Strongly Interacting Fermi Gases: Universal Thermodynamics, Spin Transport, and Dimensional Crossover

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Ultracold atomic Fermi Gases

Ideal test-bed for Many-Body physics

- Tunable, resonant interactions
- Fermi mixtures with controllable spin composition
- All relevant parameters precisely known

I. Realize idealized models of many-body physics
   → Benchmarking the many-body problem
   Need high precision to discriminate between theories
   - Equation of State of Unitary Fermi Gas
   - Evolution from 3D to 2D

II. “Beyond the Standard Model”
   Non-Equilibrium Physics, New States of Matter
   - Spin Transport
A Fermi gas collides with a cloud with resonant interactions.

From Magnetism to Superfluidity

Harmonic Trap

A. Sommer, M. Ku, G. Roati, MWZ, Nature 472, 201 (2011)
Little Fermi Collider

No Interactions, $a=0$
Evolution without interactions
Little Fermi Collider

Unitary Interactions, $1/a=0$
First collision
initial 20 ms

Time (1ms per frame)
Much later times

![Graph showing distance vs. time with a peak at 450 ms.](image)
Universal Spin Transport

Relaxation of spin current only due to $\downarrow \uparrow$ collisions

Resonant scattering cross section: $\sigma \sim \frac{1}{k_F^2}$

Mean free path: $l = \frac{1}{n\sigma} \sim \frac{1}{k_F} = \text{interparticle spacing}$

“A perfect liquid”

Spin drag coefficient ($\propto$ Collision rate)

$\Gamma_{SD} \sim n\sigma v \sim \frac{\hbar}{m} k_F^2 \sim E_F / \hbar$

Diffusion constant:

$D \sim \frac{(\text{mean free path})^2}{\text{collision time}} \sim \frac{\hbar}{m}$

$\frac{\hbar}{m} = \frac{(100 \ \mu m)^2}{1 \ s}$
Universal Spin Transport

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Mean free path: $l = \frac{1}{n\sigma} \sim \frac{1}{k_F} = \text{interparticle spacing}$

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Spin drag coefficient ($\propto$ Collision rate)

$$\Gamma_{SD} \sim n\sigma v \sim \frac{\hbar}{m} k_F^2 \sim \frac{E_F}{\hbar}$$

Diffusion constant:

$$D \sim \frac{(\text{mean free path})^2}{\text{collision time}} \sim \frac{\hbar}{m}$$

Spin conductivity:

$$\Sigma_s = \frac{n}{m\Gamma_{SD}} = \frac{1}{m\sigma v} \sim \frac{k_F}{\hbar}$$
Tunable spin conductivity

• Spin conductivity in cold Fermi gases is tunable over a wide range

\[ \Sigma_{\text{spin}} = \frac{n}{m} \frac{1}{\Gamma_{SD}} = \frac{1}{m \sigma v} \]

\[ \Sigma_{\text{spin}} \approx \frac{k_F}{\hbar} \frac{1 + ck_F^2 a^2}{a^2} \]

“Giant Magneto Spin Resistance”
Spin Diffusion vs Temperature

\[ j_s = n_{tot} \dot{d} = n_{tot} \Gamma_{sd} d = -D_s \left( \frac{dn_{\uparrow}}{dx} - \frac{dn_{\downarrow}}{dx} \right) \]

Quantum Limit of Spin Diffusion

Fit: $6.3(3) \frac{\hbar}{m} \left( \frac{T}{T_F} \right)^{3/2}$
Spin Diffusion for imbalanced mixtures

- highly imbalanced mixture: Fermi liquid

A. Sommer, M. Ku, MWZ, NJP 13, 055009 (2011)
Spin Diffusion in Presence of Superfluid

- Slight imbalance: Normal + Superfluid

Total density

Difference density

Intricate motion of normal unpaired atoms around(?) the superfluid core

Lot’s of Andreev reflections

A. Sommer, M. Ku, MWZ, NJP 13, 055009 (2011)
Thermodynamics

Equilibrium Thermodynamics as baseline for non-equilibrium studies

→ Establish Equation of State

Classical gas

Generally need

Or fixing chemical potential

Or replacing

\[ P = nk_B T \]

\[ P(n, T) \]

\[ P(\mu, T) \]

\[ n = \left. \frac{\partial P}{\partial \mu} \right|_{T, a, ...} \]

\[ n(\mu, T) \]
Spin $\frac{1}{2}$ - Fermi gas at a Feshbach resonance

- **Normal state:**
  - Is it a Fermi liquid?
  - Are there preformed pairs (pseudogap regime)?

