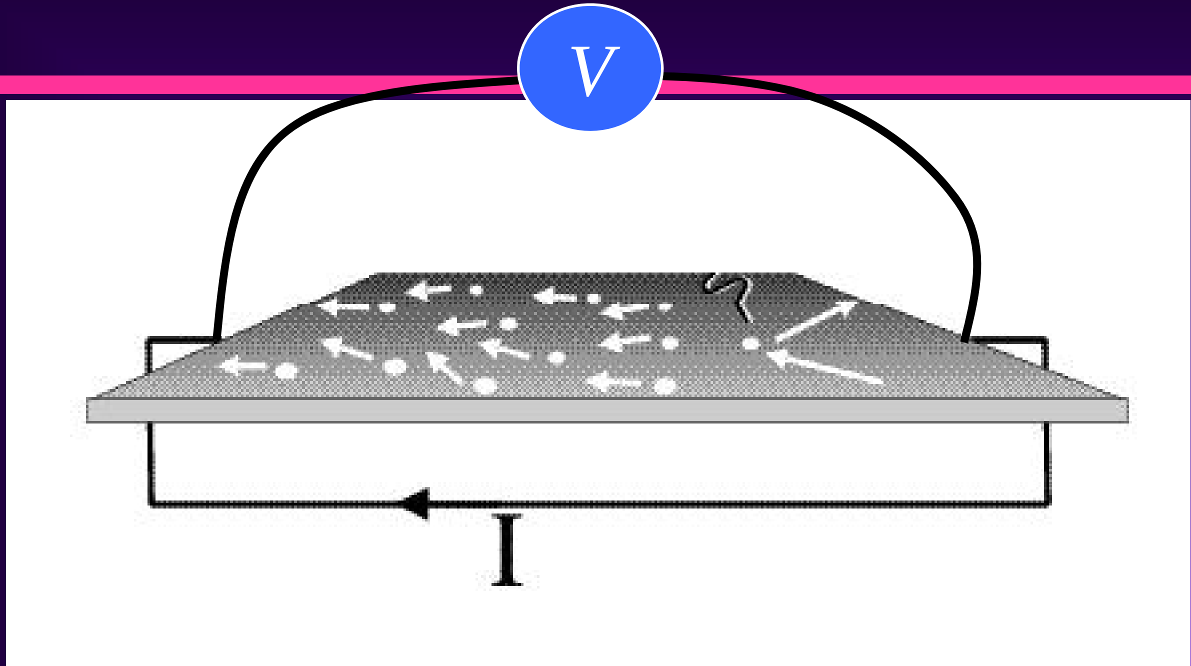
$$n\vec{v}(t) = \int \frac{d\vec{k}}{(2\pi)^3} f\left(\vec{k},t\right)\frac{\hbar\vec{k}}{m}$$

*Newton's equation of motion+dissipation:*

$$nm_e \frac{d\vec{v}}{dt} = -ne\vec{E} - \frac{nm_e\vec{v}}{\tau_{tr}}$$

*relaxation*

# Electronic transport (III)




$$nm_e \frac{d\vec{v}}{dt} = -ne\vec{E} - \frac{nm_e\vec{v}}{\tau_{tr}}$$

$$\vec{j}_e = -ne\vec{v} = \frac{\vec{E}}{\rho_e} = \sigma_e \vec{E}$$

steady state: $\frac{d\vec{v}}{dt} = 0$

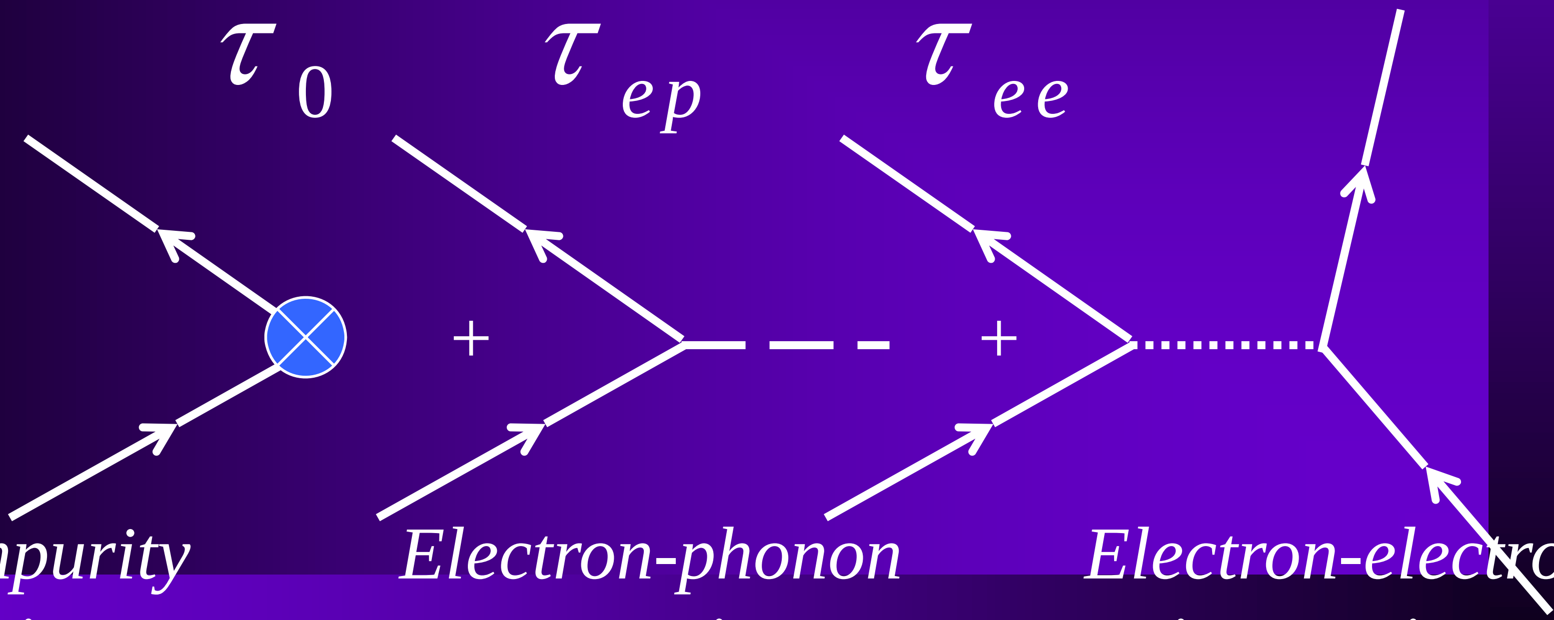
$$\rho_e = \frac{m}{ne^2\tau_{tr}}$$

*Drude formula*

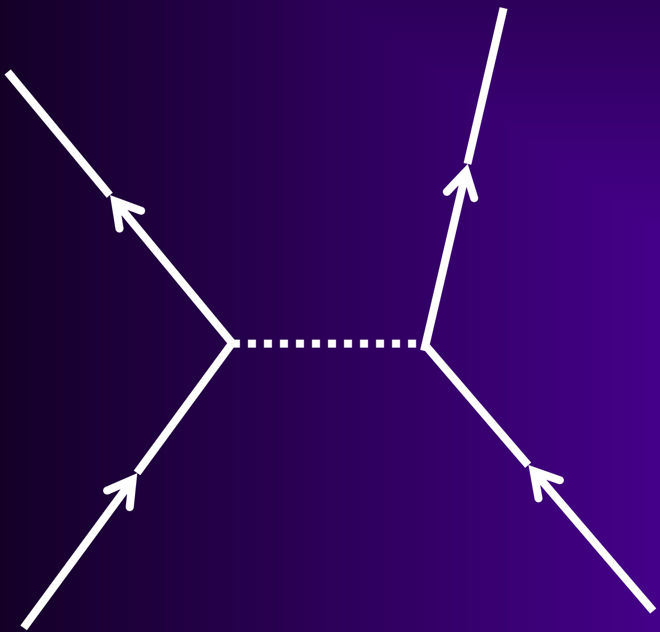
# Transport relaxation time

$\tau_{tr} \sim \mathrm{ps}$ *(metals)*

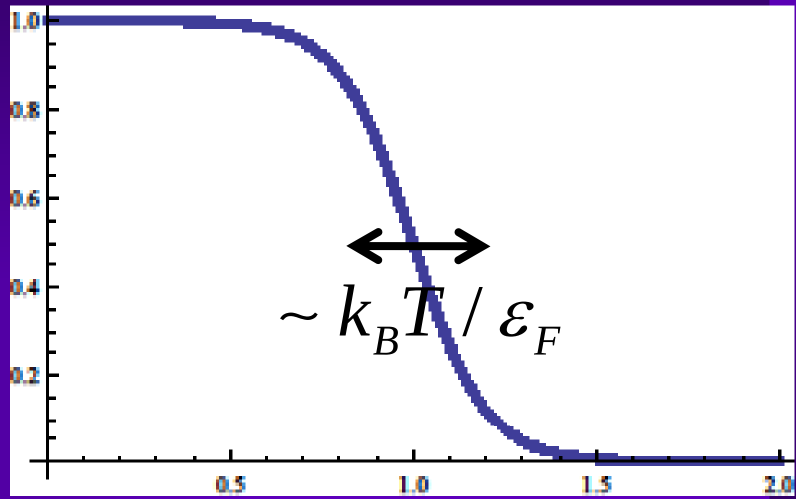
$$\frac{1}{\tau_{tr}} = \frac{1}{\tau_0} + \frac{1}{\tau_{ep}} + \frac{1}{\tau_{ee}} + \cdots$$



