Theory of Spin-Orbit-Coupled Cold Atomic Systems

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Outline

- Synthetic spin-orbit coupling schemes, old and new and many more
- A zoo of spin-orbit-coupled BECs: (mostly) theory
- How to create vortices in spin-orbit-coupled BECs?
- Topological optical lattices & lattice quantum Hall states
- Practical applications: Quantum gravimetry and interferometry
Spin-Orbit-Coupled Systems

Spin-orbit-coupled 2D electron gas: \( \hat{\mathcal{H}} = \frac{p^2}{2m} + \mathbf{h}_p \cdot \mathbf{\hat{\sigma}} \)

- Rashba: \( \mathbf{h}_p^R = \alpha \mathbf{p} \times \mathbf{\hat{z}} \); Dresselhaus: \( \mathbf{h}_p^{D_1} = -\beta_1 (p_x, -p_y) \)
Synthetic Spin-Orbit Couplings
in Cold-Atom Systems
Atom in a $r$-dependent laser field. Tripod scheme

Hamiltonian: $\hat{H} = \hat{H}_{\text{kin}} + \hat{V}_{\text{trap}} + \hat{H}_{a-1}$

Atom-laser interaction:

$$\hat{H}_{a-1} = -[\Omega_1(r)|0><1| + \Omega_2(r)|0><2| + \Omega_3(r)|0><3|] + \text{h. c.}$$

Two equivalent descriptions

To get the effective Hamiltonian...
1. Diagonalize $\hat{H}_{a-l}$ via a unitary rotation, $\hat{H}_{a-l} \rightarrow \hat{U}(r)\hat{H}_{a-l}\hat{U}(r)$
2. Project the result onto the dark subspace

$$\hat{H}_{\text{eff}} = \hat{P}_{\text{dark}}\hat{U}(r) \left( -\frac{\hbar^2 \nabla^2}{2m} \right) \hat{U}(r)\hat{P}_{\text{dark}}$$

- **Picture I**: Particle in a non-Abelian “gauge field”

$$\hat{H}_{\text{eff}} = \frac{1}{2m} \left[ -i\hbar \nabla - A^i(r)\hat{\sigma}_i \right]^2$$

- **Picture II**: Spin-orbit coupled particle

$$\hat{H}_{\text{eff}}(p) = \frac{p^2}{2m} + b(p) \cdot \hat{\sigma}$$

For the tripod scheme in 2D: $b(p) = (0, vp_x, v'p_y)$
Loop scheme for Rashba SOC

\[ \hat{H}_{SO} = \frac{p^2}{2m} + \nu (p_x \hat{\sigma}_x + p_y \hat{\sigma}_y) \]

D. Campbell, G. Juzeliunas, & I. Spielman, PRA 84, 025602 (2011)
Chose lasers, so that "momentum flux" through every loop vanishes.

\[ \hat{\mathcal{H}}_{\text{SO}} = \frac{p^2}{2m} + \nu (p_x \hat{\sigma}_x + p_y \hat{\sigma}_y + p_z \hat{\sigma}_z) \]

B. Anderson, G. Juzeliunas, I. Spielman, & V. Galitski, tbp
A different choice of phases in the tetragonal scheme leads to a Hamiltonian that can NOT be spanned by Pauli matrices alone, but may be spanned by $3 \times 3$ Gell-Mann matrices

$$[\hat{\lambda}_i, \hat{\lambda}_j] = i f_{ij}^k \hat{\lambda}_k$$

Gell-mann matrices are generators of $su(3)$.

$su(3)$ spin-orbit coupled system:

$$\mathcal{H}_{su(3)} = \frac{p^2}{2m} + \sum_{i=1}^{8} b_i(p)\hat{\lambda}_i$$

No analogue in condensed matter (or any other matter)...