- **Superfluid properties:**
  - Transition temperature
  - Critical Entropy
  - Energy of the superfluid
  - ...

- Thermodynamics of the Unitary Fermi Gas

  - High-$T$
  - Classical gas
  - Fermi Liquid
  - Preformed pairs?

  - Low-$T$
  - Superfluid

  $T_c$
Equation of State

Equation of state from density distribution in a trap

\[ V(\mathbf{r}) \]

\[ \mu_0 \]

Local chemical potential

\[ \mu(\mathbf{r}) = \mu_0 - V(\mathbf{r}) \]

Local density

\[ n(V) = n(\mu_0 - V, T) \]

The density profile provides a scan through the equation of state.
1. Cylindrical Symmetry: Inv. Abel
   \[ \rightarrow \text{Get } n(\rho, z) \]
2. Equidensity lines = Equipotential lines
   \[ \rightarrow \text{Get } V(\rho, z) \text{ as we know } V(0, z) \]

Experimental \( n(V) \) from single profile
How to get $\mu$ and $T$?
How to get $\mu$ and $T$?

**Solutions:**

- Fit wings to high-temperature theory (Virial expansion, Hartree-Fock, etc...)

$$n \lambda^3 = 2(e^{\beta \mu} + 2b_2 e^{2\beta \mu} + 3b_3 e^{3\beta \mu} +...)$$

$$b_2 = \frac{3\sqrt{2}}{8}, \quad b_3 = -0.29095295$$

That’s what we all do.

3D Bose gas (see recent Cambridge expt.)

2D Bose Gas: Chicago, ENS, JILA

3D Unitary Fermi Gas: ENS (T fixed via $^7$Li thermometer)
No-fit equation of state

We want: \( n(\mu, T) \)

We know \( \mu = \mu_0 - V \)

Thus \( \frac{d\mu}{dV} = -1 \)

**Local compressibility**

\[
\kappa = \frac{1}{n^2} \frac{dn}{d\mu} = -\frac{1}{n^2} \frac{dn}{dV}
\]

**Compressibility Equation of State**

\( \kappa(n, P) \)

No fit at all!

Mark J. H. Ku, Ariel T. Sommer, Lawrence W. Cheuk, Martin W. Zwierlein

No-fit Equation of State

Normalize compressibility & pressure via known density:

\[
\tilde{\kappa} = \frac{K}{\kappa_0} = \frac{d\varepsilon_F}{d\mu} = -\frac{d\varepsilon_F}{dV}
\]

\[
\tilde{P} = \frac{P}{P_0}
\]

\[
\kappa_0 = \frac{3}{2} \frac{1}{n\varepsilon_F}
\]

\[
P_0 = \frac{2}{5} n\varepsilon_F
\]

For a scale-invariant system

\[
\tilde{\kappa} = \tilde{\kappa}(\tilde{P})
\]
At $T=0$:  
\[ \mu = \xi \varepsilon_F \]
\[ \kappa / \kappa_0 = 1 / \xi \]
\[ P / P_0 = \xi \]
Compressibility Equation of State

\[ \kappa(n, P) \longrightarrow \kappa(n, T) \]