# Electron-electron interactions (I)



*Fermi-Dirac distribution function*

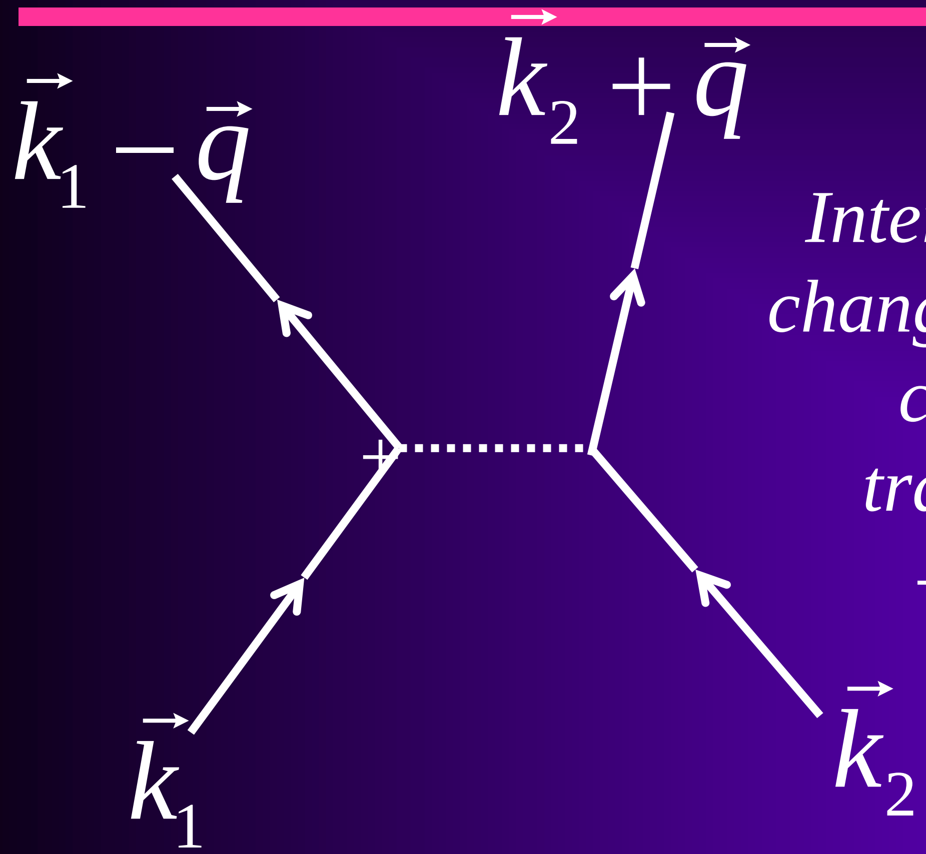



$$\Delta\rho \propto \frac{1}{\tau_{ee}} \propto T^2 \ \text{for}\ T/T_F \to 0$$

*Due to Pauli blocking*

*Hallmark of a Fermi liquid*

# Electron-electron interactions (II)




*Interactions do not contribute to change in drift velocity and do not contribute to resistivity in translation invariant system – need underlying lattice*

*Galilean invariance: resistivity zero*

# Electro-chemical potential

*Closed circuit:*

$E \longrightarrow$

$j=0$

$L$

$$\text{Voltage} = \mu(L) - \mu(0) = EL$$

*Current*

$$j_e = \sigma_e\left(E - \nabla\mu_e\right)$$

$$\nabla \cdot j_e = 0$$

*Conservation of charge*

$$\mu_e = Ex + cst$$

*Need:*

$$L >> \ell \sim v_F \tau_{tr}$$

# Recap (I): Lecture - outline

- Solid-state electronics and spintronics
  *electric current, resistivity/conductivity, electro-chemical potential, hydrodynamics*

- Cold atoms

- Spintronics with cold-atom systems

# What is spintronics?

- Removing the "factor of two for spin degeneracy"

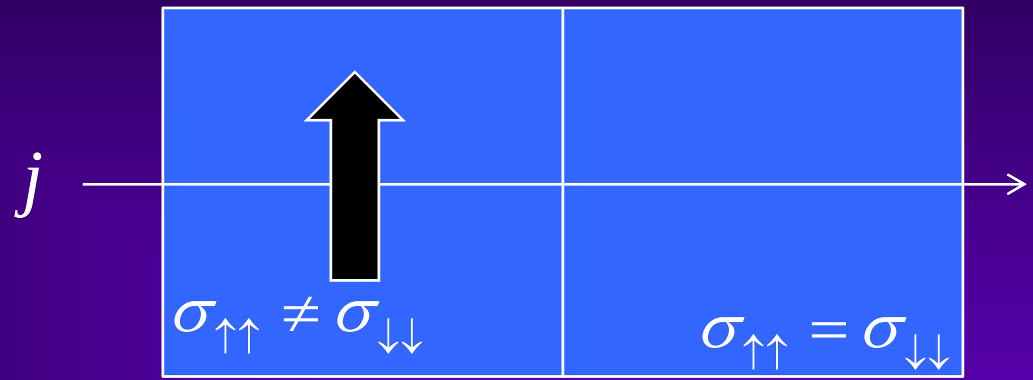
$$\begin{pmatrix} j_\uparrow \\ j_\downarrow \end{pmatrix} = \begin{pmatrix} \sigma_{\uparrow\uparrow} & \sigma_{\uparrow\downarrow} \\ \sigma_{\downarrow\uparrow} & \sigma_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} E - \nabla\mu_\uparrow \\ E - \nabla\mu_\downarrow \end{pmatrix}$$

*Spin drag (Orenstein talk?)*
*– for now ignore (should go like $T^2$)*

- (This ignores non-collinear magnetization dynamics and strong spin-orbit coupling, heat current...)

# magnet-normal metal interface

*ferromagnet* *Normal metal*



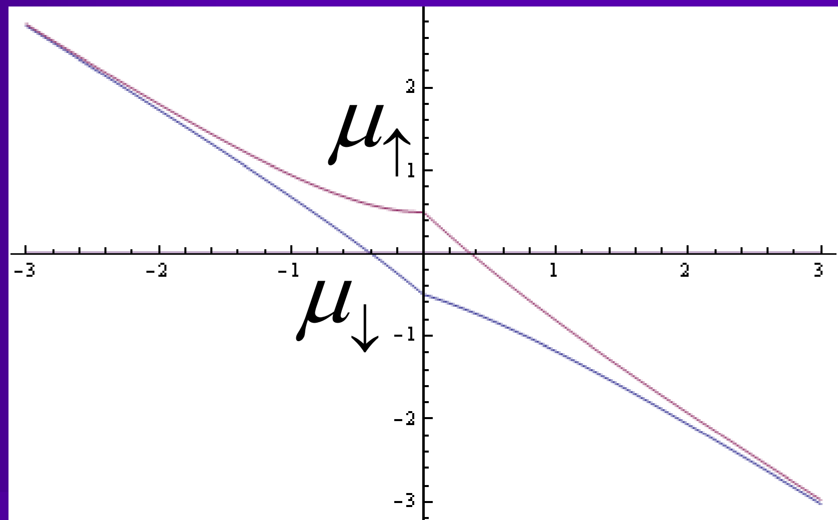

*Charge conservation:*

$$\mu_e = \mu_\uparrow + \mu_\downarrow$$
$$\nabla \cdot j_e = 0$$

*Spin accumulation*

$$\mu_s = \mu_\uparrow - \mu_\downarrow$$

$$\nabla^2 \mu_s = \frac{\mu_s}{\ell_{sf}^2}$$




*Spin-flip rel. length*

*Spin accumulation at interface*

# Recap (II): Lecture - outline

- Solid-state electronics and spintronics
  *electric current, resistivity/conductivity, electro-chemical potential, hydrodynamics, everything spin-resolved, spin accumulation*

- Cold atoms

- Spintronics with cold-atom systems

# Spintronics with cold atoms

- Does it exist?
- What can we learn from it?
- What is it good for?