G. Boyd, B. Anderson, & V. Galitski, tbp
Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential


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Gaithersburg, Maryland, 20899, USA
(Received 17 September 2008; published 30 March 2009)

- “Only” “Abelian” SOC (∼ persistent spin helix type) has been realized so far
  \[ \hat{H} = \frac{p^2}{2m} + v p_x \hat{\sigma}_z + \Omega \hat{\sigma}_x + \delta \hat{\sigma}_z \]

- The loop SOC (∼ Rashba) may probably be realized soon
- The tetragonal SOC scheme (∼ Weyl or \( su(3) \)) is realistic
Many-Body Physics of Spin-Orbit-Coupled Cold Atoms

Spin-Orbit-Coupled Bose-Einstein Condensates
Spin-Orbit-Coupled Fermions and Bosons

- Spin-orbit-coupled fermions form two Fermi surfaces.
- Spin-1/2 (!) spin-orbit-coupled bosons condense.

New type of Bose-Einstein condensate

- Time-of-flight for the usual BEC

\[ |\Psi\rangle = e^{i\phi_0} |\text{Zero-momentum state}\rangle \]

- ToF for a spin-orbit coupled BEC

\[ |\Psi\rangle = \text{Linear combination of } |\text{Left}\rangle \text{ and } |\text{Right}\rangle \]
Bose-Einstein Condensate in a Uniform Light-Induced Vector Potential


Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA
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(a) Experimental layout

(b) Level diagram
Non-interacting spin-orbit-coupled BEC

If there are no interactions and we do not require momentum to be a good quantum number, then the degeneracy of the many-body ground state is enormous – \((N + 1)\)-fold.

\[ \left| \Psi_N \right\rangle = c_1 + c_2 + c_3 + c_4 + \ldots \]

General many-body wave-function

\[ \left| \Psi_N \right\rangle = \sum_{n=0}^{N} \frac{c_n}{\sqrt{n!(N-n)!}} \left( \bar{B}_L^\dagger \right)^n \left( \bar{B}_R^\dagger \right)^{N-n} \left| \text{vac} \right\rangle, \]

\[ \sum_n |c_n|^2 = 1 \]

Interactions lift the huge degeneracy at the many-body level and reduce it to a two-fold degenerate state...
“Order-by-disorder:” Selecting a ground state

• Density-density interaction model (in terms of the original bosons)

\[ \hat{H}_{\text{int}} = \frac{1}{2V} \sum_{p,p',q} V_{\text{int}}(q) \tilde{b}_{\alpha p}^\dagger \tilde{b}_{\alpha p} + q \tilde{b}_{\beta p'}^\dagger \tilde{b}_{\beta p'-q} \]

• Interaction term for pseudo-spin bosons

\[ \hat{H}_{\text{int}} = \frac{1}{2V} \sum_{p,p',q} \sum_{\{\sigma_i\}} V_{\text{int}}(q) \tilde{B}_{\sigma_1 p}^\dagger \tilde{B}_{\sigma_2 p+q} \tilde{B}_{\sigma_3 p'}^\dagger \tilde{B}_{\sigma_4 p'-q} \]
\[ \times U_{\sigma_1 \alpha}(p) U_{\alpha \sigma_2}(p+q) U_{\sigma_3 \alpha'}(p') U_{\alpha' \sigma_4}(p-q), \]

Matrices \( \tilde{U} \) represent momentum-space rotations (Berry’s phases).

• Bogoliubov theory (\( \tilde{\beta}'s \) below are Goldstone modes)

\[ \hat{H} = \sum_{n=0}^{N} \tilde{P}_{N_L,N_R} \left[ \epsilon_0(N_L,N_R) + \sum_{q,\sigma} \Omega_{\sigma}(n,q) \tilde{\beta}_{\sigma,q}^\dagger \tilde{\beta}_{\sigma,q} \right] \tilde{P}_{N_L,N_R} \]

\( \tilde{P}_{N_L,N_R} \) projects on the subspace with \( N_L \) left- and \( N_R \) right-movers. The ground state energy is subspace-dependent!
**N00N state**

Energy of the ground state as a function of the density of the left-movers, $N_L/N$.

![Graph showing the energy as a function of density.

Energy is minimized if all particles are moving to the left or to the right. The ground state

$$|\psi_N\rangle = \frac{1}{\sqrt{N!}} \left[ \sqrt{w_L} e^{i\phi_L} + \sqrt{w_R} e^{i\phi_R} \right]$$
Measuring a Spin-Orbit-Coupled “Qubit”

- A way to measure the cat-state ($N00N$-state) of a trapped BEC

$$\left| \Psi_N \right> = \frac{1}{\sqrt{N!}} \left[ \sqrt{w_L} e^{i\phi_L} |N \ 0\rangle + \sqrt{w_R} e^{i\phi_R} |0 \ N\rangle \right]$$

is via time-of-flight expansion.