Getting the temperature scale:

\[
\frac{d\left(\frac{P}{P_0}\right)}{d\left(\frac{T}{T_F}\right)} = \frac{5}{2} \frac{T_F}{T} \left(\frac{P}{P_0} - \frac{\kappa_0}{\kappa}\right)
\]

\[
\frac{T}{T_F} = \frac{T_i}{T_F} \exp \left\lbrace \frac{2}{5} \int_{\tilde{p}_i}^{\tilde{p}} d\tilde{p} \frac{1}{\tilde{p} - \frac{1}{\tilde{\kappa}}} \right\rbrace
\]
Compressibility Equation of State

Compressibility

Sudden rise and fall
Heat capacity

At unitarity:

\[ P = \frac{2}{3} \frac{E}{V} \]

\[ C_V = \frac{d\left(\frac{E}{Nk_B}\right)}{dT} \bigg|_{N,V} = \frac{d\left(\frac{P}{nE_F}\right)}{d\left(\frac{T}{T_F}\right)} = \frac{3}{2} \frac{T_F}{T} \left( \frac{P}{P_0} - \frac{\kappa_0}{\kappa} \right) \]
Heat capacity

At unitarity:

\[
P = \frac{2E}{3V}
\]

\[
C_V = \frac{d(E/Nk_B)}{dT}
\]

\[
= \frac{d(P/nE_F)}{d(T/T_F)} = \frac{3}{2} \frac{T_F}{T} \left( \frac{P}{P_0} - \frac{\kappa_0}{\kappa} \right)
\]

Unitary Fermi Gas
At unitarity:

\[ P = \frac{2}{3} \frac{E}{V} \]

\[
\frac{C_V}{N k_B} = \frac{d\left(\frac{E}{N k_B}\right)}{dT} \bigg|_{N,V} = \frac{d\left(\frac{P}{n E_F}\right)}{d\left(\frac{T}{T_F}\right)} = \frac{3}{2} \frac{T_F}{T} \left( \frac{P}{P_0} - \frac{\kappa_0}{\kappa} \right)
\]

A lambda-like feature in the specific heat
Direct observation of the superfluid transition at $T_C/T_F = 0.167(13)$
Going back to Density Equation of State

\[ \tilde{\kappa} = \frac{d\varepsilon_F}{d\mu} = -\frac{T_F^2}{T^2} \frac{d(T/T_F)}{d(\beta\mu)} \]

\[ \beta\mu = \beta\mu_i - \int_{T_i/T_F}^{T/T_F} d(T/T_F) \frac{T_F^2}{T^2} \frac{1}{\tilde{\kappa}} \]

with \( \beta\mu_i \) and \( T_i \) known at high temperatures
New value for $\xi$: $\xi = 0.376(5)$[8]
Entropy vs Temperature

On resonance:

\[ S = \frac{1}{T} (E + PV - \mu N) \]

\[ \frac{S}{Nk_B} = \frac{5}{2} \frac{P}{nk_BT} - \mu \beta \]

Critical Entropy

\[ S_c = 0.73(13) Nk_B \]
Density and Pressure

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Strongly Interacting Fermi Gases in coupled quasi-2D layers

- Access physics of layered superconductors
- Evolution of Fermion Pairing from 3D to 2D
- Berezinskii-Kosterlitz-Thouless transition in deep 2D limit

Making quasi-2D Fermi gases

- Confine tightly in 1 direction
- 1D lattice (retro-reflected 1064nm)
- Lattice depths up to $30 \, E_R$
- 2D-ness tuned by lattice depth
- Deep lattice: $\frac{\varepsilon F}{\hbar \omega} \sim 0.1$, aspect ratio of $\sim 1:1000$
Binding Energy from 3D to 2D (1D lattice)

BCS

$V_0 = 0 \, E_R$

BEC

Orso & Wouters. PRL 95 060402 (2005)
Binding Energy from 3D to 2D
(1D lattice)

$V_0 = 5 \, E_R$

Orso & Wouters. PRL 95 060402 (2005)
Binding Energy from 3D to 2D (1D lattice)

\[ V_0 = 10 E_R \]

Orso & Wouters. PRL 95 060402 (2005)
Binding Energy from 3D to 2D
(1D lattice)