# First: introducting cold atoms

# Introducing cold atoms (I)

**Electrons in metals**

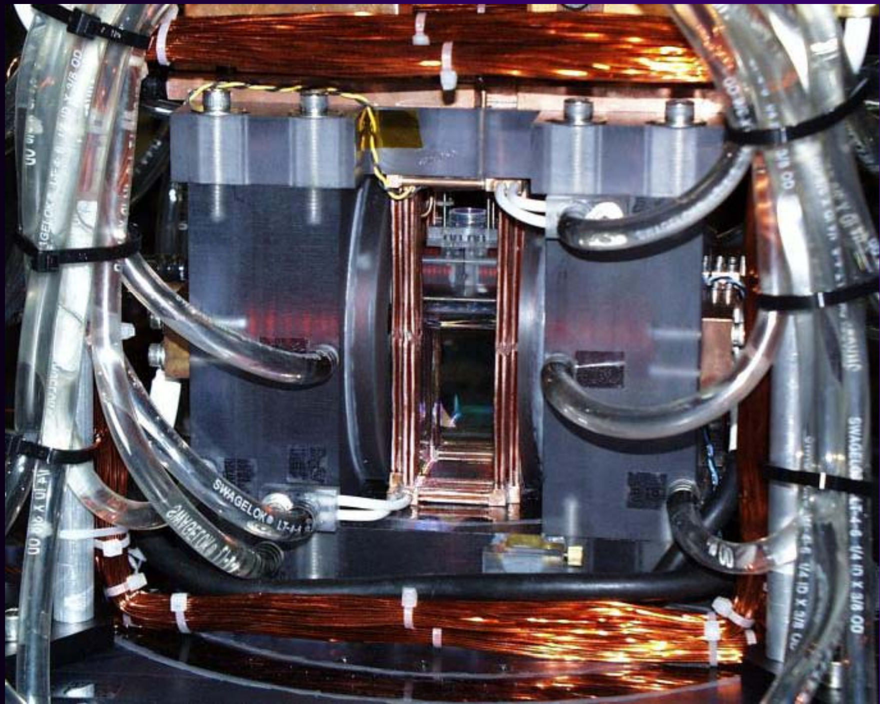
- charge *–e*
- Spin *S=1/2*
- fermion
- ionic lattice
- disorder
- phonons
- spin-orbit coupling
- long range *e*-*e* interactions

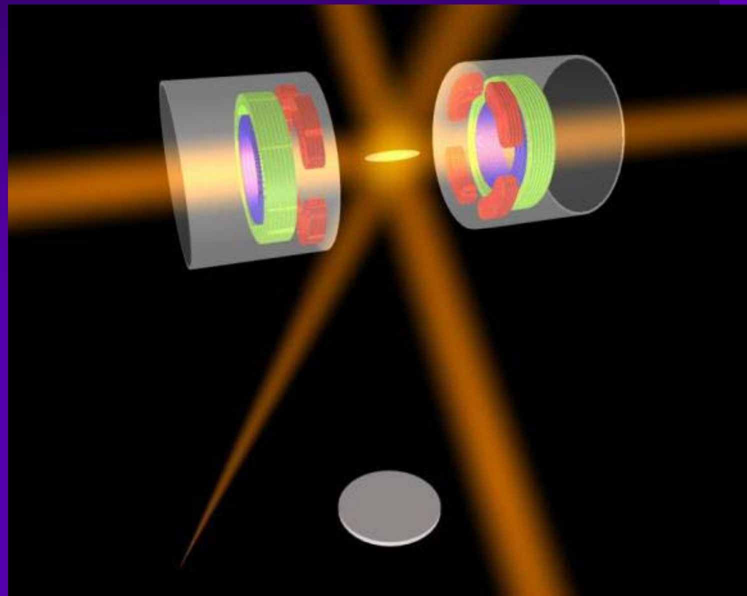
**Trapped cold atoms**

- neutral, e.g., Rb-87, Li-6/7, ...
- (hyperfine) spin *F*=...
- Bosons and/or fermions
- magnetic and/or optical trap
- disorder
- phonons
- spin-orbit coupling
- short-range atomic interactions

*can be engineered*

# Introducing cold atoms (II)





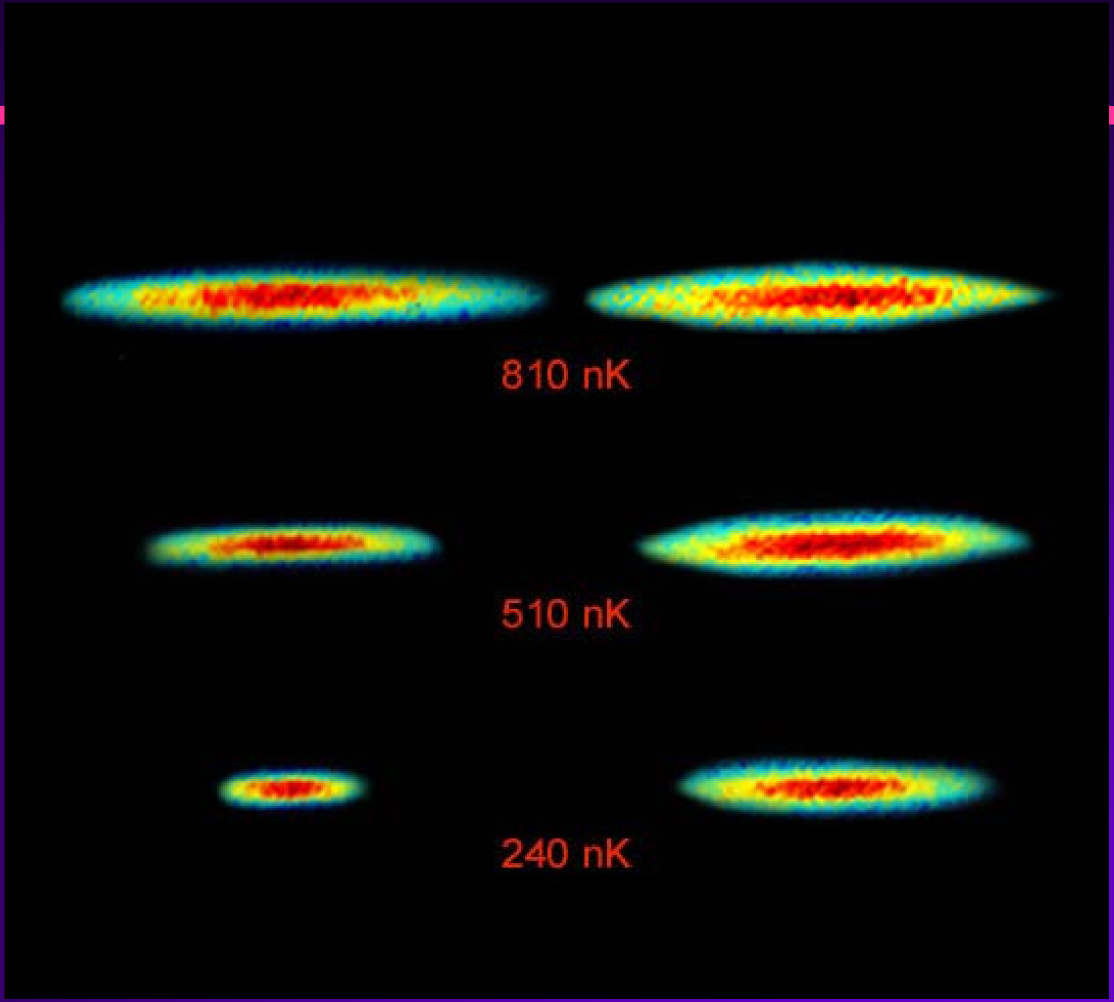
*Laser cooling, harmonic confining potential, evaporative cooling,absorption imaging*