- Velocities at the minima vanish, $\langle \frac{\partial \mathcal{H}}{\partial p})_{\pm p_0} = 0$. To observe the condensate(s), we need to turn off both the trap and SO-couplings.

- The result of the measurement is intrinsically impossible to predict with certainty. There are two possibilities:

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Left-moving state

Right-moving state

Time
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Topological Bose-Einstein Condensates

- Energetics analysis selects a “double-degenerate” BEC:

$$\| \Psi_N \rangle = \frac{1}{\sqrt{N!}} \left[ \sqrt{w_L} e^{i\phi_L} + \sqrt{w_R} e^{i\phi_R} \right]$$

- The Heisenberg uncertainty principle, $\delta x \delta p \gtrsim \hbar$, provides intuition: Bosons repel each other in $r$-space ("attract" each other in $p$-space).

- More exotic states appear for infinite-degenerate (Rashba) BEC:

   (b) Spontaneous symmetry breaking ("Higgs" physics)
   (c) Topologically distinct states
Vortices in Spin-Orbit

Bose-Einstein Condensates

PHYSICAL REVIEW A 84, 063604 (2011)

Vortices in spin-orbit-coupled Bose-Einstein condensates

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(Received 24 August 2011; published 5 December 2011)
How to create vortices in a BEC?

- Rotation: If an ordinary BEC is stirred by a laser “spoon” or the anisotropic trap is rotated:
  - The Hamiltonian in the rotating frame is time-independent:
    \[ H_{RF} = H_0 - \Omega \cdot L \]
  - Equilibrium stat. mechanics apply
  - Vortex lattice appears

- Synthetic magnetic field for neutral atoms leads to an effective magnetic field for dressed states

Rotating the trap generally does NOT work for spin-orbit BECs, as there is no rotating frame where the Hamiltonian is time-independent (non-equilibrium physics, heating). However, to combine synthetic spin-orbit coupling with synthetic magnetic field should work.
Spin-orbit coupling + spatially-dependent detuning

- Introducing spatially-dependent detuning $\delta(r)$ in I. Spielman’s existing scheme creates an effective gauge field

$$H_{\text{eff}} = \frac{p^2}{2m} + vp_x \hat{\sigma}_z + \Omega \hat{\sigma}_x + \delta(y) \hat{\sigma}_z$$

- The combination of the SOC and synthetic gauge field yields two main effects:
  1. Spatial separation of the left- and right-movers
  2. Synthetic mag. field for each component and vortex nucleation
Due to symmetry of the effective gauge field with respect to reflection about the $y = 0$ axis and almost spin-independent interactions, an interesting parity effect is observed (in GPE simulations): the number of vortices is the same in both components.
Topological Optical Lattices

PHYSICAL REVIEW A 79, 053639 (2009)

Topological insulators and metals in atomic optical lattices
Tudor D. Stanescu,1 Victor Galitski,1 J. Y. Vaishnav,2 Charles W. Clark,2 and S. Das Sarma1

PHYSICAL REVIEW A 82, 013608 (2010)

Topological states in two-dimensional optical lattices
Tudor D. Stanescu,1,2 Victor Galitski,1 and S. Das Sarma1
- Kinetic energy of a moving particle: \( E = \frac{mv^2}{2} = \frac{p^2}{2m} \).
  
  Free particle plane-wave wave-function, \( \psi_p \propto e^{i\mathbf{p} \cdot \mathbf{r}} \).

- Now consider electrons moving in a crystal lattice.
  1. Band structure changes. I.e., \( E \neq \frac{p^2}{(2m)} \) in a solid
  2. Discrete translational symmetry demands that \( \psi(x) = \psi(x + na) \)
     and lattice momentum is defined modulo \( 2\pi \hbar / a \). Topologically momentum space is a torus in 2D with complex wave-functions \( \psi_p \)
     associated with each point. Complicated “topological” stuff!