Orso & Wouters. PRL 95 060402 (2005)
Evolution of Fermion Pairing from 3D to 2D

Density of States: 3D \( \sim \sqrt{\varepsilon} \)  
2D \( \text{const.} \)

Direct consequence: Always a bound state in 2D

RF spectrum:

RF Spectrum: \( I(\omega) \sim \rho(\varepsilon_k) |\psi(k)|^2 |\varepsilon_k = \varepsilon(\omega) \)

3D: \( I(\omega) \sim \sqrt{\omega - \omega_{th}} / \omega^2 \)

2D: \( I(\omega) \sim \theta(\omega - \omega_{th}) / \omega^2 \)

Interactions in final state:

\[
\frac{\ln^2 \left( \frac{E_b^f}{E_b} \right)}{\ln^2 \left( \frac{\omega - E_b}{E_b^f} \right) + \pi^2}
\]
Evolution of Fermion Pairing from 3D to 2D

3D

2D

Atom transfer [a.u.]

RF Offset [kHz]

$V_0 = 2 \, E_R$

$V_0 = 5 \, E_R$

$V_0 = 6 \, E_R$

$V_0 = 10 \, E_R$

$V_0 = 12 \, E_R$

$V_0 = 19 \, E_R$

$V_0 = 20 \, E_R$
Evolution of Fermion Pairing from 3D to 2D

3D

2D

Resonance
d/a = 0

3D BCS
d/a = -1.2

Evolution of Fermion Pairing from 3D to 2D

Atom transfer [a.u.]

RF Offset [kHz]
Logarithmic Corrections in Spectra in 2D

2D nature of interactions strongly influence “naïve” spectra

\[ I_{Pair}(\omega) \sim \frac{\theta(\omega - \omega_{th})}{\omega^2} \]

\[ \frac{\ln^2 \left( \frac{E_b^f / E_b}{E_b^f} \right)}{\ln^2 \left( \frac{(\omega - E_b) / E_b^f}{E_b^f} \right) + \pi^2} \]

MIT RF spectrum at 690 G, 18 E_R

Fit \( \theta(\omega - \omega_{th})/\omega^2 \)

Fit including log-corrections
Logarithmic Corrections in Spectra in 2D

2D nature of interactions strongly influence “naïve” spectra

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\[ \frac{\ln^2 \left( \frac{E_b^f}{E_b} \right)}{\ln^2 \left( \frac{\omega - E_b}{E_b^f} \right) + \pi^2} \]
Evolution of Fermion Pairing from 3D to 2D

On Resonance (in harmonic trap):

\[ E_{B,\text{th}} = 0.244\hbar\omega_z \]

Resonance
\[ d/a = 0 \]
3D BCS
\[ d/a = -1.2 \]
3D BCS
\[ d/a = -2.7 \]

Deep 2D regime
Comparison with mean-field BEC-BCS in 2D

Prediction: Randeria et al. (1989)
Many-body bound state energy
= 2-body bound state energy

Deep 2D regime
Comparison with mean-field BEC-BCS in 2D

Prediction: Randeria et al. (1989)
Many-body bound state energy
= 2-body bound state energy

Only small deviation observed between many-body and 2-body bound-state energy

Revealing the Superfluid Lambda Transition

- Compressibility, Density, Pressure → No-Fit EoS
- Specific Heat, Chemical potential, Entropy, Energy etc.
- Observe transition at $T_c = 0.167(13)$ $T_F$

Fermi gases in 2D

- Are those pairs superfluid?
- Study coherence, thermodynamics, rotation…
BEC I
*BEC-BCS Crossover*
2D Fermi Gases
Ariel Sommer
Mark Ku
Lawrence Cheuk
Dr. Waseem Bakr
Dr. Tarik Yefsah

Fermi I
LiNaK
Fermi-Fermi mixtures
Cheng-Hsun Wu
Ibon Santiago
Jee Woo Park
Dr. Peyman Ahmadi
Dr. Sebastian Will

Fermi II
Fermi Gas Microscope
Thomas Gersdorf
Takuma Inoue
Melih Okan
Dr. Waseem Bakr