# Introducing cold atoms (III)

*Hulet group (talk@ 13:30)*

*This experiment: few million atoms*

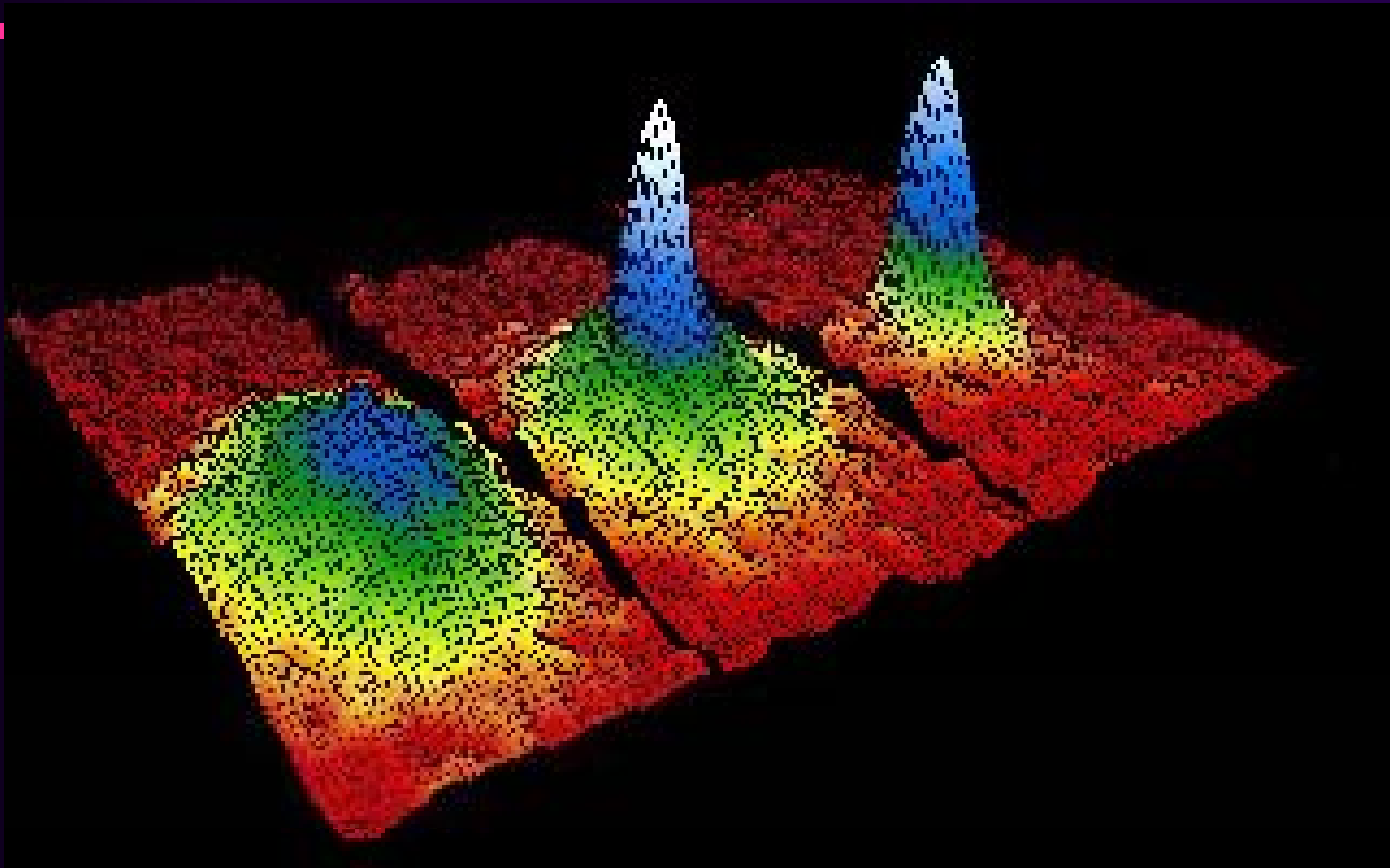
*(Generally ranges from $10^4$-$10^9$)*




*Lithium-7*
*(Boson -> Bose Condensation!)*

*Lithium-6*
*(Fermion, T/T_F~0.1)*

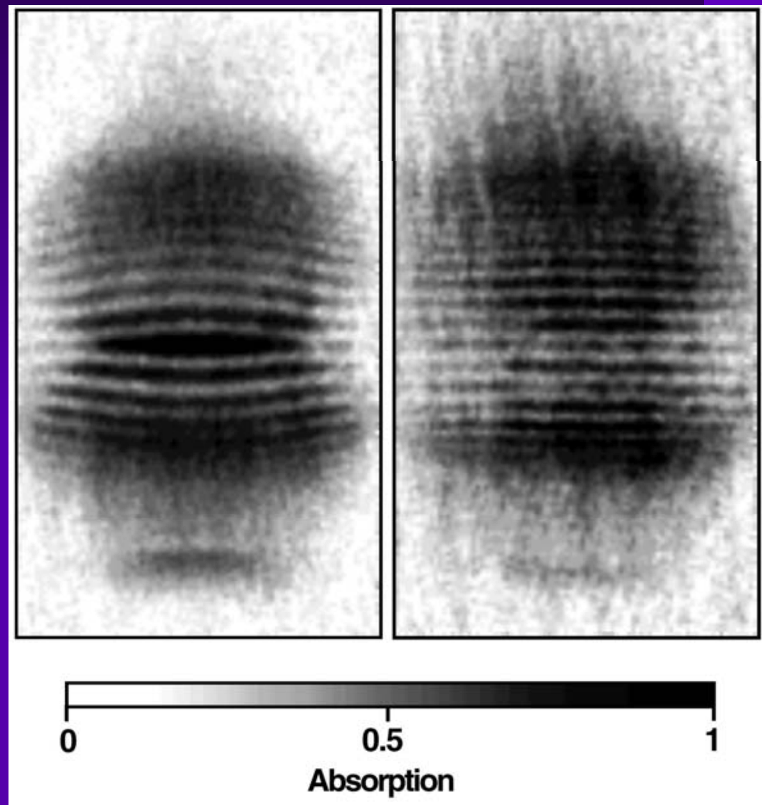
# First Bose condensate (1995)