Quantum wave-functions on a torus

Classical world, where we measure things (resistance, etc.)

This correspondence can be classified by integer topological indices (Chern numbers associated with bands).
Topological Optical Lattices

- $T$-invariant topological insulators are akin having two replicas of QHE. They are of interest in solids, because no $B$-field is needed. It is not relevant in AMO: Lattice QHE is easier to realize.

- Canonical Haldane model of a top. insulator with broken $T$-reversal:

\[ \hat{\mathcal{H}} = \frac{1}{2m} \left[ \hat{p} - A_{\text{synt}}(r) \right]^2 + V_0 \sum_{i=1}^{3} \cos^2 (k_i \cdot r) + U_{\text{trap}}(r) \]
Topological Spectrum and Wave-Function: an Example

Exact single-particle spectrum and density of states

Geometry: A finite-size disk sample
Topological Spectrum and Wave-Function: an Example

Exact single-particle spectrum and density of states

Exact \(|\text{wave-function}|\) profile

1. Exact single-particle spectrum and density of states

2. Orbital momentum

DOS ($\alpha = 2$)

A

B

C

A

B

C
Viewpoint

Quantum liquids move to a higher dimension

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Selected for a Viewpoint in Physics

PRL 107, 077201 (2011)

Kaleidoscope of Exotic Quantum Phases in a Frustrated XY Model

Christopher N. Varney, Kai Sun, Victor Galitski, and Marcos Rigol

FIG. 1 (color online). (a) Phase diagram of the model in Eq. (1) as a function of $J_2/J_1$, (b) antiferromagnetic ordering
Practical Applications of the Synthetic Gauge Fields for Precision Quantum Interferometry
Existing quantum devices: Atomic interferometers

- Key idea: To take advantage of the wave-like nature of particles

- Separating and recombining particle beams leads to interference

- Dynamics of QM phases depends on external fields (gravity, acceleration, magnetic field, etc.). So, we can measure them!
Measurement of gravitational acceleration by dropping atoms

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Laser-cooling of atoms and atom-trapping are finding increasing application in many areas of science. One important use of laser-cooled atoms is in atom interferometers. In these devices, an atom is placed into a superposition of two or more spatially separated atomic states; these states are each described by a quantum-mechanical phase term, which will interfere with one another if they are brought back together at a later time. Atom

From Steven Chu group web-site at Stanford:

http://www.stanford.edu/group/chugroup/

Monitoring of local gravity using $T = 400$ ms fringes

Offset measured $g$ (µGal = $10^{-3}$ m s$^{-2}$)

- Experimental data from $T = 400$ ms fringes
- Solid earth tide and ocean loading (model II)
Existing gravimeters do not probe time-dependent gravity/acceleration, \( \dot{\mathbf{a}}(t) \). But it is possible with synthetic gauge fields [B. Anderson, J. Taylor, & VG, Phys. Rev. A 83, 031602(R) (2011)].

Atoms with \( e \pm 1 \) in a field:

\[
H = \frac{(p - \sigma m \omega c e z \times r)^2}{2m} + \frac{1}{2} m \omega^2 r^2 - m \delta g(t) \cdot r
\]

Trajectories for “spin-up” and “spin-down” differ depending on time-dependent gravity!

Phase-difference: \( \langle \hat{S}_z \rangle \propto \sin \left( 2 \int_0^T d\mathbf{r}(t) \cdot \mathbf{g}(t) \right) \)
Summary of the recent progress

- Dressed states with cold atoms potentially host a much richer variety of spin-orbit structures than that available in solids (Rashba, Dresselhaus, “Weyl,” su(3)-SOC, ...)
- Synthetic spin-structures may be practically useful for quantum interferometry
- Abelian spin-orbit BECs have already been observed. Theory predicts macroscopically-entangled states in non-Abelian SO-BECs (staying tuned for new experiments...)
- Synthetic SOC + synthetic magnetic field = new vortex structures
- SO coupling on a lattice may lead to lattice quantum Hall effect
- Future work: Non-equilibrium physics + synthetic fields. Cold atoms may be ideal candidates to realize Floquet topological insulators.