*C. Wieman, E. Cornell (Nobel prize, 2001)*

# Two interfering Bose condensates

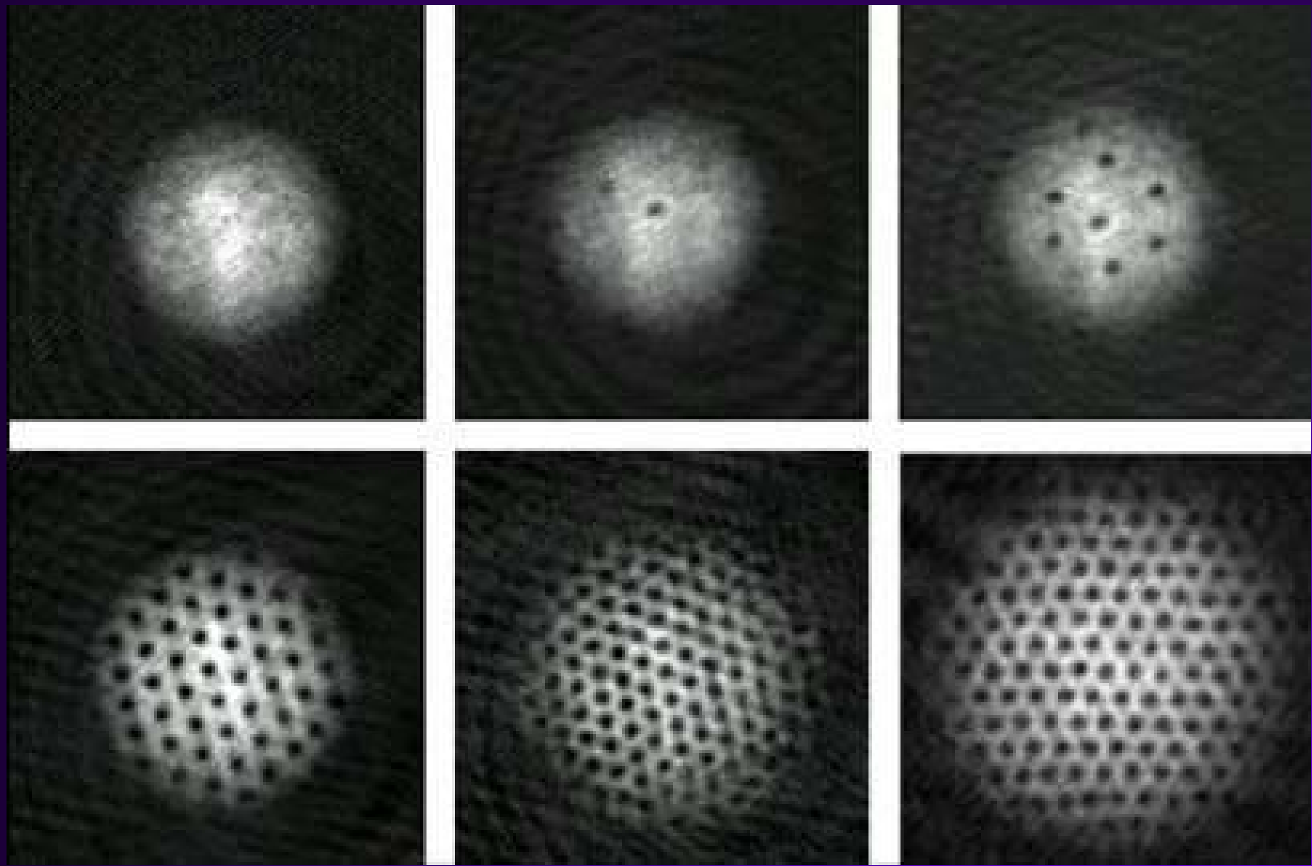

*Classical waves*




*"matter waves"*

*W. Ketterle (Nobel prize, 2001)*

# Rotation: quantum vortices

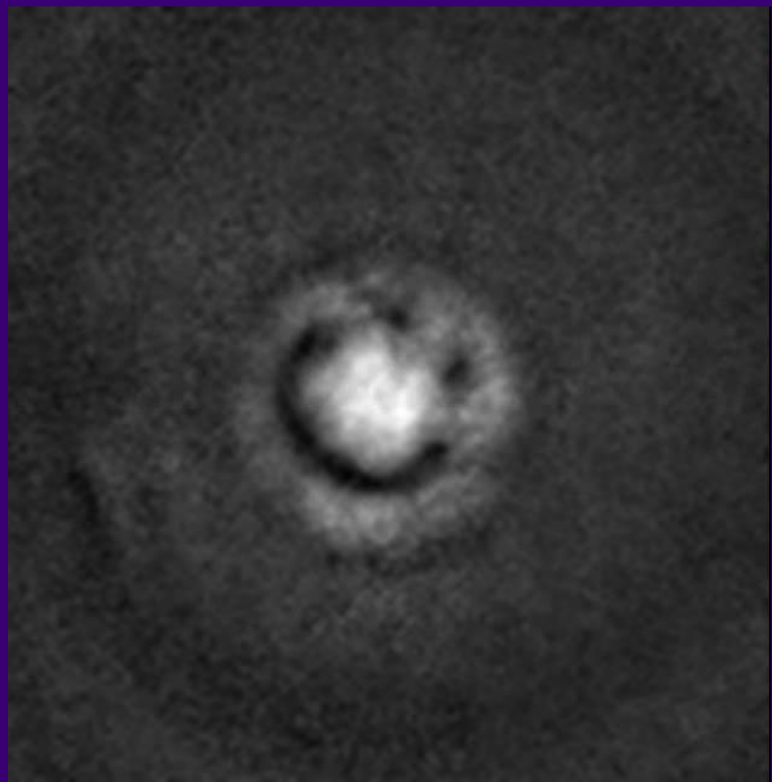


*NB: rotation acts like magnetic field!!*

http://jilawww.colorado.edu/bec

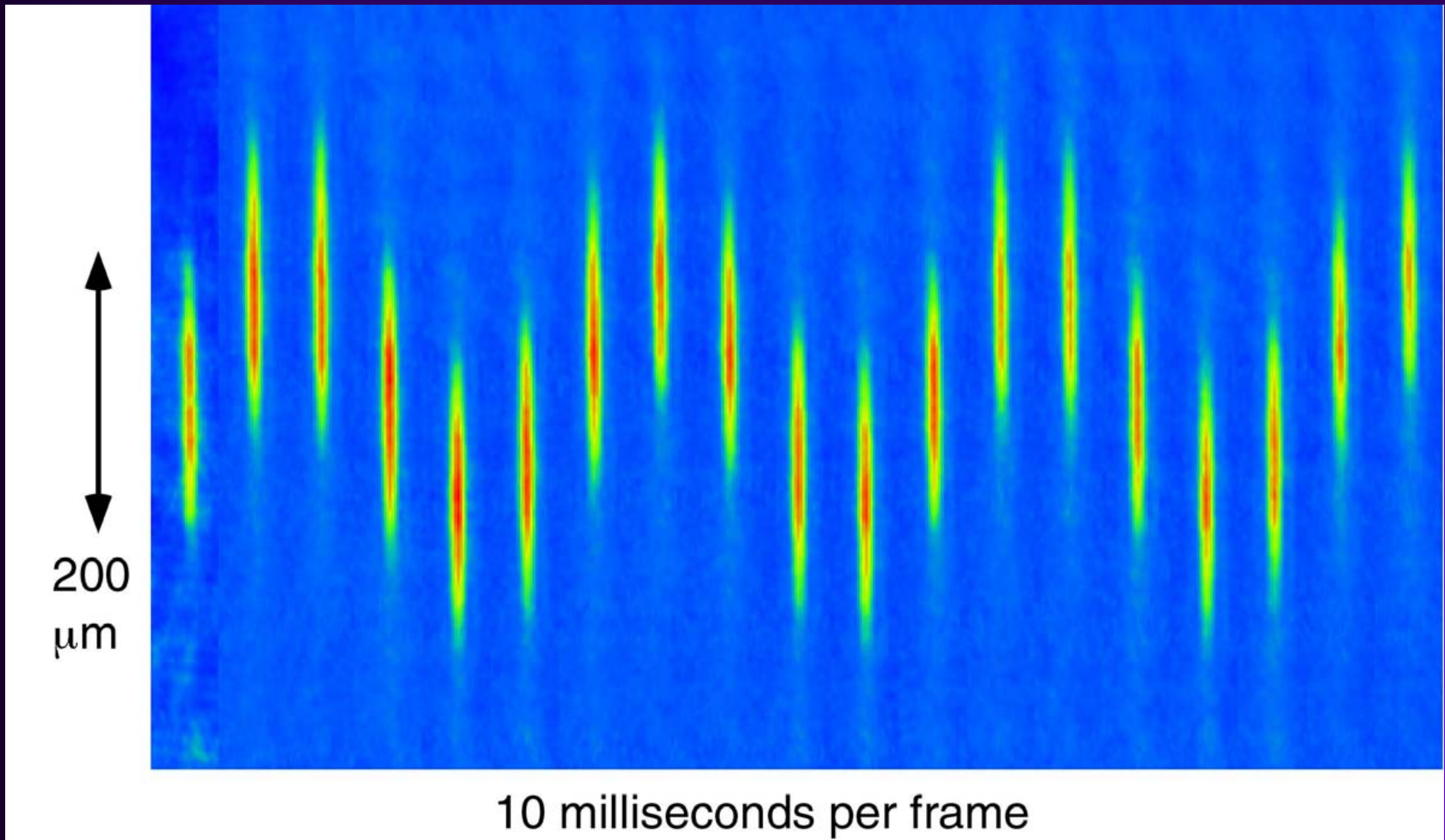
# Introducing cold atoms (IV)

- + many more (mostly equilibrium) results...e.g:



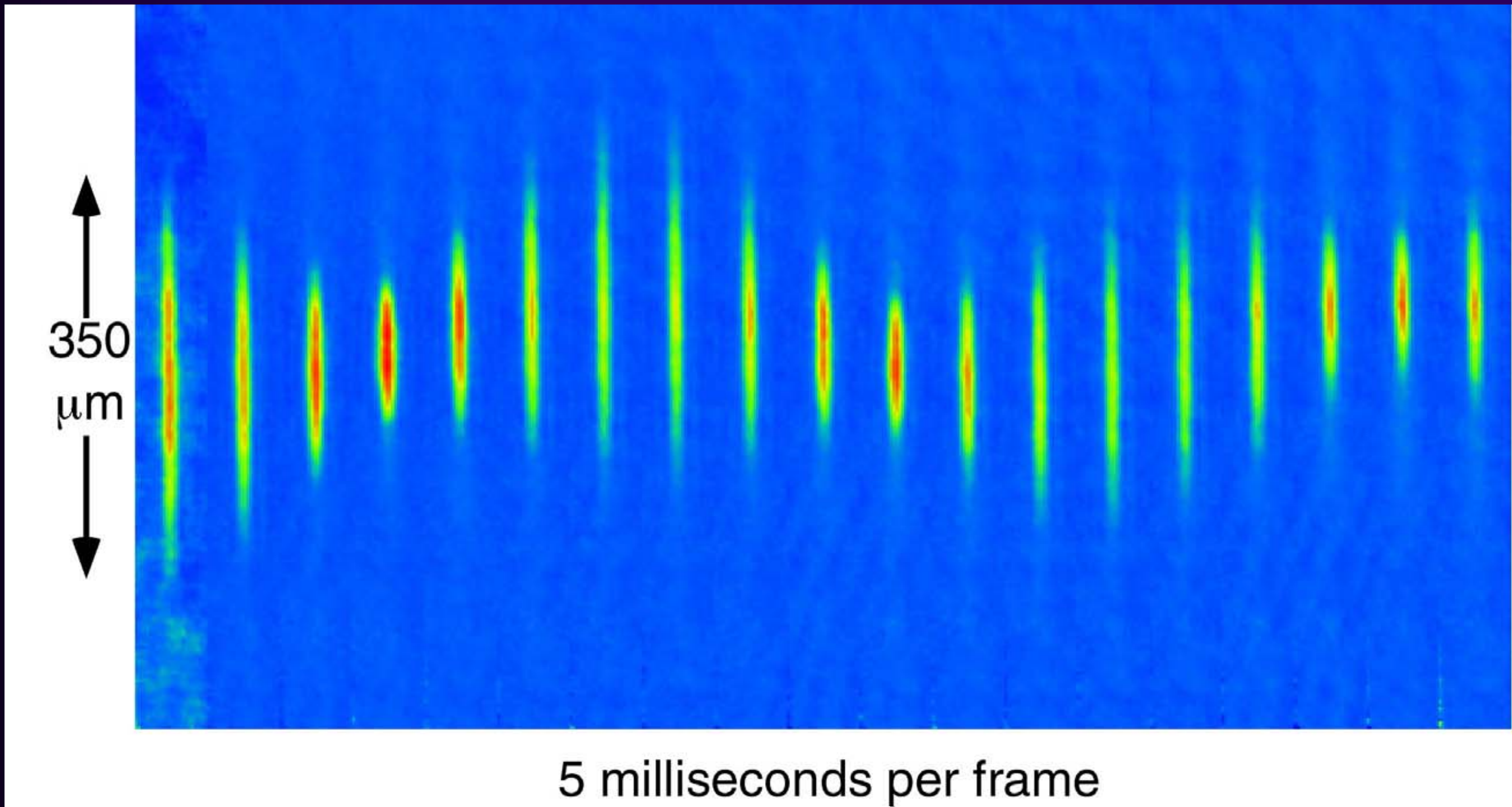
*happy Bose condensate*

# Collective modes (I)




*Dipole oscillation*

# Collective modes (II)




*Breathing mode*

# Spintronics with cold atoms

- Does it exist?
- What can we learn from it?
- What is it good for?

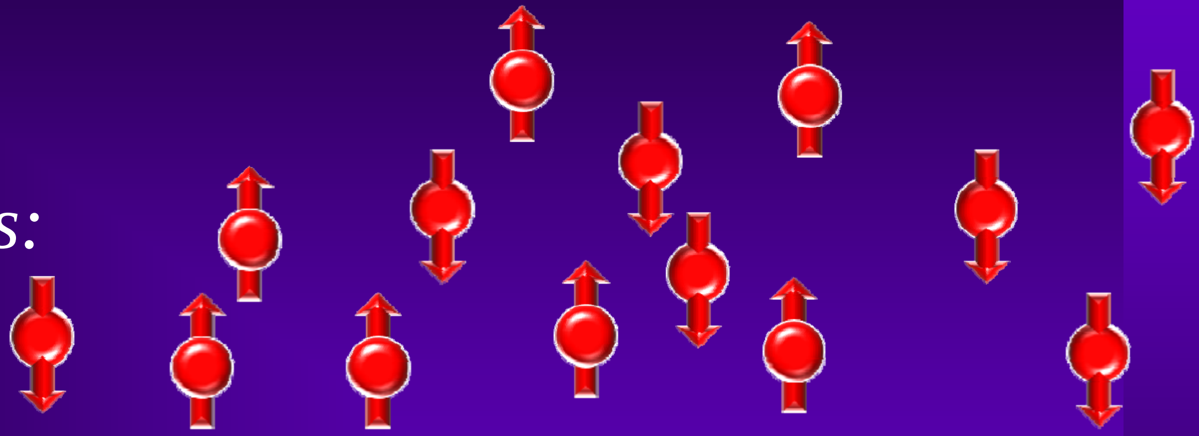
# Homogeneous cold-atom system (I)

*Two spin states*

$n_\uparrow = n_\downarrow = n$

*Spin-dependent forces:*

$F_\uparrow \neq F_\downarrow$



*No condensate*

$$nm\frac{d\vec{v}_\uparrow}{dt} = -nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + nF_\uparrow$$

$$nm\frac{d\vec{v}_\downarrow}{dt} = +nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + nF_\downarrow$$

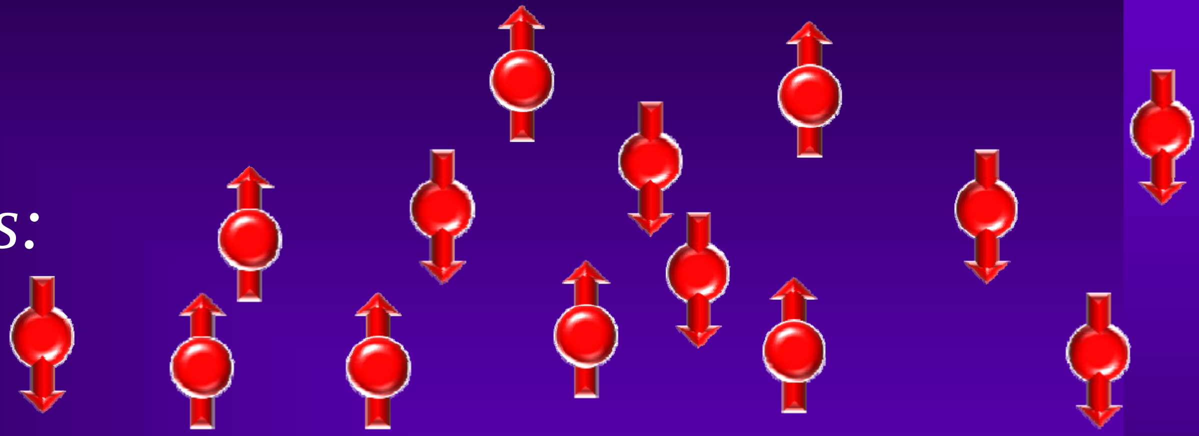
*No disorder/phonons; Only interactions that lead to spin drag*

# Homogeneous cold-atom system (II)

*Two spin states*
$n_\uparrow = n_\downarrow = n$

*Spin-dependent forces:*

$F_\uparrow \neq F_\downarrow$



*No condensate*

$$nm\frac{d\vec{v}_\uparrow}{dt} = -nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + n\vec{F}_\uparrow$$

$$nm\frac{d\vec{v}_\downarrow}{dt} = +nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + n\vec{F}_\downarrow$$

*Spin-resolved particle current*

$$\vec{j}_\alpha = n\vec{v}_\alpha$$

$$\begin{pmatrix} j_\uparrow \\ j_\downarrow \end{pmatrix} = \begin{pmatrix} \infty & \infty \\ \infty & \infty \end{pmatrix} \begin{pmatrix} F_\uparrow \\ F_\downarrow \end{pmatrix}$$

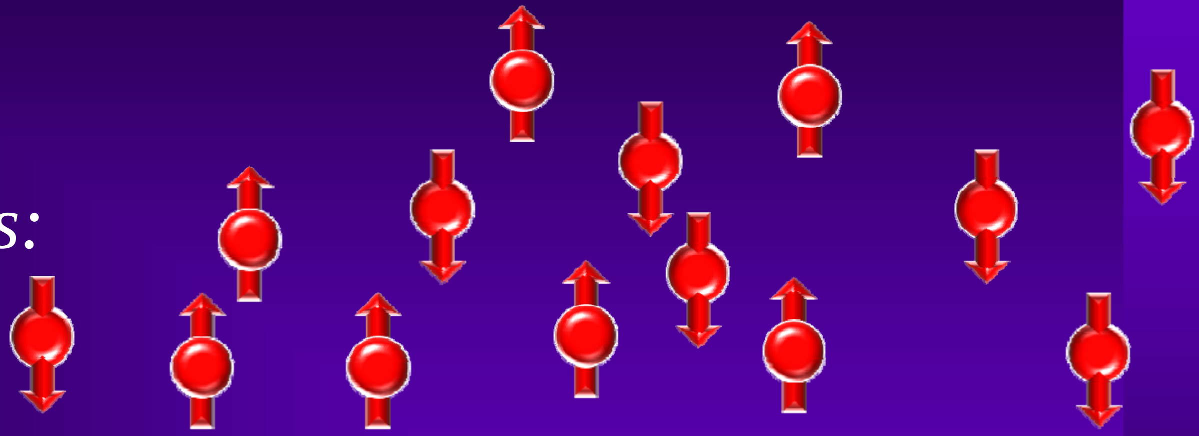
# Homogeneous cold-atom system (III)

*Two spin states*

$n_\uparrow = n_\downarrow = n$

*Spin-dependent forces:*

$F_\uparrow \neq F_\downarrow$



*No condensate*

$$\vec{j}_s = n\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)$$

$$\vec{j} = n\left(\vec{v}_\uparrow + \vec{v}_\downarrow\right)$$

$$nm\frac{d\vec{v}_\uparrow}{dt} = -nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + n\vec{F}_\uparrow$$

$$nm\frac{d\vec{v}_\downarrow}{dt} = +nm\frac{\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)}{\tau_{\uparrow\downarrow}} + n\vec{F}_\downarrow$$

$$\begin{pmatrix} j \\ j_s \end{pmatrix} = \begin{pmatrix} \infty & 0 \\ 0 & \sigma_s \end{pmatrix} \begin{pmatrix} F_\uparrow + F_\downarrow \\ \left(F_\uparrow - F_\downarrow\right)/2 \end{pmatrix}$$

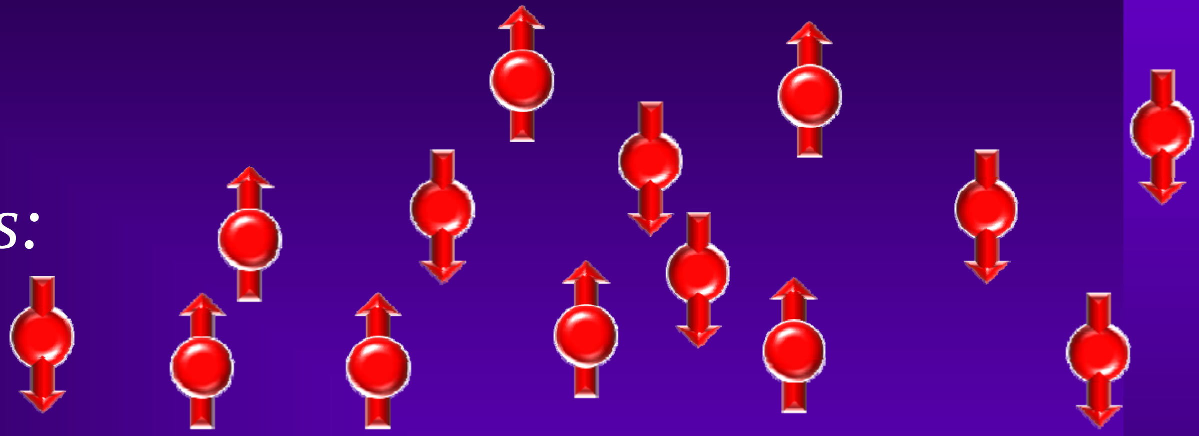
# Homogeneous cold-atom system (IV)

*Two spin states*

$n_\uparrow = n_\downarrow = n$

*Spin-dependent forces:*

$F_\uparrow \neq F_\downarrow$



*No condensate*

$$\vec{j}_s = n\left(\vec{v}_\uparrow - \vec{v}_\downarrow\right)$$

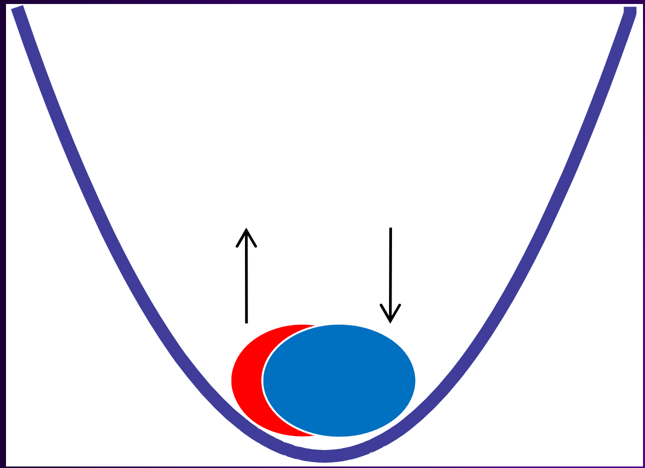
$$\vec{j} = n\left(\vec{v}_\uparrow + \vec{v}_\downarrow\right)$$

$$\boxed{\begin{pmatrix} j \\ j_s \end{pmatrix} = \begin{pmatrix} \infty & 0 \\ 0 & \sigma_s \end{pmatrix} \begin{pmatrix} F_\uparrow + F_\downarrow \\ \left(F_\uparrow - F_\downarrow\right)/2 \end{pmatrix}}$$

$$\sigma_s = \frac{n\tau_{\uparrow\downarrow}}{m}$$

*Spin conductivity determined by interactions (spin drag)*

# Trapped cold atom systems (I)




*Bulk transport: Size of clouds should be larger than mean free path*

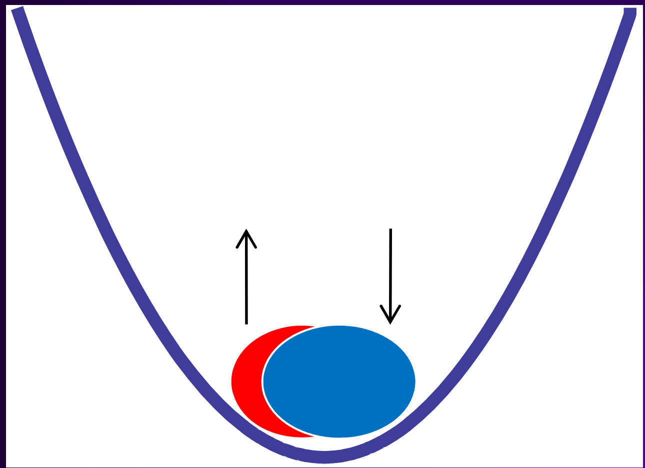
Size of clouds :$L$~$\mu m$-$mm$
relaxation time: $\tau_{\uparrow\downarrow} \sim$ ms

*Both collisionless and hydrodynamic regimes possible*

$$nm\frac{d^2 x_\uparrow}{dt^2} = -\frac{nm}{\tau_{\uparrow\downarrow}}\frac{d\left(x_\uparrow - x_\downarrow\right)}{dt} - nm\omega^2 x_\uparrow$$

$$nm\frac{d^2 x_\downarrow}{dt^2} = +\frac{nm}{\tau_{\uparrow\downarrow}}\frac{d\left(x_\uparrow - x_\downarrow\right)}{dt} - nm\omega^2 x_\downarrow$$

# Trapped cold atom systems (II)



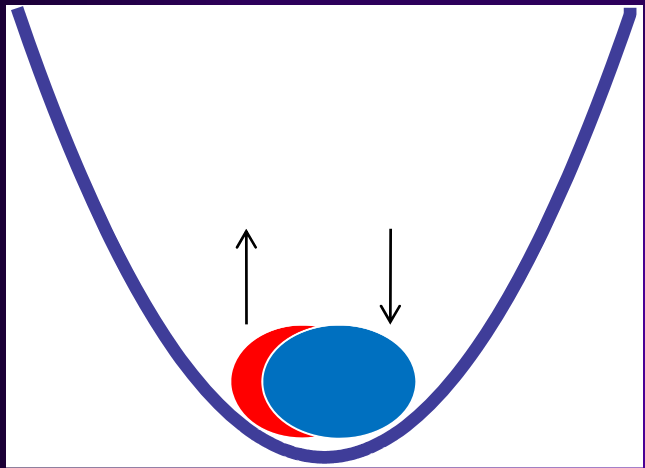

$$X = \left(x_\uparrow + x_\downarrow\right)/2$$

$$x = x_\uparrow - x_\downarrow$$

$$nm\frac{d^2 x_\uparrow}{dt^2} = -\frac{nm}{\tau_{\uparrow\downarrow}}\frac{d\left(x_\uparrow - x_\downarrow\right)}{dt} - nm\omega^2 x_\uparrow$$

$$nm\frac{d^2 x_\downarrow}{dt^2} = +\frac{nm}{\tau_{\uparrow\downarrow}}\frac{d\left(x_\uparrow - x_\downarrow\right)}{dt} - nm\omega^2 x_\downarrow$$

# Trapped cold atom systems (II)



$$nm\frac{d^2X}{dt^2} = -nm\omega^2 X$$

$$nm\frac{d^2x}{dt^2} = -\frac{2nm}{\tau_{\uparrow\downarrow}}\frac{dx}{dt} - nm\omega^2 x$$

$\omega_d = \omega$ *Dipole-mode undamped (Kohn's theorem)*

$$\omega_{sd} = \omega - \frac{i}{\tau_{\uparrow\downarrow}} \equiv \omega - \frac{ni}{m\sigma_s}$$

*Measure spin conductivity from damping spin-dipole mode*

*Talk by M. Zwierlein, Friday @ 14:40*

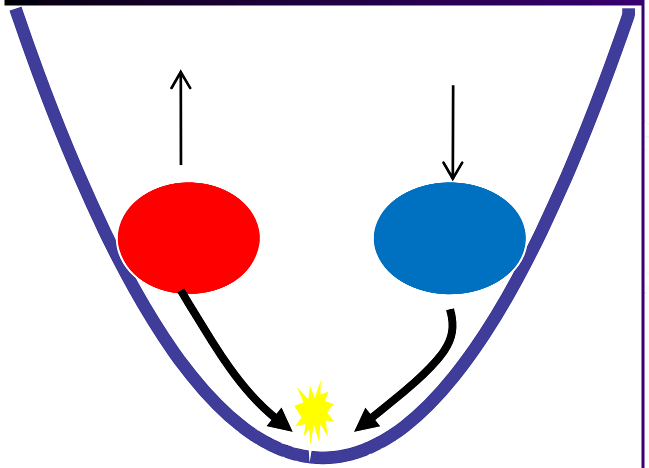
# Experiments on cold fermions

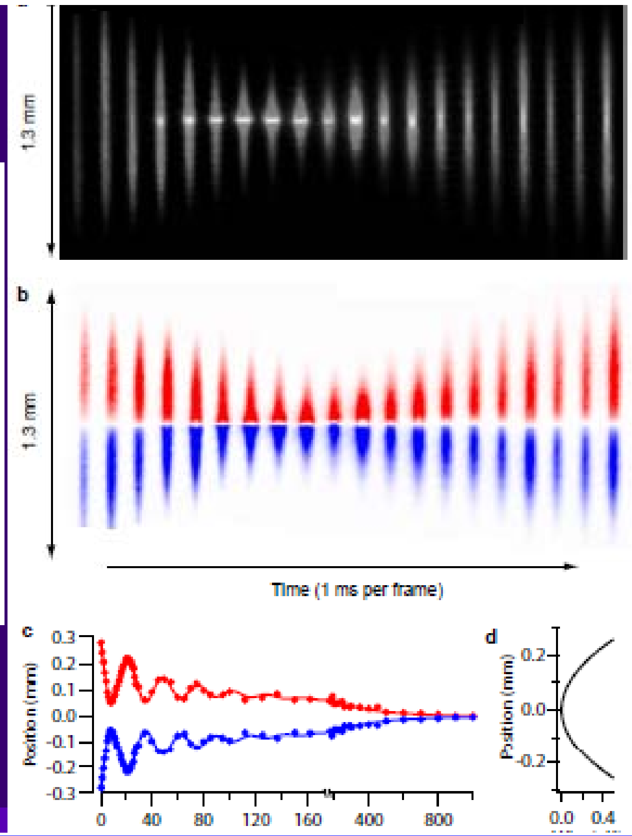
LETTER

doi:10.1038/nature09989

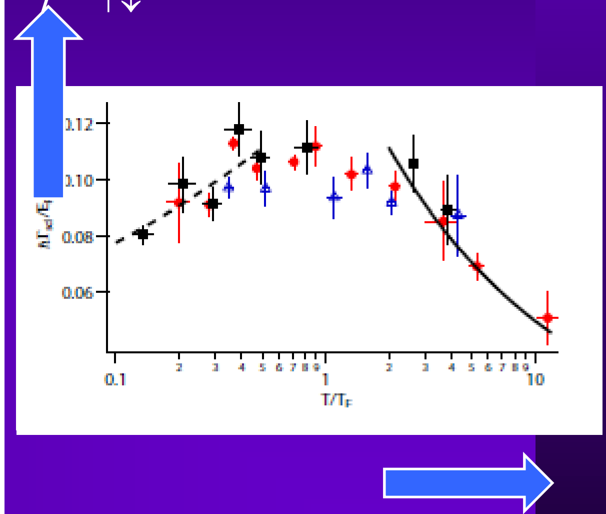
## Universal spin transport in a strongly interacting Fermi gas

Ariel Sommer[1,2,3], Mark Ku[1,2,3], Giacomo Roati[4,5] & Martin W. Zwierlein[1,2,3]






$1/\tau_{\uparrow\downarrow}$




*temperature*

*Experiments with bosons: P. van der Straten (Utrecht)*

# Recap (III): Lecture - outline

- Solid-state electronics and spintronics
  *electric current, resistivity/conductivity, electro-chemical potential, hydrodynamics, everything spin-resolved, spin accumulation*
- Cold atoms
  *spin currents, spin conductivity, determined from collective modes*
- Spintronics with cold-atom systems

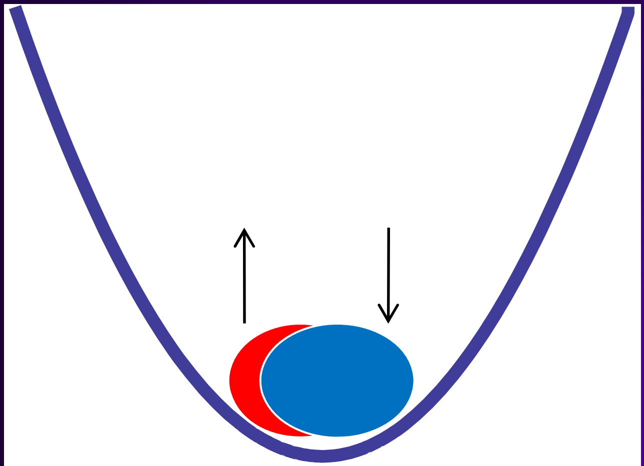
# Spintronics with cold atoms

- ☑ Does it exist?
- What can we learn from it?
- What is it good for?

# Recap (III): Lecture - outline

- Solid-state electronics and spintronics
  *electric current, resistivity/conductivity, electro-chemical potential, hydrodynamics, everything spin-resolved, spin accumulation*
- Cold atoms
  *spin currents, spin conductivity, determined from collective modes*
- Spintronics with cold-atom systems
  *spin conductivity: bosons vs. fermions*

# Spin transport properties cold gases (I)




$$\omega_{sd} = \omega - \frac{i}{\tau_{\uparrow\downarrow}} \equiv \omega - \frac{ni}{m\sigma_s}$$

*Measure spin conductivity from damping spin-dipole mode*

*Boltzmann equation leads to:*

$$\frac{1}{\tau_{\uparrow\downarrow}} \sim a^2 \sum_{\text{all momenta}} k\left\{\left[1\pm f_\uparrow(k)\right]\left[1\pm f_\downarrow(k_2)\right] f_\uparrow(k_3)\, f_\downarrow(k_4)\right.$$

$$\left. - f_\uparrow(k)\, f_\downarrow(k_2)\left[1\pm f_\uparrow(k_3)\right]\left[1\pm f_\downarrow(k_4)\right]\right\} + \text{momentum/energy conservation}$$

*Pauli blocking vs. Bose enhancement*

# Spin transport properties cold gases (II)

$\sigma_s^{-1} \sim 1/\tau_{\uparrow\downarrow} \sim T^2$ (fermions "blocking" in 3D) $10^{-9}$ $\Omega$m

$\sigma_s^{-1} \sim 1/\tau_{\uparrow\downarrow} \sim T^{-2}$ (bosons "lasing" in quasi-1D) $10^{-5}$ $\Omega$m (use charge=$e$)

Enhancement of spin resistivity
(reduction of spin conductivity)
upon approaching
critical temperature for Bose condensation!

# Spintronics with cold atoms

- ☑ Does it exist?
- ☑ What can we learn from it?
  *Fundamentals of spin transport, different regimes w.r.t. electrons in solids (interactions vs. disorder/phonons)*
- What is it good for?

# More examples/teasers:

PRL **107**, 195302 (2011) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2011

**Anomalous Hall Conductivity from the Dipole Mode of Spin-Orbit-Coupled Cold-Atom Systems**

E. van der Bijl and R. A. Duine
*Institute for Theoretical Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands*
(Received 11 July 2011; published 1 November 2011)

*Poster Erik van der Bijl*

*Posters Clement Wong, Martijn Mink,*
*Hedwig van Driel, Alice Bezett*

Spin Caloritronics in Noncondensed Bose Gases

C. H. Wong,* H.J.van Driel, R. Kittinaradorn, H.T.C. Stoof and R.A. Duine
*Institute for Theoretical Physics, Utrecht University,*
*Leuvenlaan 4, 3584 CE Utrecht, The Netherlands*

# Spintronics with cold atoms

- ☑ Does it exist?
- ☑ What can we learn from it?
  *Fundamentals of spin transport, different regimes w.r.t. electrons in solids (interactions vs. disorder/phonons)*
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# Spintronics with cold atoms

- ☑ Does it exist?

- ☑ What can we learn from it?
  *Fundamentals of spin transport, different regimes w.r.t. electrons in solids (interactions vs. disorder/phonons)*

- ☑ ✗ What is it good for?
  *Understanding magnon transport/Bose condensation magnons in magnetic insulators*

# Quasi-equilibrium magnons

- "Magnons with nonzero chemical potential:"

nature Vol 443|28 September 2006|**doi:10.1038/nature05117**

LETTERS

## Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

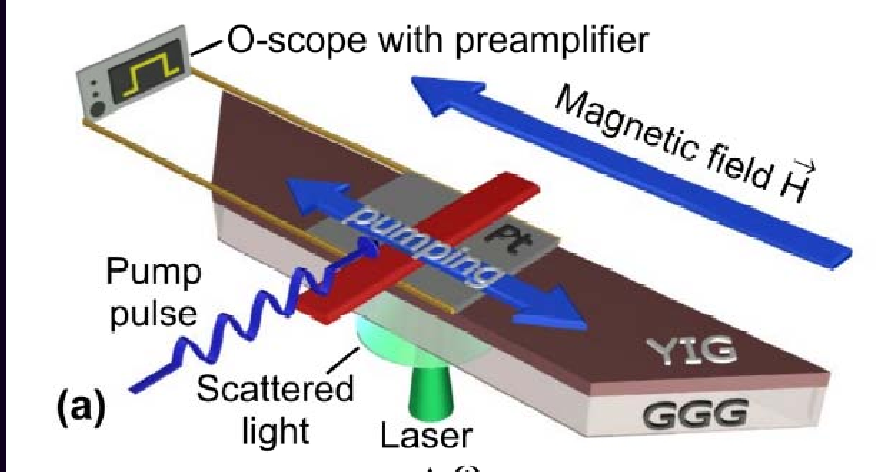
S. O. Demokritov[1], V. E. Demidov[1], O. Dzyapko[1], G. A. Melkov[2], A. A. Serga[3], B. Hillebrands[3] & A. N. Slavin[4]

# Magnon spin transport

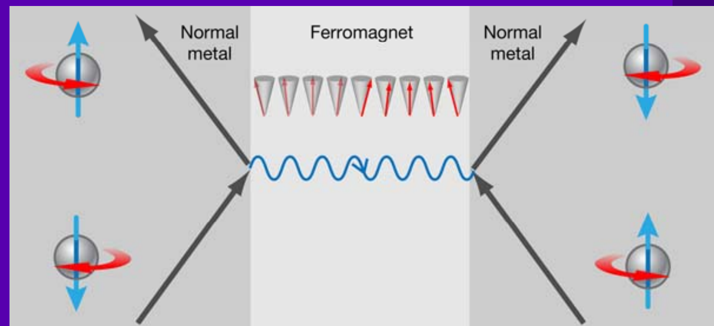
PRL **106,** 216601 (2011) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 27 MAY 2011

**Spin Pumping by Parametrically Excited Exchange Magnons**

C. W. Sandweg,[1,*] Y. Kajiwara,[2] A. V. Chumak,[1] A. A. Serga,[1] V. I. Vasyuchka,[1] M. B. Jungfleisch,[1]
E. Saitoh,[2,3,4] and B. Hillebrands[1]




*Tserkovnyak/Bauer*




*Spin transport accross normal metal –*
*magnetic insulator interface*

# One more teaser: Electrically driving magnon BEC:

Electronic Pumping of Quasiequilibrium Bose-Einstein Condensed Magnons

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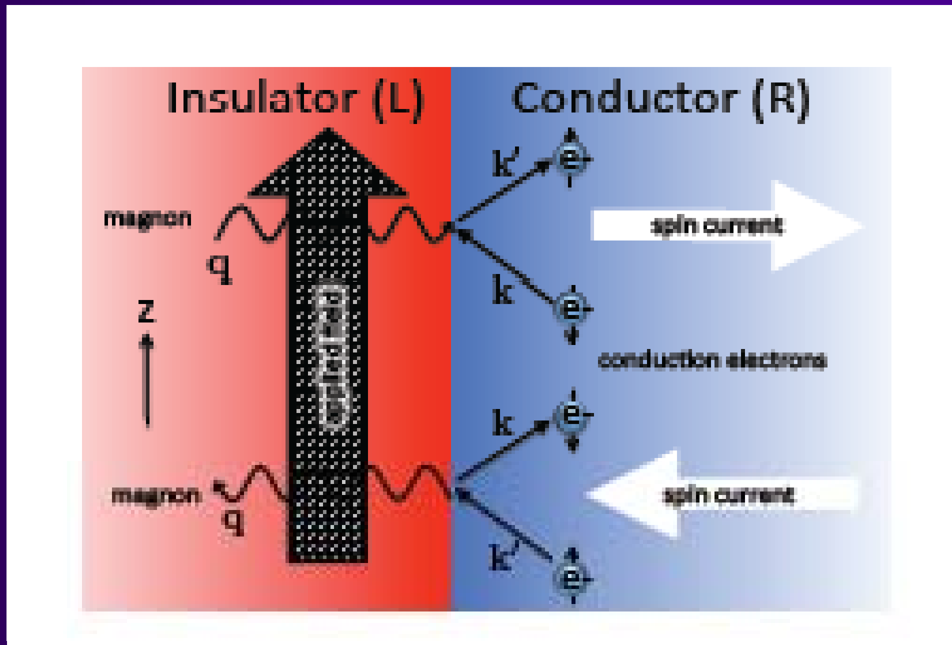
(Dated: November 11, 2011)




*Yaroslav Tserkovnyak/Scott Bender*

# Spintronics with cold atoms

- ☑ Does it exist?

- ☑ What can we learn from it?

  *Fundamentals of spin transport, different regimes w.r.t. electrons in solids (interactions vs. disorder/phonons)*

- ☑ ✗ What is it good for?

  *Understanding magnon transport/Bose condensation magnons in magnetic insulators*